Non-perturbative renormalization with improved staggered fermions

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Abstract

This is a class B proposal, asking for continuation of time on the clusters to calculate matching ("Z") factors using non-perturbative renormalization (NPR) for the quark mass, bilinear and four-fermion operators with staggered quarks. (The web page for this project is `http://www.phys.washington.edu/users/sharpe/usqcd/index.html`.)

Due to a variety of factors, we have made less progress during the present (2008-9) allocation year than anticipated, and are likely only to have completed the calculation of the mass renormalization for asqtad quarks (as well as those for related bilinears) on coarse and fine MILC lattices. We propose here to extend this calculation to HYP-smeared staggered valence quarks, for both the bilinears and four-fermion operators. These matching factors are needed to reduce and control the systematic error in the calculation of B_K and other kaon matrix elements using HYP-fermions. Matching is, or is likely soon to be, the dominant source of error in this calculation. We propose to use the standard NPR methodology, adapted to staggered fermions, which our results to date indicate will be successful with relatively small ensembles. We use the Chroma package, augmented by the addition of momentum sources. We are asking for time on clusters, and prefer to continue running at Fermilab.

We are requesting 500,000 6n-equivalent node-hours on the clusters.

1 Scientific Background

With the exception of (partially) conserved currents, hadronic matrix elements obtained from lattice QCD require matching factors to relate bare lattice quantities to those in conventional continuum schemes. Such matching is essential for electroweak matrix elements because the effective Hamiltonian (including overall Wilson coefficients) has been determined in continuum schemes. Early calculations used perturbative matching, but this inevitably introduces truncation errors. For example, one-loop matching leads to errors of $O(\alpha_s^2) \approx 10\%$. This is small enough for some applications, but not for precision tests of the standard model (SM). For these, one needs ultimately to use non-perturbative renormalization (NPR) [1]. This method has been widely applied with considerable success, most notably with Wilson-like [2] and domain-wall fermions (DWF) [3]. In the latter case, percent-level accuracy of matching factors for B_K have been attained.

We are undertaking a project to implement NPR for mass and matrix element calculations using improved staggered fermions. To our knowledge, there has been only one previous application of NPR to staggered fermions [4], although other(s) are underway. Reference [4] used unimproved (quenched) staggered fermions, and calculated matching factors for the quark mass. This work found large discretization errors but successfully determined Z_m. We expect that the discretization errors will be reduced in our calculation since we use improved staggered fermions, and so expect that the NPR method will be easier to apply.

The overall goals of our project are twofold:

- To calculate the matching factor for the quark mass, Z_m, with asqtad fermions, so as to improve the precision of the determination of the light quark masses from the existing determination of the bare quark masses.[1] Quark-mass determinations with asqtad quarks presently use the two-loop matching factor of Ref. [5], and the largest, and least well controlled, systematic is the corresponding truncation error. Quark masses are fundamental parameters of the standard model, and their determination is one of the goals of USQCD.
- To calculate matching factors for the operators needed to determine B_K and related kaon matrix elements using HYP-smeared [6] improved staggered fermions. The calculation of the bare matrix elements of these operators are underway using USQCD resources (see proposal by Lee, Jung and Sharpe), but accurate matching factors are needed to provide precision results. As has been repeatedly stressed in USQCD proposals and white papers, a precision result for B_K allows a strong test of the SM and places significant constraints on new physics.

Both calculations will be done, in the first place, on the coarse and fine MILC lattices, though we envisage extension to yet finer lattices if the project is successful.

Last year's proposal (ultimately allocated the maximum class B allocation of 500,000 6n-equivalent node-hours), aimed to reach both goals. In practice, due to a slower start up than anticipated, and other issues discussed below, we expect only to be able to achieve the first goal. For the same reasons, we are unlikely to use our full allocation in 2008-9.

[1] We understand from conference reports that the MILC collaboration is also implementing NPR for this purpose, although we do not know the methodological details. We think that two independent calculations of this important quantity are warranted.

Our focus in this proposal for 2009-10 is to attain the second goal. Given the relatively early stage of these calculations, and the fact that they are relatively computationally inexpensive, we have kept this proposal at the class B level.

2 Methodology

The NPR method is well established and we give here only a brief recap focusing on special features associated with the application to staggered fermions (features that are common to both asqtad and HYP-smeared variants).

