1) A large rubber balloon is filled with air; its volume is $V = 8.00 \times 10^{-3}$ m$^3$. The air inside the balloon has a pressure of 1.05 atmospheres. The air both inside and outside the balloon has $T = 20°$ C.

a) How many moles of air are in the balloon?

$$n = \frac{pV}{RT} = \frac{(1.05 \text{ atm})\left(\frac{1.01\times10^5 \text{ N/m}^2}{1 \text{ atm}}\right)(8 \times 10^{-3} \text{ m}^3)}{(8.32 \text{ J/K mol})(293\ K)} = 0.348 \text{ mol}$$

b) Suppose you want to hold this balloon entirely just under the surface of a pool of water, as shown. (The water also has $T = 20$ C.) *Calculate the force* you must exert to do so. [Work only to two significant figures.]

The forces on the balloon are those of pressure (which come from all sides, but we easily summarize as the "buoyant force" $F_b$) and the weight of the balloon and the air in the balloon. The force needed to hold the balloon steady is such that the net force on the balloon is zero; therefore

$$F = F_b - m_{air}g - m_{balloon}g$$

We are not given the mass of the balloon! But we are told to work only to two significant figures. The buoyant force equals the weight of the water displaced by the balloon, namely

$$F_b = \rho_{H_2O}V_{balloon}g = (10^3 \text{ kg/m}^3)(8 \times 10^{-3} \text{ m}^3)(9.8 \text{ m/s}^2) = 78 \text{ N}$$

to two significant figures. Now the weight of the air and of the balloon together can only change this answer if $m_{air}g + m_{balloon}g$ is at least 1 Newton (otherwise 78 remains 78). This would require that their mass be 0.1 kg. But we know a balloon weighs only a few grams (either from class, or from common sense) and we know that $\rho_{air} \sim 1$ kg/m$^3$, so the mass of the air in the balloon is only of order 8 grams (actually 9 or so, but nowhere near 100.) So the weight of the air and the balloon can be ignored, and thus $F = F_b = 78$ N to two sig. figs.

c) Suppose you drag the balloon 20 meters underwater and hold it there. Is the force required to hold it in place *greater than, less than, or the same* as your answer to part (b)? *Explain* your answer as *clearly and completely* as possible, in at most two or three brief sentences. **Note you are not required to calculate the force.**

I graded your answer for content, completeness and clarity (though I was tougher on the first point.)

Your answer needed somehow to contain the following logical points:
(1) The *pressure* in the water is much greater 20 meters below the surface.
(2) The *volume* of the balloon will shrink considerably, until the pressure inside the balloon essentially balances the pressure outside (to a good approximation.)
(3) The *density* of the water does not change very much despite the increased pressure (to a good approximation.)
(4) Therefore the buoyant force — equal to the weight of the water displaced by the balloon — is much less than near the surface.

**Minor errors:** left off point 3 — logically incomplete.

**Major errors:** left off point 2 — didn't remember what determines the volume of a balloon.

**Profound errors:** "Greater pressure implies greater bouyant force" (it does not) and "The weight of the water overhead pushes down on the balloon, making the required force less" (but the weight of the water overhead increases the pressure, so it increases the pressure force pushing down *and* the pressure force pushing up.)

d) **Hard!** Let's bring the balloon back out of the water into the atmosphere. Suppose two holes are opened in the balloon, as shown. Each one has area 1 $\text{cm}^2$. *Estimate the time* required for the balloon to deflate to half its volume. [Of course, you must ignore viscosity, since we have not learned how to include it.]

You can't do this exactly; the pressure inside the balloon changes as it deflates. The best you can do is get a rough idea — but that's all I asked you to do.

If you knew the flow rate (by volume) out of the balloon, and the flow rate were constant, you'd be done:

$$\frac{\Delta V}{\Delta t} = \text{Flow Rate} \Rightarrow \Delta t = \frac{\Delta V}{\text{Flow Rate}}$$

But the flow rate $A_1 v_1 + A_2 v_2$ (where 1 and 2 label the two holes) is not constant. So what do you do? Approximate. How about

$$\Delta t \approx \frac{\Delta V}{\text{Average Flow Rate During Deflation}}$$

So now what? Well, $\Delta V = \frac{1}{2}V$ for the balloon to deflate half way. Also, $A_1 = A_2 = A$, and finally (by various possible arguments, the simplest being that the two holes are identical) $v_1 = v_2$ so they both have the same average $\bar{v}$ during deflation. Thus the average flow rate is something like $(2A)\bar{v}$:

