

**The average score on the examination was 61.8, and the standard deviation 14.2**

1. **Average 15.6, standard deviation 5.4** *A glass bottle weighs 0.50 kg when empty, weighs 1.50 kg when full of water, and 0.30 kg when it is full and completely under water. The words “weighs  $x$  kg” are used here to mean the force needed to support it is equal to the force needed to support an  $x$  kg mass in air. Ignore the buoyant force due to air displaced by the bottle.*

- (a) [7 points] *What are the volumes of the glass of which the bottle is made, and of the water inside the full bottle?* The difference between the weight of the bottle and the weight of the bottle under water is .20 kg, so this must represent the weight of water displaced by the glass bottle. The volume of the glass must therefore be 0.20 l, or  $0.20 \times 10^{-3} \text{ m}^3$ , while the 1.00 kg extra weight of the filled bottle show that the volume of water it holds is 1.00 l, or  $1.00 \times 10^{-3} \text{ m}^3$ .
- (b) [5 points] *What is the density of the glass?* The density of the glass is the ratio of its mass 0.50kg to its volume 0.20 l,

$$2.5\text{kg/l} = 2.5 \times 10^3 \text{ kg m}^3 .$$

- (c) [6 points] *If the open bottle, initially full of air at pressure 1.00 atmospheres, is pushed neck downwards, so that no air escapes, to a depth of roughly 10 m, where the total pressure is 2.00 atmospheres, and it comes to thermal equilibrium at the same temperature as it was at the surface, what force is needed to hold it in position? You may express the answer in terms of the weight of a certain mass, but be sure to say whether the required force is up or down.* The buoyant force on the bottle is due to the water that the glass displaces, .20 l, and the volume that the compressed air in the bottle displaces. At the surface the volume of air in the bottle is 1.00 l, but under 2.0 atm pressure the air will displace 0.50 l of water, making a total of .70 l. The buoyant force is therefore equivalent to the force needed to support 0.70 kg of water, whereas the mass of the bottle is only 0.50 kg. The mass of the air is about .0012 kg, so can be ignored in comparison with the .50 kg of the bottle, which is .20 kg less than the mass of the displaced water. A downward force of  $0.20g$ , where  $g$  is the gravitational acceleration, is therefore needed to hold the bottle in position.
- (d) [7 points] *Describe clearly how you would do the calculation if the time taken to push the bottle down was sufficiently short that no heat flowed out of the compressed air in the bottle to the surrounding glass*

and water. Will this force be less than or more than the force in part (c)? If the air does not have time to equilibrate the compression is adiabatic, so it satisfies  $pV^\gamma = \text{constant}$  rather than  $pV = \text{constant}$ , with the ratio of specific heats  $\gamma$  equal to 1.4 for air. The volume is therefore reduced by a factor  $2^{5/7}$  from the volume at the surface, which gives a larger volume than for the isothermal compression, and means that an increased force is needed to counterbalance the buoyant force.

## 2. Average 16.7, standard deviation 5.4

- (a) [6 points] *Explain clearly why ice sometimes forms on grass on a still night when the air temperature a meter above the ground is above freezing temperature.* During the night solid and fluid surfaces emit infrared radiation out into space faster than it can be replaced by thermal conduction from lower down in the ground, so they get colder than the air above them. Since the air is still a temperature gradient is formed by conduction through the air, with the cold dense air forming a stable layer. If the relative humidity of the air a meter or so above the grass is fairly high, the air at the level of cut grass may well be cooled below both its dew point and the freezing point of water, in which case water vapor will condense out of the air and form ice on the grass, which we call “frost”.

I gave some credit to people who said that the cooling was due to evaporation or respiration, but the more I thought about it the less I believed it. The problem is that as the temperature approaches the dew point from above the evaporation rate drops rapidly. Have you tried drying clothes outside on a day with no sunshine, no wind, and high relative humidity?

- (b) [6 points] *What consequences does equipartition of energy have for the typical speeds of different species of molecules in equilibrium with one another in a mixed gas such as air?* Equipartition of energy means that the molecules all have the same average translational kinetic energy. As a result of this the mean square velocities of different molecules are inversely proportional to the molecular masses, and so the root mean square speeds of each sort of molecule are inversely proportional to the square root of its molecular mass.
- (c) [6 points] *Give an example of the use of forced convection of heat, and an example of the use of thermal radiation, in the preparation of food.* Stirring a pot of thick soup or oatmeal to spread the heat evenly and to prevent it from burning on the bottom is an example of forced convection. The fan used to spread hot air evenly round a convection oven is a more high-tech example of the same process. I asked specifically about forced convection, and many people gave me examples of natural convection, in which the currents in the fluid are

produced by the expansion of the hot fluid. Perhaps I did not stress the difference between forced and natural convection enough in class. Thermal radiation from a heated element above the food is the main mechanism in that type of broiler. A lot of the heat from a slowly burning charcoal broiler is also radiant, although some heat is convected upwards by the air and carbon dioxide. A microwave oven is not a good example of the use of thermal radiation, because the microwave uses resonant radiation with a wavelength of the order of 20-30 cm, while thermal radiation has a very broad bandwidth where the wavelengths are of the order of microns.

- (d) [7 points] *Why does the proportion of different molecules in an isothermal atmosphere vary with height? Give one reason why that does not happen much at low altitudes in the earth's atmosphere.* In the earth's gravitational potential  $gz$  the Boltzmann distribution gives the probability density for a molecule of mass  $m_j$  at height  $z$  as proportional to  $\exp(-m_jgz/k_B T)$ . The height over which molecules fall off by a factor of  $e$  in an isothermal atmosphere is therefore  $k_B T/m_j g$ , inversely proportional to the molecular mass, so the concentration of heavier molecules will drop more rapidly than the concentration of lighter molecules in an equilibrium isothermal atmosphere. You can, as many people did, argue that the lighter molecules are going faster, so will go further up than the heavier ones.