The essence of NPR is to match short distance, gauge-fixed correlators to their continuum perturbative forms. With staggered fermions this matching is non-trivial because the taste and spin degrees of freedom are spread over a 2^4 hypercube. In momentum space, this corresponds to breaking up the lattice Brillouin zone ($-\pi \le p_\mu a < \pi$) into 2^4 pieces,

$$p_\mu = p'_\mu + B_\mu \frac{\pi}{a}\,, \tag{1}$$

where B_μ is a hypercube vector (composed of 0's and 1's). p'_μ is the physical momentum, which lies in the reduced range $-\pi/2 \le p'_\mu a < \pi/2$. The 16 different values of p associated with a single physical p' correspond to the spin-taste degrees of freedom. This formulation in momentum space, which was developed by Ref. [7] and elaborated by Ref. [8], naturally incorporates the lattice symmetries. This is shown by the free propagator, which takes the form (setting $a = 1$):

$$S^{-1}[p' + \pi A, -(q' + \pi B)] = \bar{\delta}(q' - p') \left[m\overline{\overline{(1 \otimes 1)}}_{AB} - i \sum_\mu \sin(q'_\mu)\overline{\overline{(\gamma_\mu \otimes 1)}}_{AB} \right]\,, \tag{2}$$

where the over-barred spin-taste matrices (defined in Ref. [8]) are unitarily equivalent to the usual forms. In other words, apart from the discretization error $q'_\mu \to \sin q'_\mu$, the propagator is just that for four degenerate fermions with a full $SU(4)$ taste symmetry. The translation and rotation symmetries of staggered fermions guarantee that the spin-taste structure of the propagator holds non-perturbatively, although each term can have a more general, though hypercubic invariant, momentum dependence.[2]

The first step in an NPR calculation is to determine S^{-1} on Landau-gauge fixed lattices. Following Ref. [4], and many other works, we use momentum sources [i.e. $\exp(ip_\mu n_\mu)$ on the entire lattice] rather than point sources. The resultant volume-averaging greatly reduces the noise, although this comes at the cost of having to calculate a new propagator for each momentum (rather than Fourier transforming the free end of a point-source propagator). Since we are interested in precision, and since we can build upon previous work to choose optimal momenta, we think that the advantages outweigh the disadvantages. Furthermore, we need to contract propagators with hypercube operators, which involve quark and antiquark fields at different positions on a 2^4 hypercube. Thus we would need 16 point sources were we to use them, and we would lose the volume averaging at the operator insertion.

S^{-1} is a 16×16 matrix, and thus requires 16 inversions for each physical momentum. We package these together in our analysis program, and Fourier transform on the free end,

[2] This is not true in the approach of Ref. [4], which does did not use simple momentum space fields, but rather Fourier transformed the position-space hypercube fields. This leads to additional taste-breaking terms in the propagator (even in the non-interacting theory).

producing the desired matrix. We then decompose this matrix into the spin-taste basis. Only the parts proportional to the matrices $(1 \otimes 1)$ and $(\gamma_\mu \otimes 1)$ should be non-zero after averaging over configurations—and we have checked that this holds within errors. Removing the (periodic) delta-function with the definition $S^{-1}[p'] \equiv S^{-1}[p', -p']/V$, the wave-function renormalization is given in the "RI′ scheme" by (with $a = 1$)

$$\frac{1}{Z'_q(p')} = -i\frac{1}{12N_T}\sum_\mu \frac{p'_\mu}{p'^2}\mathrm{Tr}\left[\overline{\overline{(\gamma_\mu \otimes 1)}}S^{-1}[p']\right] . \tag{3}$$

where $N_T = 4$ is the number of tastes. $Z'_q(p')$ is needed for subsequent parts of the calculation, and can also be compared, after chiral extrapolation, to perturbative results.[3] In practice we use the free form $p'_\mu = \sin p'_\mu + (1/6)\sin^3 p'_\mu$.

The mass renormalization factor Z_m may then be determined using

$$M(p') \equiv \frac{1}{12N_T}\mathrm{Tr}\left[\overline{\overline{(1 \otimes 1)}}S^{-1}[p']\right] = \frac{1}{Z_q(p')}\left[Z_m(p')m + C_1\frac{\langle\bar{q}q\rangle}{p'^2} + \ldots\right] . \tag{4}$$

More precisely, we must extrapolate Z_m to the chiral limit in order to make this a mass-independent renormalization constant. The second term in eq. (4), which can be derived using the operator-product expansion [9], is an example of the non-perturbative effects that are always present at some level even in short distance correlation functions. It must be removed by fitting, and makes this a sub-optimal method for determining Z_m.

