7 Superconductivity

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For some years after 1908, Leyden was the only place where helium had been liquefied, and Kammerlingh-Onnes' laboratory there was the only place in the world where low temperature physics could be done. In the course of a study of the decrease in the electrical conductivity of metals with decreasing temperature, Kammerlingh-Onnes found that, below a critical temperature T_c characteristic of the particular metal, the electrical resistance would become too small to measure. Current could be left flowing round a superconducting loop for several months without any detectible loss in the current. Critical temperatures for the elements range from 9.2 K for niobium and 7.2 K for lead down to 26 mK for beryllium and 15 mK for tungsten. The Group I elements, such as sodium and copper, do not seem to be superconducting, nor do the elements in Group IIA, such as calcium, but many of the metals in Groups IIB, III, IV and V are superconducting.

Superconductivity remained mysterious after the advent of quantum mechanics, although attempts were made to understand it by Werner Heisenberg, Felix Bloch, and many others. In the 1950s John Bardeen and his colleagues made a determined effort to understand the problem, and a successful theory, known as the *BCS theory*, was published in 1957 by Bardeen, Cooper and Schrieffer. The basic picture that they developed was a detailed elaboration of the idea that superconductivity was due to pairs of electrons (with opposite spin) condense into a single state at low temperatures, in a way similar to the condensation of helium atoms into a single state below 2 K, or to the condensation of dilute clusters of alkali atoms below one μK, which was first observed in 1995 at JILA in Boulder by Cornell and Wieman.

The ability of superconductors to transmit current without dissipation of energy seemed useful for a number of purposes. It could cut down the energy losses involved in the transmission of electrical power between the power plant and the consumer. It could enable electromagnets to be maintained without supplying power. It could allow electronic memories to be maintained without losses. The third of these has been explored by several large electronic companies, including Phillips, IBM, and a consortium of big Japanese companies, but nothing competitive with silicon technology has been achieved by any of these attempts. Superconducting power transmission has been used on a small scale, but has not been used extensively, except for the high powered wave guides which are used to accelerate charged particles in accelerators. The construction of magnets, particularly for those in high energy accelerators and in MRI machines, has been the only large-scale use of superconducting materials.

7.1 Magnetic fields and the two types of superconductors

For many possible applications of superconductors the influence of magnetic fields on superconductivity is extremely important. It was found by Meissner and collaborators (Zeits. Technische Physik, **15**,1934, 507-514) that for low magnetic fields the magnetic flux is completely excluded by a superconductor. It was also found that above some critical applied magnetic field $B_c(T)$ the superconductivity is completely destroyed. For pure metals these critical fields are quite low; even lead, with its high critical temperature, can only sustain a field of 0.08 T. When a superconducting sphere is put into a weak uniform magnetic field, the field lines are bent around the sphere, so that the field lines are bent around the sphere without penetrating it, so that the field strength is maximum at the equator of the sphere (the great circle perpendicular to the direction of the field). If the strength of the magnetic field is increased it attains the critical value B_c on the equator before it reaches the critical value anywhere else; it is a standard exercise in potential theory to show the it attains the value B_c on the equator when its asymptotic strength is $2B_c/3$. When the field reaches the value B_c everywhere it penetrates the sphere completely, destroying all the superconductivity. In the intermediate region $2B_c/3 < B < B_c$ the sphere is divided into cylindrical regions of superconductivity alternating with cylindrical regions of normal metal carrying field of strength B_c. This state, with interleaved normal field-carrying regions and superconducting field-free regions, is known as the *intermediate state* of the superconductor.

This behavior is found in most pure metals, and such superconductors are known as *Type I superconductors*. In 1935 a completely different behavior was found some metallic alloys by Shubnikov and collaborators in Moscow (Rjabinin, J.N.; Schubnikow, L.W., Phys. Zeits. Sowjetunion, **7**, 1935, 122-125; Schubnikow et al., Phys. Zeits. Sowjetunion, **10**, 1936, 165-192). This behavior is widespread in alloys, and is also found in pure niobium and vanadium. Such superconductors are called *Type II superconductors*. In a Type

II superconductor the magnetic field does not penetrate the superconductor until it reaches the *lower critical field* $B_{c1}(T)$. There is an *upper critical field* $B_{c2}(T)$ above which the superconductivity is destroyed. In the range $B_{c1}(T) < B < B_{c2}(T)$ both the magnetic field and superconductivity coexist, and this state is known as the *mixed state* of the Type II superconductor.