In the lower levels of the earth's atmosphere (the troposphere) there are rising and falling air currents that mix the air from different levels, and this stirring makes the chemical composition more uniform than it would be in equilibrium.

3. **Average 11.8, standard deviation 5.8** *Atoms of mass  $m$ , very weakly interacting with one another, are held in a parabolic trap with potential  $U_0 = (1/2)K_0(x^2 + y^2 + z^2)$ , so that they oscillate about the center of the trap with angular frequency  $\omega = (K_0/m)^{1/2}$ . They are in thermal equilibrium with one another at a temperature  $T_0$  kelvin, where  $k_B T_0 > \hbar\omega$  and  $h$  is Planck's constant.*

This question was not done well, and my wording of part (c) could have been better. I was disappointed by how many people did not think about equipartition when answering this question, or remembered it for the potential energy, but forgot that in a harmonic oscillator the energy is equally shared between kinetic energy and potential energy. It is not so long since we were discussing vibrational modes of molecules in class. These atomic traps can be thought of as enormous molecules with  $3N$  vibrational modes.

- (a) [7 points] *What are the average kinetic and potential energies of the atoms in this trap?* From the equipartition law each mode of oscillation will contribute  $k_B T_0/2$  to the average kinetic energy of an atom and  $k_B T_0/2$  to the average potential energy, and there are

three modes of vibration  $(x, y, z)$ , so each atom averages  $3k_B T_0/2$  of kinetic energy and  $3k_B T_0/2$  of potential energy.

- (b) [6 points] *If the trap potential is suddenly changed to*

$$U_1 = (1/2)K_1(x^2 + y^2 + z^2) ,$$

*with  $K_1 \ll K_0$ , without changing the instantaneous positions and momenta of the atoms, how does the average energy of the atoms change?* When the trap is suddenly switched to a lower strength the atoms will have no sudden change of position or velocity, but their potential energy drops by a factor of  $K_1/K_0$ , because this is the factor by which the spring constant is reduced. The average energy therefore changes by a factor of  $(K_0 + K_1)/2K_0$ .

- (c) [6 points] *What determines the new equilibrium temperature  $T_1$ ?* I should have separated the two steps in the argument more clearly. Part (b) can be answered by thinking about what happens instantaneously to the two components of the energy. Immediately after the change in spring constant is made the high speed of the atoms will result in oscillation of larger amplitude in the weaker potential. Initially the atoms will all be more or less in phase with one another, but eventually energy-conserving collisions cause them to get out of phase, and the system will settle down to an equilibrium temperature determined by the conserved total energy, so we should have

$$T_1 = \frac{K_0 + K_1}{2K_0} T_0 .$$

- (d) [6 points] *What is the significance of the condition  $k_B T_0 > h\omega$ ?* Equipartition only holds for a harmonic oscillator if the average energy in each mode is well above  $h$ , since the allowed energy levels of an oscillator are spaced  $h\omega$  apart. A number of people gave me stuff I did not understand about the escape velocity. Indeed real traps are not parabolic all the way, and evaporative cooling by the escape of faster atoms is one way of preparing a really cold sample, but I do not think this has anything to do with Planck's constant. If you can explain this answer to me you deserve more credit than I gave you.

4. **Average 17.7, standard deviation 5.0** This was the best done question and the easiest to grade.

- (a) [6 points] *Show that the work done by a mole of ideal gas during the reversible isothermal expansion from volume  $v_1$  to volume  $v_2 > v_1$  is  $RT \ln(v_2/v_1)$ , where  $T$  is the Kelvin temperature.* The pressure is given by  $pv = RT$ , so the work done by the mole of gas expanding from  $v_1$  to  $v_2$  is

$$W = \int_{v_1}^{v_2} \frac{RT}{v} dv = RT \ln \frac{v_2}{v_1} .$$

This is six points for very little, but some people seemed to want to give me something different.

- (b) [6 points] *How much heat must be supplied to the gas in this process, and how do you know this?* The internal energy of an ideal gas does not change at constant temperature, so, from the conservation of energy, the heat supplied to the gas must balance the work done, and is also

$$Q = RT \ln \frac{v_2}{v_1} .$$

- (c) [6 points] *What is the entropy change of the gas in this process?* In a reversible process such as this the heat is given by  $Q = T\Delta S$ , so the entropy increase of the gas is

$$\Delta S = \frac{Q}{T} = R \ln \frac{v_2}{v_1} .$$

- (d) [7 points] *What would be the entropy change of the ideal gas if the expansion occurred by a free expansion, in which a wall of a well-insulated container of volume  $v_1$  is suddenly removed and the gas allowed to fill a larger container of volume  $v_2$ , where the portion of volume  $v_2 - v_1$  was originally evacuated? No work is done in this process, and no heat transferred.* In an ideal gas the internal energy is a function of temperature only, so if there is no work or heat transfer in a free expansion process there is not change of the internal energy, and therefore no temperature change. Since the final state is the same state as was obtained in parts (a), (b), (c), the entropy must be the same, so the increase in entropy is again

$$\Delta S = R \ln \frac{v_2}{v_1} .$$

This caught a lot of people who thought that no heat flow means no entropy change, even though the process is obviously irreversible,