A better method is to calculate the matching factor for the scalar, taste singlet bilinear, Z_s, and use $Z_m = Z_s^{-1}$, which follows from the approximate axial symmetry of staggered fermions. More generally, we want to calculate the insertion of staggered bilinear operators $\mathcal{O}$ having arbitrary spin-taste. These operators are spread out over a 2^4 hypercube, and generically involve gauge links. For the taste singlet scalar, however, the operator is ultra-local and does not involve gauge links. By contracting together two momentum-source propagators (one appropriately conjugated) we can construct a vertex function $\Lambda^{\mathcal{O}}[p', -p']$.[4]

To obtain the matching factor for the operator, we first amputate,

$$\Gamma^{\mathcal{O}}[p'] = S[p']^{-1}\Lambda^{\mathcal{O}}[p', -p']S[p']^{-1} , \tag{5}$$

where we stress that all quantities in this equation are 16×16 matrices. We now obtain the matching factor $Z_{\mathcal{O}}$ by projection,[5]

$$\frac{1}{Z_{\mathcal{O}}(p')} = \frac{Z_q(p')}{12N_T}\mathrm{Tr}\left\{\overline{\overline{(\gamma_{\mathcal{O}})}}^\dagger\Gamma^{\mathcal{O}}[p']\right\} , \tag{6}$$

where $\gamma_{\mathcal{O}}$ is the appropriate spin-taste matrix. To obtain the final result one must extrapolate to the chiral limit, multiply by the appropriate perturbative renormalization factor to obtain a nominally scale independent quantity, ensure that non-perturbative contributions are suitably

[3]We can convert from the RI′ scheme to the more conventional RI scheme (i.e. from Z'_q to Z_q) using perturbation theory, and this is assumed in the formulae below (although we have not implemented this small effect yet in practice).

[4]In general the momenta can be different (and this has been used to good effect in DWF studies [3] to avoid exceptional momenta), but, for simplicity in this presentation we assume that the incoming and outgoing momenta are the same.

[5]In general, one must also include operator mixing, but this is not the case for most bilinears, and we do not include it here so as to avoid cluttering the notation.

small by studying the p' dependence, and attempt to extrapolate away discretization errors proportional to $(ap')^2$. These steps are non-trivial, but have been carried out successfully in previous NPR calculations.

Two Ward identities provide a check on our computations:

$$\frac{M(p')}{m} = \frac{1}{12N_T}\mathrm{Tr}\left\{\overline{(\gamma_5\otimes\xi_5)}\,\Gamma^{(\gamma_5\otimes\xi_5)}[p']\right\}, \tag{7}$$

$$\frac{\partial M(p')}{\partial m} = \frac{1}{12N_T}\mathrm{Tr}\left\{\Gamma^{(1\otimes 1)}[p']\right\}. \tag{8}$$

These hold for unimproved, HYP-smeared and asqtad fermions, and allow one to show that $Z_m = Z_S^{-1} = Z_P^{-1}$. The first identity can be checked directly—any disagreement between the two sides indicates incomplete convergence of the propagators. The second we can check only semi-quantitatively, since it involves a derivative, but it provides an important check that we have normalized quantities correctly.

Our resulting matching factors will not, strictly speaking, be mass independent, as we cannot extrapolate the strange sea-quark mass to zero (it being fixed in the MILC configurations to close to its physical value). We will estimate the resulting systematic along the lines followed in Ref. [3], and expect this to be small. We can also study this numerically by using the coarse MILC lattices with different values for the strange mass.

The same method applies to four-fermion operators, except that one has, of course, four external legs, and that all cases involve operator mixing.

Finally, we note that, since we use the MILC asqtad configurations, we must assume that the use of a rooted fermion determinant leads to the correct continuum limit. This is plausible, based on the extensive numerical results and theoretical work summarized in Refs. [10, 11, 12].

3 Code details

We have chosen to carry out our calculations using the Chroma software package because of its portability, flexibility, and accessibility. The Chroma system is portable so that code written and tested on our native scalar computers can be run on parallel workstations with minimal modification. Chroma uses an xml input system to specify its calculations, which leads to a great amount of flexibility in running jobs. There is no need to recompile code for every modification, and a complex series of manipulations can be specified from a single input file. Chroma is accessible to us in the sense that many useful algorithms can be run "out of the box." This allows us to focus code development on the specific needs of our project. In particular, we use the built in Landau gauge fixing, HYP-smearing and asqtad inversion options.

Because we want to invert directly on momentum sources (as discussed above) we have written routines to construct such sources for staggered propagators, and integrated them into the "factories" of Chroma so that they can be used in the standard way. We have written routines to Fourier transform these propagators and output them in an xml format, which is then preprocessed by a python script into a form convenient for analysis with Mathematica. Our analysis is done locally.

We have production code on the Fermilab cluster for calculating momentum-source propagators and scalar/pseudoscalar renormalizations for asqtad fermions using equal incoming and outgoing momenta.

Further code development is required to complete the calculations outlined above. Our development path is as follows:

- Implement the contraction of two propagators with a staggered bilinear operator of arbitrary spin-taste, which in general involves the inclusion of gauge links.
- Implement inversions with unimproved staggered fermions in Chroma, as needed for HYP-smeared staggered fermions. We expect that generating these inversions will require stripping down some previously written code and reintegrating it into the libraries. (The bilinear code for asqtad fermions will work also for HYP-smeared fermions.)
- Extend the bilinear calculations to the non-exceptional case with of different incoming and outgoing momenta in the bilinears. This may reduce systematic errors and will certainly provide an important check.
- Develop code to contract four propagators together with staggered four-fermion operators of arbitrary spin and taste and involving gauge-links. This is required to calculate matching factors for kaon mixing matrix elements.