$$\Delta t \approx \frac{\frac{1}{2}V}{2A\bar{v}}$$

We still have to estimate $\bar{v}$. As the balloon deflates the velocity of the air $v_{in}$ inside the balloon is much less than the velocity $v_{out}$ of the air squirting out the holes. We can try Bernoulli

$$p_{in} + \frac{1}{2}\rho v_{in}^2 = p_{out} + \frac{1}{2}\rho v_{out}^2$$

(since the air is moving horizontally we can ignore the $\rho g y$ term) and dropping the $v_{in}$ term we can obtain

$$v_{out} \approx \sqrt{\frac{2\Delta p}{\rho_{air}}}$$

Now, unfortunately $v_{out}$ might change with time as the pressure difference inside and out changes with time; but the balloon is only shrinking to half its volume, so we shouldn't expect (from our own experience with blowing up a balloon) that the pressure difference should change by more than a factor of, say, three or so. (Meanwhile $\rho_{air}$ is the density of the air outside, so it doesn't change much.) And notice the square root; the variation in $\sqrt{\Delta p}$ is even smaller yet. But we're making an estimate anyway (and viscosity is probably a much bigger effect) so we simply are worrying about something that isn't very important. If we say

$$\bar{v} \approx \sqrt{\frac{2\overline{\Delta p}}{\rho_{air}}}$$

where $\overline{\Delta p}$ is the average pressure difference, then the only thing left to guess is $\overline{\Delta p}$. Now if you said $\overline{\Delta p} = 0.05$ atm (the initial $\Delta p$) or twice this or half this, I was perfectly happy to say that is a decent estimate.

Putting this all together, $\bar{v} \approx 100$ m/s and

$$\Delta t \approx \frac{\frac{1}{2}V}{2A}\sqrt{\frac{\rho_{air}}{2\overline{\Delta p}}} \approx \frac{.008 \text{ m}^3}{4(.0001 \text{ m}^2)} \frac{1 \text{ kg/m}^3}{2(0.05 \text{ atm})(10^5 \text{ N/m}^2/\text{atm})} \approx 0.2\ s$$

which is maybe a little fast but is physically sensible — consistent with what you know from life. Now if you put $\overline{\Delta p} = 0.1$ atm or 0.2 atm or 0.025 atm, you would still have gotten an reasonably good answer. But $\Delta p$ is certainly not 1 atm!

For those who obtained answers greater than 10 seconds — I wonder where you have been buying your balloons! (For answers much shorter than .01 seconds the situation is more ambiguous – even though you know that is too fast, perhaps viscosity might slow things down?...)

2) A container at $T = 320$ K contains 3.0 moles of $O_2$ gas (and nothing else). The pressure in the container is measured to be $30,000$ N/m$^2$.

a) What is the mass density of the oxygen inside the container? [Hint: You do not need to know the volume of the container.]

Since an oxygen atom has mass about 16 amu, the mass per mole of oxygen gas $O_2$ is about 32 grams. You can write this in various ways:

$$\rho = \frac{m_{O_2} p}{k\,T} = \frac{m_{O_2} N_A p}{N_A k\,T} = \frac{(\text{mass per mole of } O_2)\; p}{R\,T}$$

$$= \frac{(0.032kg)(3 \times 10^4 \text{ N/m}^2)}{(8.32 \text{ J/K})(320\text{K})} = 0.36 \text{ kg/m}^3$$

b) Mix in 1.0 mole of $H_2O$ vapor (also at $T = 320$ K.) What is now the total pressure on the walls of the container? [Hint: If this seems hard, remember how we dealt with air, which is also a mixture of gasses.]

We now have four moles of a mixture of gasses. We know that $pV = NkT$, independent of the average mass of the gas molecules. Since $V$ hasn't changed, and $T$ hasn't changed, we know that the pressure $p$ is simply proportional to the number of moleculues $N$, no matter what their masses.

And so, without calculating $V$, we know

$$\frac{p_2}{p_1} = \frac{N_2}{N_1} = \frac{4}{3}$$

and therefore

$$p_2 = \frac{4}{3} p_1 = 4.0 \times 10^4 \text{ N/m}^2$$

Alternatively, we could determine the volume of the container by using $V = N_1 kT/p_1 = 0.026$ m$^3$ from part (a). Then we could use $p_2 V = N_2 kT$. This is correct, but not efficient, since it requires far more computation than necessary.

$$V = \frac{(3N_A)kT}{p_1} = \frac{3(6.0 \times 10^{23})(1.38 \times 10^{-23} \text{ J/K})(320 \text{ K})}{3 \times 10^4 \text{ N/m}^2} = 0.26 \text{ m}^3$$

$$p_2 = \frac{(4N_A)kT}{V} = 4.0 \times 10^4 \text{ N/m}^2$$

c) When the oxygen and water vapor are mixed as in part (b), do you expect condensation to form on the walls of the container? (The saturated vapor pressure of water vapor at $T = 320$ K is known to be $1.20 \times 10^4$ N/m$^2$.) Explain your answer.

