QCD HW IV

1. Let us focus briefly on the Drell-Yan process, the production of a virtual photon in hadron-hadron collisions via the annihilation of a quark and antiquark (here ignore the possibility of Z production). The short distance ("hard") process is the time reversed version of the electron-positron annihilation process. Thus $e^+e^- \to q\overline{q}$ becomes $q\overline{q} \to \mu^+\mu^-$ (or $q\overline{q} \to e^+e^-$), but where the specific choice of the muon pair typically rises from the desire to employ a lepton pair that is "easily" detected. This cross section must then be convoluted with the appropriate parton distribution functions. In terms of the scaled virtual photon mass $\tau = Q^2/s$ and the photon rapidity $y = \frac{1}{2}\ln\left[\left(q_0 - q_z\right)/\left(q_0 + q_z\right)\right]$, the "scaling" or parton model version of the cross section looks like

$$s\frac{d\sigma}{d\sqrt{\tau}\,dy} = \frac{8\pi\alpha^2}{3\sqrt{\tau}}\,g\left(\sqrt{\tau}e^{y}, \sqrt{\tau}e^{-y}\right),$$

where the (LO) parton "luminosity" function has the form

$$g\left(x_a, x_b\right) = \frac{1}{3}\sum_f e_f^2\left\{q_f^a\left(x_a\right)\overline{q}_f^b\left(x_b\right) + \overline{q}_f^a\left(x_a\right)q_f^b\left(x_b\right)\right\}.$$

The label *a*,*b* correspond to the 2 incident hadrons. The explicit factor of 1/3 is required because the conventional normalization of the *pdf*s ($q,\overline{q}$) includes an implicit sum over colors. Here the quark-antiquark pair that annihilates must be of the same color. Thus the annihilation occurs for only 1/3 of the possible pairs.

Contrary to the collider physics we have focused on in the Lectures, consider now the case of pion beams incident on a nuclear target, *i.e.*, composed of the canonical nucleon $N = (p+n)/2$. If we focus on large $\tau = Q^2/s$ so that we can safely assume that the interaction is dominated by the valence quarks (and antiquarks), determine the expected (and observed) value of the ratio

$$R_{DY} \equiv \frac{\sigma\left(\pi^+ N \to \mu^+\mu^- X\right)}{\sigma\left(\pi^- N \to \mu^+\mu^- X\right)}.$$

2. In this exercise we want to become familiar with various features of collider kinematics. As noted in Lecture the "real" rapidity and the pseudo-rapidity are defined by

$$\text{rapidity } = y_J = 0.5 \ln\left(\frac{E + p_z}{E - p_z}\right)$$

$$\text{pseudo-rapidity } = \eta = -\ln\left(\tan\left(\frac{\theta}{2}\right)\right)$$

where the z direction is the direction of the beam.

a) Verify, as stated in Lecture 4, that for any particle of mass M we can write

$$E = \sqrt{M^2 + p_T^2} \cosh y,\ p_z = \sqrt{M^2 + p_T^2} \sinh y,$$
$$p_T^2 = p_x^2 + p_y^2.$$

b) Prove that $\tanh \eta = \cos\theta$ and thus that η is easy to measure.

c) If particles are produced uniformly in longitudinal phase space with a differential distribution that looks like

$$dN = C\frac{dp_z}{E},$$

with C a constant, find the corresponding distribution in y, dN/dy.

d) Prove that the rapidity equals the pseudo rapidity, $\eta = y$, for a massless particle (and thus approximately for a relativistic particle, E ≫ M).

e) Prove that for a Lorentz transformation (boost) in the beam (z) direction, the rapidity, y, of every particle is shifted by a constant y_0, which is related to the boost velocity. Recall the form of such a boost to a reference frame moving in the z direction with velocity u (with respect to the original frame and with c = 1)

$$E' = \gamma\left(E - \beta p_z\right),$$
$$p_z' = \gamma\left(p_z - \beta E\right),$$
$$p_x' = p_x,\ p_y' = p_y,$$
$$\beta = u,\ \gamma = \frac{1}{\sqrt{1-\beta^2}}.$$

f) Consider a Z boson that is produced on-shell at the LHC in a $q\bar{q}$ annihilation process. The velocity of the Z boson is along the beam direction. What are the conditions relating x_1 and x_2, the momentum fractions of the quark and anti-quark? (Compare to the expressions in the previous exercise.)