We our aiming to complete the first and two steps during the present allocation year, with the remaining two steps occurring during the next allocation year. We stress that, since we save all propagators and gauge-fixed lattices, the extension from bilinears to four-fermion operators will not require further inversions (which dominates our use of CPU time). As code is developed we can go back and reuse propagators.

4 Status of 2008-9 running and Preliminary Results

Our running has gone more slowly than anticipated due to two factors. First, we needed to spend more time than anticipated understanding systematics and checking results before proceeding to production runs (which is only starting now). Second, we have encountered problems running on the Fermilab cluster. In particular, copy failures wiped out jobs, including a 2 week delay while one problem was fixed. Although for the most part the Fermilab site has been responsive, we estimate that we have lost at least 1 month of running from these problems. We have also not been able to push jobs through the queues as expediently as we had hoped—probably due to our inexperience with the system.

The net effect is that we are unlikely to be able to use our full allocation for this year (500,000 6n-equivalent node-hours), mainly because we will not be able to push through enough jobs. We also do not think it is prudent to rush into our proposed work using HYP-smeared quarks, so that part of the project is being postponed until the present proposal. Since we have relatively little to show in terms of results at this stage we think that it is appropriate that this should remain a class B proposal, although it is well aligned with the USQCD collaboration goals.

Our exploratory running to date has mainly taken place on the coarse lattices with $a \approx 0.125\,\mathrm{fm}$ of size $20^3 \times 64$ and with $m_\ell : m_s = 0.01 : 0.03$. We have investigated a wide range of momenta (both near the "diagonal" of the Brillouin zone and off-diagonal), and valence masses ranging from $am_v = 0.005 - 0.05$. We also have less extensive running on fine lattices. We have found that the use of momentum sources leads to results which vary little between configurations, and that as few as 4 configurations (or perhaps even 1!) are sufficient for systematic errors to dominate over statistical.

As examples of our results we show $M(p)$, $Z'_q(p)^{-1}$ and $Z_S(p)^{-1}[= Z_m(p)]$ from 4 coarse lattices for $am_v = 0.01$, 0.02 and 0.03 in Figs. 1, 2 and 3, respectively. For these lattices, with $1/a \approx 1.6\,\mathrm{GeV}$, the "window" in which non-perturbative and discretization effects are small ($|p| \gg \Lambda_{\mathrm{QCD}}$ and $|ap| \ll 1$) should lie around $(ap)^2 = 1$. In this region the results for M and $Z_q'^{-1}$ have very small statistical errors. After linear extrapolation to $m_v = 0$, M has a non-zero contribution that fits well, for $(ap)^2 \geq 0.8$, to the expected $1/p^2$ form of the non-perturbative contribution [see Eq. (4)]. $Z_q'^{-1}$ is essentially mass-independent for $ap \approx 1$, and has a value close to unity, with the slow growth with p being qualitatively consistent with the sign of the anomalous dimension. Finally, Z_m shows more substantial errors and grows as m_v decreases, attaining a value $\approx 2.3 - 2.4$ in the chiral limit for $ap = 1$. Its decrease with p is again of the expected sign.

Clearly we have much work to do in the analysis of this data—taking out the perturbative logarithmic dependence on ap to expose any underlying $(ap)^2$ terms, carefully extrapolating to the chiral limit, etc.. We also are aware that using partially quenched (PQ) data, as in these figures, can lead to additional systematics [such as a $1/(mp^2)$ term in Z_m from PQ chiral logs in the condensate]. But we have now thoroughly checked these results and are ready to extend them in production runs on more lattices, different sea quark masses, and to the fine lattices.

We were initially surprised at the large size of Z_m. How can this be consistent with the expectation that asqtad quarks are improved? To answer this, we note that there is an accumulation of factors of ~ 1.2 that builds up to an expected value close to 2:

$$Z_m(RI, 1.6\,\mathrm{GeV}) \equiv am(RI, 1.6\,\mathrm{GeV})/(am_0) \tag{9}$$

$$= \underbrace{\frac{m(RI, 1.6\,\mathrm{GeV})}{m(RI, 2\,\mathrm{GeV})}}_{\approx 1.15} \underbrace{\frac{m(RI, 2\,\mathrm{GeV})}{m(\overline{\mathrm{MS}}, 2\,\mathrm{GeV})}}_{\approx 1.25} \underbrace{\frac{m(\overline{\mathrm{MS}}, 2\,\mathrm{GeV})}{m_0}}_{1.171/u_0} \tag{10}$$