Condensation occurs when the partial pressure of $H_2O$ exceeds the saturated vapor pressure of $H_2O$. The partial pressure is

$$p_{H_2O}^{partial} = p_{tot} \frac{N_{H_2O}}{N_{tot}} = p_{tot} \frac{n_{H_2O}}{n_{tot}} = \frac{1}{4} p_{tot}$$

Therefore the correct answer is "yes" if $p_{tot}$ is more than four times greater than $1.20 \times 10^4$ N/m$^2$, and "no" otherwise.

If you did (b) correctly, then you should therefore have said "no".

(If you did (b) incorrectly, but drew the appropriate conclusion given your mistake in (b), I gave you full credit.)

d) **VERY HARD!!!** Suppose we instead mixed in a container 3.0 moles of $O_2$ at $T = 320$ K and 1.0 mole of $N_2$ at $T = 500$ K. What would be the *total pressure* on the walls of the container after the system comes to equilibrium? [**We have not learned how to do a problem of this type. Can you find a way?** You can assume the walls do not play an important role in determining the equlibrium temperature and pressure, and that oxygen and nitrogen molecules, both being diatomic molecules, behave similarly in collisions.]

Well, as you proved to me, this wasn't that hard.

The oxygen and nitrogen are at different temperatures, and they must come to a common temperature at equilibrium. To determine the final pressure we must know the final temperature. How do we find this?

This is a conservation of energy issue. Since oxygen and nitrogen are both simple diatomic molecules, we can assume safely that the kinetic energy per molecule is the same fraction, for both molecules, of their total energy. The nitrogen starts off with a larger energy per molecule than the oxygen, but over time that larger energy will be distributed evenly over all of the molecules of nitrogen and oxygen, until (from kinetic theory) the average energy of nitrogen molecules equals the average energy of oxygen molecules.

Since the average kinetic energy of a gas molecule is $\frac{3}{2}kT$, the total kinetic energy in nitrogen gas molecules is initially $\frac{3}{2}N_{N_2}kT_{N_2}$. The total kinetic energy in oxygen is $\frac{3}{2}N_{O_2}kT_{O_2}$. And once they come to equilibrium at $T_{eq}$, the total kinetic energy should be $\frac{3}{2}(N_{N_2}+N_{O_2})kT_{eq}$. Thus energy conservation says

$$\frac{3}{2}N_{N_2}kT_{N_2}+\frac{3}{2}N_{O_2}kT_{O_2}=\frac{3}{2}(N_{N_2}+N_{O_2})kT_f \qquad (*)$$

Now, if you want, you can solve for $T_f$

$$T_f=\frac{N_{N_2}T_{N_2}+N_{O_2}T_{O_2}}{N_{N_2}+N_{O_2}}=365\text{K}$$

and then obtain the pressure (using $V$ calculated in part (b)) from

$$p_{eq}=\frac{n_{tot}RT}{V}=4.6\times10^4\text{N/m}^2$$

What I didn't notice (and a few of you did) is that you can go back to the earlier equation marked (*) and write it as

$$[p_{N_2}V]_{at\ T=500\ K}+[p_{O_2}V]_{at\ T=320\ K}=[p_{eq}V]_{at\ T_{eq}}$$

and so (since all the $V$'s are the same)

$$p_{eq}=[p_{N_2}]_{at\ T=500\ K}+[p_{O_2}]_{at\ T=320\ K}$$

which is just about the simplest thing you might have guessed, even if you had no idea about energy conversation. Notice it does not even require you to calculate $T_{eq}$! Now you can just use $p=nRT/V$ repeatedly, using $V$ calculated in part (b). Or, in fact, since you know from part (a) that

$$[p_{O_2}]_{at\ T=320\ K}=3\times10^4\ \text{N/m}^2$$

you can eliminate the volume

$$\frac{[p_{N_2}]_{at\ T=500\ K}}{[p_{O_2}]_{at\ T=320\ K}}=\frac{[n_{N_2}T]_{at\ T=500\ K}}{[n_{O_2}T]_{at\ T=320\ K}}=\frac{1\ \text{mol}}{3\ \text{mol}}\,\frac{500\ \text{K}}{320\ \text{K}}=0.52$$

to find

$$[p_{N_2}]_{at\ T=500\ K}=1.56\times10^4\ \text{N/m}^2$$

giving the answer $p_{eq}=4.6\times10^4\ \text{N/m}^2$.