In 1950, before the BCS theory was developed, Landau and Ginzburg developed a phenomenological but still very useful theory of superconductivity based on the existence of a complex *condensate wave function* $\Psi = |\Psi|\exp(iS)$ that vanishes in the normal state and is a function of position in the superconducting state. Vitaly Ginzburg received the 2003 Nobel Prize for this work. I am not giving the derivation of this theory, which is elegant but quite demanding, but will just give the two essential equations; the derivation can be found in any textbook on superconductivity, such as *Superfluidity and superconductivity*, by David R. Tilley and John Tilley. The equation for the magnitude of Ψ is

$$\alpha|\Psi|^2 - \frac{\hbar^2}{2m}|\Psi|\nabla^2|\Psi| + \frac{m}{2e^2\mu_0^2}\frac{(\mathbf{curl\,B})^2}{|\Psi|^2} + \beta|\Psi|^4 = 0\,, \tag{1}$$

where this has a positive solution for $|\Psi|^2$; when there is no positive solution $|\Psi| = 0$. Here m, e are the mass and charge of the electron, μ_0 is the permeability constant of the vacuum (1.26×10^{-6} henry/meter), $\alpha(T)$ is a parameter that is negative below T_c and positive above it, and β is a positive parameter characterizing the material. The equation for the phase S is

$$\frac{e}{m}(\hbar\mathbf{grad}S + 2e\mathbf{A})|\Psi|^2 + \frac{1}{\mu_0}\mathbf{curl\,B} = 0 \tag{2}$$

where $\mathbf{A}$ is the *vector potential* whose curl gives the magnetic field $\mathbf{B}$. The term proportional to $|\Psi|^2$ in this equation is just the electric current density, and so this equation is just a statement of Faraday's law.

An explanation of the difference between the two types of superconductivity was given by Brian Pippard (Proc. Cambridge Phil. Soc. **47**, 1951, 617-625). There are two length scales involved in the interface between a superconductor and the normal metal carrying a magnetic field. If a magnetic field should penetrate a superconducting region, the field bends the orbits of the superconducting electrons in such a way that the field these electrons generate cancels the external field over a *penetration depth* λ. The penetration depth is inversely proportional to the square root of the number of superconducting pairs. This result was derived by Fritz London (Phys. Rev. **74**, 1948, 562-573), and can be derived straightforwardly from the Landau-Ginzurg equations (1), (2). The second length ξ_p became apparent from Pippard's experimental work, but is now interpreted as the intrinsic size of an electron pair, in accordance with the BCS theory. Pippard observed that for $\xi_p >> \lambda$ the magnetic field falls to zero long before the condensate wave function has built up to its full magnitude, so there is a surface region in which both the magnetic field and the condensate wave function are depleted from their equilibrium vales in the uniform normal and superconducting states. This gives a free energy in the interface region which is more positive than it is in either of the uniform states. If, however, we have $\xi_p << \lambda$, the condensate wave function builds up quickly but the magnetic field penetrates far into the superconductor, so there is a surface region which has both a magnetic field and uniform condensate density, and the free energy of the interface region is negative. It is therefore thermodynamically favorable for the magnetic field and superconductivity to coexist.

Although I have just quoted an argument for what happens when one of these length parameters is much bigger or smaller than the other, there is actually a sharp distinction between the two cases. When the ratio $\kappa = \lambda/\xi_p < 1/\sqrt{2}$, the metal is a Type I superconductor, and for $\kappa > 1/\sqrt{2}$ it is Type II. As κ approaches the limit $1/\sqrt{2}$ from above, the critical fields B_{c1}, B_{c2} of a Type II superconductor approach one another, and merge at the thermodynamic critical field B_c of a Type I superconductor.

7.2 Superconducting magnets, transmission lines, and wave guides

For these applications, the difference between Type I and Type II superconductors is crucial. Even in Pb a field of 0.08 T is enough to destroy the superconductivity however low the temperature, which means that it could not be used to construct a modest 300 mT magnet. Because the magnetic field and current density only penetrate a short distance into the superconductor, the current is concentrated in the exterior of any wire that carries the current. To get a large field it is necessary to use Type II superconductors, and the larger the parameter κ is the larger will be the ratio of the upper critical field to the thermodynamic critical field. It is also convenient to have the critical temperature high, and it would be particularly valuable to have a critical temperature well above 70 K, so that liquid nitrogen could be used as the coolant instead of liquid helium.