$$\approx 1.15 \times 1.25 \times (1.171/0.86774) \tag{11}$$

$$= 1.94\,. \tag{12}$$

Here am_0 is the bare lattice mass (using the standard MILC definition of the asqtad action) and u_0 the tadpole factor. The running of quark masses is standard, the ratio of quark masses in the RI and $\overline{\mathrm{MS}}$ schemes is the two-loop result from Ref. [14], and the ratio of $\overline{\mathrm{MS}}$ mass to bare quark mass is the two-loop result from Ref. [5]. Thus our non-perturbative result of $2.3 - 2.4$ is only 20% larger than the two-loop expectation. This is not an unreasonable or unexpected size for the combined effect of higher orders and $O(a^2p^2)$ effects.

This result, if it survives further scrutiny, is of phenomenological importance, as, on its face, it implies a strange quark mass (in the $\overline{\mathrm{MS}}$ scheme at 2 GeV) closer to 110 MeV than 90 MeV. The remainder of this allocation year's effort will be devoted to investigating this question. Most crucial to obtain results on fine lattices, since it is possible that we are introducing significant $O(a^2p^2)$ effects by using NPR. We are focusing on this now. We plan to take advantage of the need for smaller ensembles than we proposed last year (e.g. 16 instead of 50 lattices) to do a more extensive investigation of systematic errors by using more than the 3 proposed quark masses and 6 proposed momenta, and by doing both unquenched and partially quenched calculations.

5 Proposed calculations for 2009-10

Our major proposal is to carry out NPR for HYP-smeared bilinears and four-fermion operators on both coarse and fine MILC lattices.

We also hope to investigate the use of non-exceptional momenta for asqtad bilinears, and, if successful, to incorporate this into our HYP-smeared calculations as well. This second aim will involve reusing propagators and so adds little to the computational costs, although significant human time will be needed.

Our experience this year indicates that we need to be nimble and cannot determine in advance the optimal way to use the computational resources in order to minimize the overall error. For example, since this proposal involves the extension from bilinears, where there is no operator mixing in most cases, to four-fermion operators, where there is operator mixing in all cases, it is likely that statistical errors will become more of an issue. Conversely, experience gained from this year may allow us to use less momenta. In light of this we present below a possible allocation of resources, with the aim of showing that significant work can be done within the class B constraints, but propose to adapt this allocation appropriately as the calculation proceeds.

As a first comment, we note that the estimated timings given in our 2008-9 proposal have proven accurate, or, in some cases, slightly conservative. It turns out (not surprisingly) that convergence is more rapid for larger than for smaller momenta, and our timings were based on trial runs on the latter. Overall, we can give accurate determinations of timing required for asqtad fermions.

On the other hand, since we do not have the unimproved staggered code in hand, we can only estimate its speed relative to that for asqtad fermions. Given the relative simplicity of the unimproved action (with only a single link needed in the Dirac operator), we estimated last year that it would take half as long as for the asqtad action, and we use this factor below.

The main computational cost is for calculating propagators from momentum sources.[6] A possible (and we think reasonable) plan is, on both coarse ($20^3 \times 64$ at $a \approx 0.125\,\mathrm{fm}$) and fine ($28^3 \times 96$ at $a \approx 0.09\,\mathrm{fm}$) lattices, to calculate propagators for

- 3 values of the sea-quark (probably for $m_\ell : m_s = 0.2, 0.4, 0.6$—we recall that what matters for NPR is that $am \ll 1$ rather than the closeness to the chiral limit);
- 16 lattices from each ensemble;
- 3 valence quark masses (chosen to approximately match the sea quark masses);
- and 18 values of momenta (we have in mind 9 to map out the momentum dependence, and a second 9 to allow calculations at non-exceptional points).

The calculation for the 3 valence masses for a single physical momentum (which involves 16 lattice momentum sources) takes (for asqtad fermions) ≈ 90 6n-equivalent node-hours on the coarse lattices, and ≈ 590 such hours on the fine lattices.[7] Combining these factors, and including the factor of 2 estimated speed-up for HYP-smeared fermions, gives the estimates collected in Table 1

[6]Gauge fixing and HYP-smearing the lattices costs less than 1% of the time for inversions, and we do not include it explicitly in the following.

[7]The relative factor agrees with the expected proportionality to $V/(am)$.

lattices	size	total (6n-hrs)
coarse	$20^3 \times 64$	39K
fine	$28^3 \times 96$	249K
total		288K

Table 1: Estimated time for calculating HYP-smeared propagators

The calculation of bilinear and four-fermion vertices also requires significant computation time, given the large number of operators involved. Taking the example of the fine lattices, our present bilinear code requires 1.1 6n-equivalent node-hours for each choice of external *physical* momenta. We estimate that calculating for all bilinears of interest will require 10 times this, or 11 6n-equivalent node-hours, and that the four-fermion operator calculation (which combines bilinears in two different gauge invariant ways) will require another factor of 4, or 44 6n-equivalent node-hours. Using these estimates, the calculations described above would require 29000/114,000 6n-equivalent node-hours for the bilinears/four-fermion operators on the fine lattices, and about 1/4 of this on the coarse lattices, for a total of 177000 6n-equivalent node-hours.

We collect our total estimates in Table 2. We include also an estimate for the calculation of asqtad bilinear vertices at non-exceptional momenta. In total, we request **500,000 6n-equivalent node-hours**.[8] Our strong preference would be that any allocation be be granted on the Fermilab clusters.

calculation	cost (6n-hrs)
HYP-smeared inversions	288K
HYP-smeared vertex calculations	177K
Asqtad non-exceptional bilinear vertices	35K
total	500K

Table 2: Total time estimate (6n-hrs)

6 Storage requirements

As noted above, we plan on storing both the gauge-fixed lattices and the propagators from momentum sources. Only the latter require significant resources. For each lattice momentum (i.e. 1/16th of the physical propagator), the propagators require 36(145) Mbytes on the coarse(fine) lattices. Using this, our estimate of the required storage at the end of this year ("asqtad propagators"), and our likely total storage needs for this proposal ("HYP-smeared propagators"—assuming the proposed calculations described above) are given in Table 3.

We are presently using volatile disk space with long term storage on tape. In the table we include an estimate assuming that 10% of our data uses disk space at any time, which we think

[8]We also note that a small scale calculation on $48^3 \times 144$ superfine lattices is also possible, if it turns out that the coarse and fine calculations require less resources (e.g. less values of momenta). This would be of considerable interest as B_K is also being calculated on these lattices by Jung *et al.*. The relative cost compared to fine lattices, at the same quark masses, is ≈ 13. Thus a calculation with 5 momenta, on 4 lattices at each of 2 sea quark masses would require about 150,000 6n-equivalent node-hours.

is very conservative. The total for tape and disk is **30,000 6n-equivalent node-hours**.

data	total (Tbytes)	cost–Tape (hrs)	cost–Disk (6-n hrs)
Asqtad propagators	4.0	5K	5K
HYP-smeared propagators (coarse)	1.5	2K	2K
HYP-smeared propagators (fine)	6.0	8K	8K
total	11.5	15K	15K

Table 3: Total storage estimates.

7 Sharing and exclusivity

We are happy to make the lattices and propagators available to the collaboration if there is interest. We ask for exclusivity for the calculations of bilinear or four-fermion operator matching factors outlined above, for a period of 6 months after the propagator calculations have been completed.

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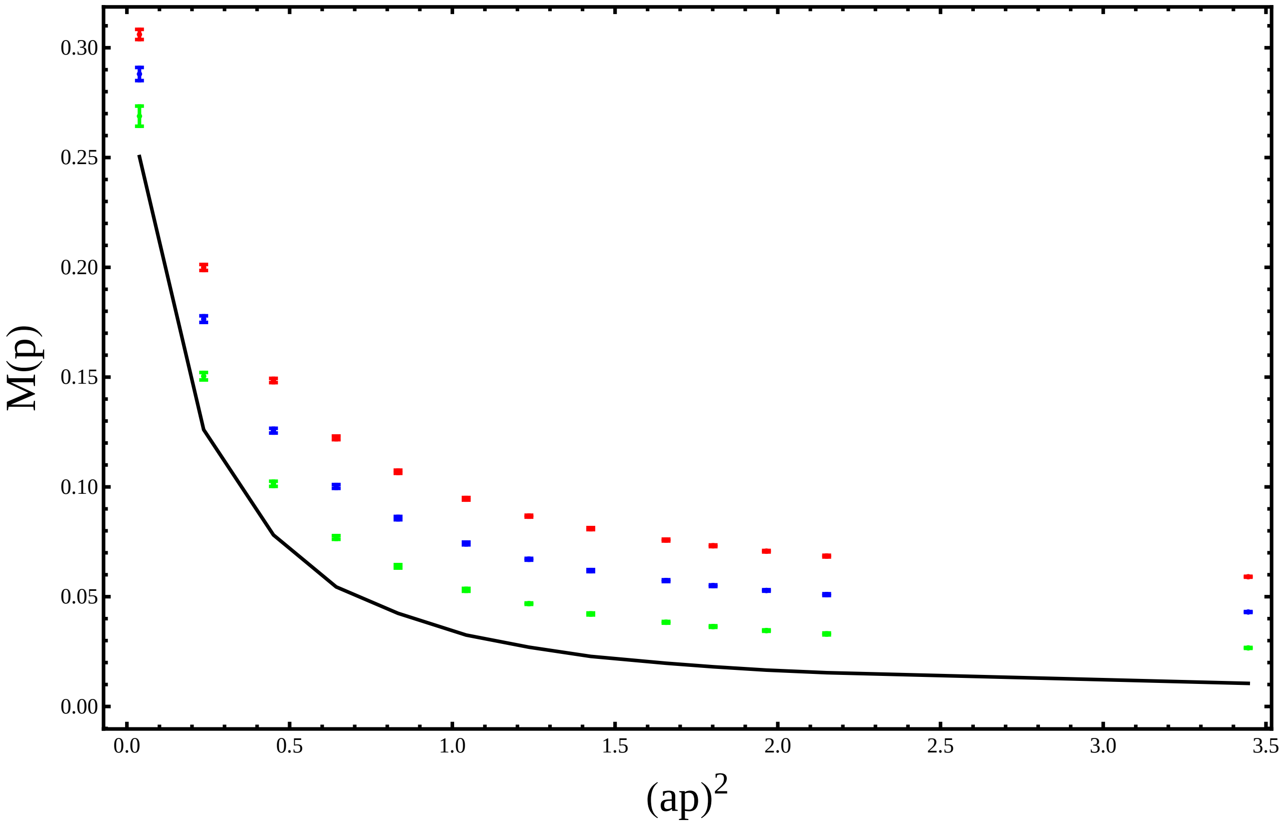