The first good superconducting magnets were developed in the mid 1950s by Bernd Matthias, originally at Bell Labs, and then at UC San Diego. Various materials of the sort that he discovered are exploited as superconducting magnets. The alloys $Ge_xNb_3Sn_{1-x}$ have critical temperatures around 18 K, and upper critical fields estimated to be around 22 T, while SiV_3 has a transition temperature of 17.0 K and a an upper critical field of 15.6 T. With the advent of cuprate superconductors, discovered by Bednorz and Müller of IBM Zurich in 1986 (Physics Nobel Prize 1987), the critical temperatures went much higher than this, starting with 39 K for La_2CuO_{4+x}, continuing with 93 K, well above the boiling point of nitrogen, for $YBa_2Cu_3O_7$, and going up to around 140 K for some other materials. The temperatures quoted are taken from the 2007-2008 edition of the *CRC Handbook of Chemistry and Physics.*

Although the cuprate superconductors have much higher transition temperatures than materials like Nb_3Sn, they are much more difficult to use as superconducting magnets. There are two major problems. The first is that they are ceramics rather than conventional metals, and are therefore much harder to work with, for example in making the multiply wound coils that are desirable for making high field electromagnets.The second reason is that a high upper critical field, which these materials undoubtedly have, is not a sufficient condition for making a low-loss electromagnet. According to the laws of electromagnetism formulated by Faraday and Henry, magnetic flux moving across a current-carrying circuit generates an electromotive force round the circuit. In a superconductor, as I discuss in the next section, flux lines are discrete, quantized, objects, each carrying flux $h/2e$. These flux lines are subject to a force, perpendicular both to the direction of the field and the direction of the current, and if the flux lines drift in the direction of this force energy will be dissipated, just as it is dissipated by the ohmic resistance in a normal metal. Good superconducting magnets have the flux lines *pinned* to defects in the material, so that they are not free to move. For the materials with T_c close to 20 K it was found how to make the materials sufficiently disordered that the flux lines were pinned. For the cuprates it proved to be disappointingly difficult to prevent this *flux flow resistivity*, which generates heat and removes the benefit of using superconducting magnets.

For the transmission of electric power in superconducting wires very similar issues rise. In order to get high power transmission it is necessary to have a high current density, since the alternative of very high voltage risks catastrophic breakdown of the insulation. A high current density necessarily generates a high magnetic field, so the upper critical field determines the maximum current that can be carried in a given geometry. For the same reason it is necessary to have low flux flow resistance, as flow of the flux lines generated by the high current will lead to energy transfer from the current flow to the thermal background, and this heated is not only costly in terms of power generation, but will produce boiling of the coolant, which is much more expensive to replace than the power loss itself would suggest.

High frequency power can be transmitted by wave guides. In this the electromagnetic radiation is guided between conducting walls,but most of the energy is contained in the space (evacuated or gas-filled) within the walls. The resistivity of the walls determines how large is the electric field that penetrates the walls, and so how much energy is lost in the walls to ohmic resistance. Superconducting walls for the wave guides are likely to save energy, and they are used in the acceleration of charged particles in such devices as linear accelerators, where the wave has to be designed to travel with a speed that matches the local speed of the particles, or in synchrotrons, where the wave has to be in phase with the charged particle each time it comes round the ring.

7.3 Flux quantization and the flux lattice

Like many other good things in the theory of superconductivity, the idea of flux quantization originated in work of Fritz London, and can be derived from the Landau-Ginzburg theory. If one looks for solutions of Eqs. (1), (2), in which the magnetic field goes to zero at large distances, one can find solutions in which there is a field confined to the region of some line, say the axis $r = 0$ of cylindrical polar coordinates. At large distances from the axis, where $\mathbf{B}$ is zero, Eq. (1) gives $|\Psi| = \sqrt{-\beta/\alpha}$ (remember that α is negative for $T < T_c$, β is positive), while

$$\mathbf{A} = -\frac{\hbar}{2e}\mathbf{grad}S\,. \tag{3}$$

The solution of these equations is not unique, since different forms of $\mathbf{A}$ can give the same values of the field $\mathbf{B} = \mathbf{curl}\,\mathbf{A}$, and one convenient solution in the large r region is

$$S = n\phi\,, \quad A_\phi = -\frac{\hbar}{2e}\frac{n}{r}\,. \tag{4}$$

Since the condensate wave function, proportional to $\exp(iS)$, has to be single valued, n in this equation has to be an integer. The magnetic flux through a disk of radius r_0 centered on the axis can be written as

$$\Phi(r_0) \int_0^{r_0} r\,dr \int d\phi B_z = \int_0^{r_0} r\,dr \int d\phi (\mathbf{curl}\,\mathbf{A})_z\,. \tag{5}$$