Figure 1: $aM(p) = \mathrm{Tr}(S(p)^{-1})$ versus $(ap)^2 \equiv a^2 \sum_\mu p_\mu^2$ for valence masses $am = 0.03$ (red), 0.02 (blue) and 0.01 (green), together with the linear extrapolation to the chiral limit (points joined by black curve–no error shown). Results are from the coarse MILC lattice with $am_\ell = 0.01$ and $am_s = 0.03$.

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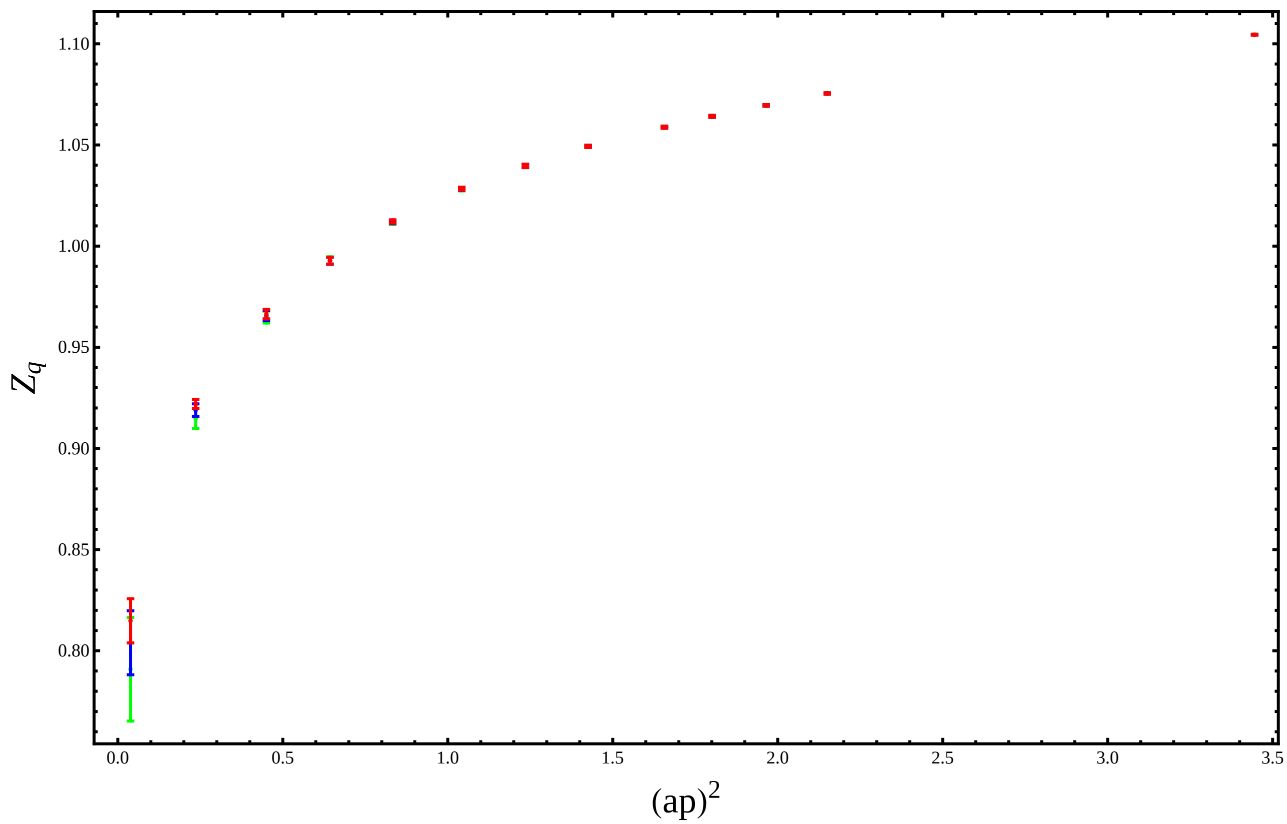

Figure 2: Quark wavefunction renormalization Z_q versus $(ap)^2$ for valence masses $am = 0.03$ (red), 0.02 (blue) and 0.01 (green). Four $(ap)^2 > 0.5$ the points lie on top of one another. Results are from the coarse MILC lattice with $am_\ell = 0.01$ and $am_s = 0.03$.

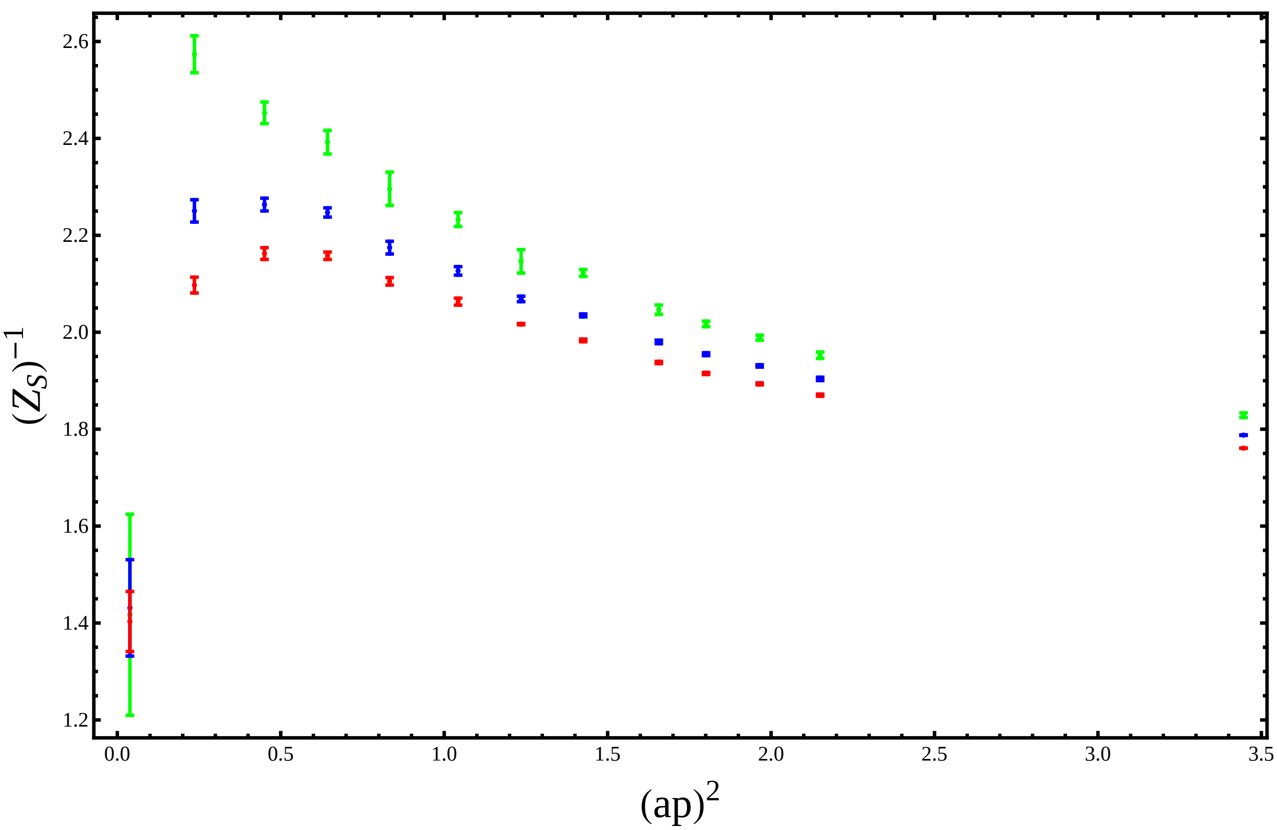

Figure 3: $Z_S^{-1} = Z_m$, determined from the scalar bilinear, plotted versus $(ap)^2$ for valence masses $am = 0.03$ (red), 0.02 (blue) and 0.01 (green). Results are from the coarse MILC lattice with $am_\ell = 0.01$ and $am_s = 0.03$.