Stokes' theorem tells us that the integral of the curl over the area can be written as an integral over the circumference

$$\Phi(r_0) = \int_0^{2\pi} r_0 d\phi\, A_\phi(r_0, \phi) = -\frac{2\pi\hbar}{2e} n\,. \tag{6}$$

This is an example of the famous result that the flux through a superconductor is quantized in integer multiples of $h/2e$

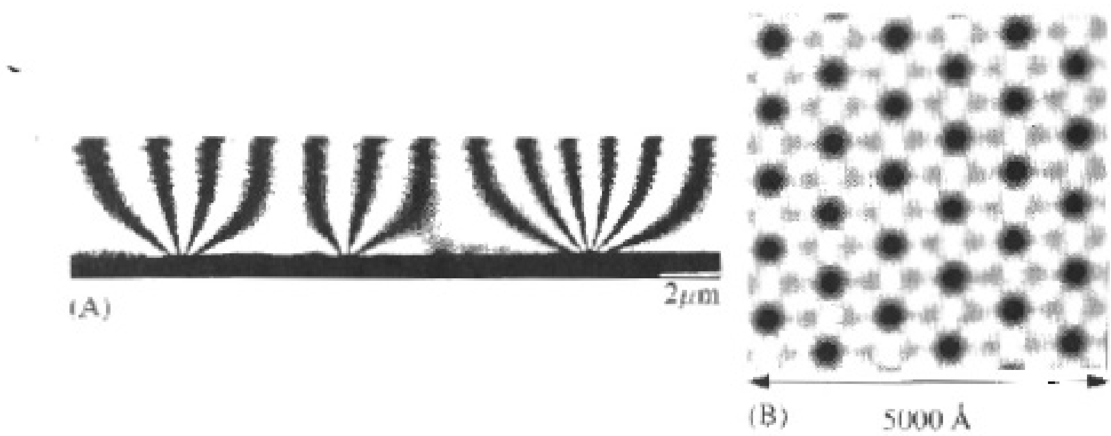

Figure 27.2. A Type II superconductor is unstable to the formation of flux tubes that penetrate the sample trying to generate a maximal area where superconductor and metal are in contact. The lowest free energy configuration is often a triangular lattice. (A) Electron hologram from the side of magnetic flux entering a lead film [Source: Tonomura et al. (1986), p. 93.] (B) Top view of an Abrikosov lattice of flux tubes in $NbSe_2$, taken with scanning tunneling microscopy. [Courtesy of S. Pan and A. de Lozanne, University of Texas.]

Figure 1: Picture from Marder, *Condensed Matter Physics, p 794*

In Type I superconductors, with positive interfacial energy between the normal metal and the superconductor, the flux quanta tend to cluster together, so that the normal regions will carry many quanta of flux. In Type

II superconductors, with negative interfacial energy, the flux quanta tend to keep apart from one another. Around 1955 Aleksei Abrikosov argued that the vortices would form a regular lattice, but his boss, Lev Landau, did not believe his arguments and did not let him publish the result, but eventually his paper appeared (Abrikosov, A.A., Zhur. Eksp. Teor. Fiz. **32**, 1957, 1442-1452), and was rewarded with the Nobel Prize in 2003. It was unfortunate that Abrikosov dropped a factor of 2 in his work, and came to the incorrect conclusion that a square lattice for the flux lines was more stable than a triangular lattice, but other theorists and experimental observations quickly corrected that error.

The Figure shows pictures, copied from Marder's book of real flux lines. On the left side you can see the uniformly spaced magnetic flux lines at the top of the picture, concentrating into tubes containing several quanta as they enter the Type I superconductor, lead. On the right side is a tunneling microscopy picture of the triangular Abrikosov lattice of singly quantized flux lines in the Type II material $NbSe_2$.

The quantization of flux lines in Type II superconductors is quite important for understanding flux pinning, because the magnetic field has its greatest value on the axis of the flux line, and the energy of those magnetic impurities is lowered by a flux line centered close to them. A very clean material is unlikely to pin flux lines, but one with a suitable concentration of magnetic impurities is likely to pin them. Grains of normal metal in the superconductor may also pin the flux lines.