Pion multiplet spectrum using improved staggered fermions

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Weonjong Lee, Lattice 2006

Collaboration

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Pion multiplet spectrum

- Motivation:

  1. Contribution from non-Goldstone pions to B_K is more important than that from Goldstone pions.
  2. We need to calculate pion multiplet spectrum for B_K.
  3. Improvement is expected to reduce splitting among pion multiplets.

- Effect:

  1. Allow full fitting to the form of staggered χPT.
  2. Perform more precise analysis of B_K data.
  3. Determine low energy constants more precisely.

Cubic Symmetry:

- It is important to implement cubic symmetry in both source and sink operators of the two-point correlation functions in order to reduce the noise.

How to implement cubic symmetry in the sources

- We choose U(1) noise sources to implement the cubic symmetry in the source side.
- Coulomb gauge fixing for all time slices.

- Cubic U(1) Source:

$$(D+m)x = y$$
$$y(t=0, 2\vec{n}+\vec{A}, c) = \xi(n,c) \in U(1)$$
$$\lim_{N\to\infty} \frac{1}{N} \sum_{\xi} \xi(\vec{n},c)\xi(\vec{n}',c') = \delta_{\vec{n},\vec{n}'}\delta_{c,c'}$$

- Cubic Wall Source:

$$(D+m)x = y$$
$$y(t=0, 2\vec{n}+\vec{A}, c) = \xi(c) \in U(1)$$
$$\lim_{N\to\infty} \frac{1}{N} \sum_{\xi} \xi(c)\xi(c') = \delta_{c,c'}$$

Operator Transcription:

How to implement cubic symmetry in the sink operator

- Kluberg-Stern method (approximate representation):

$$\mathcal{O}_{S,F} = \bar{\chi}(A)\overline{(\gamma_S \otimes \xi_F)}_{A,B}\chi(B)$$

$$\overline{(\gamma_S \otimes \xi_F)}_{A,B} \equiv \frac{1}{4}\mathrm{Tr}(\gamma_A^\dagger \gamma_S \gamma_B \gamma_F^\dagger)$$

- Golterman method (true representation):

Example: operators local in time

$$\mathcal{O}_{S,F} = \eta_{S,F}(x)\bar{\chi}(x) M_{S,F}\chi(x)$$

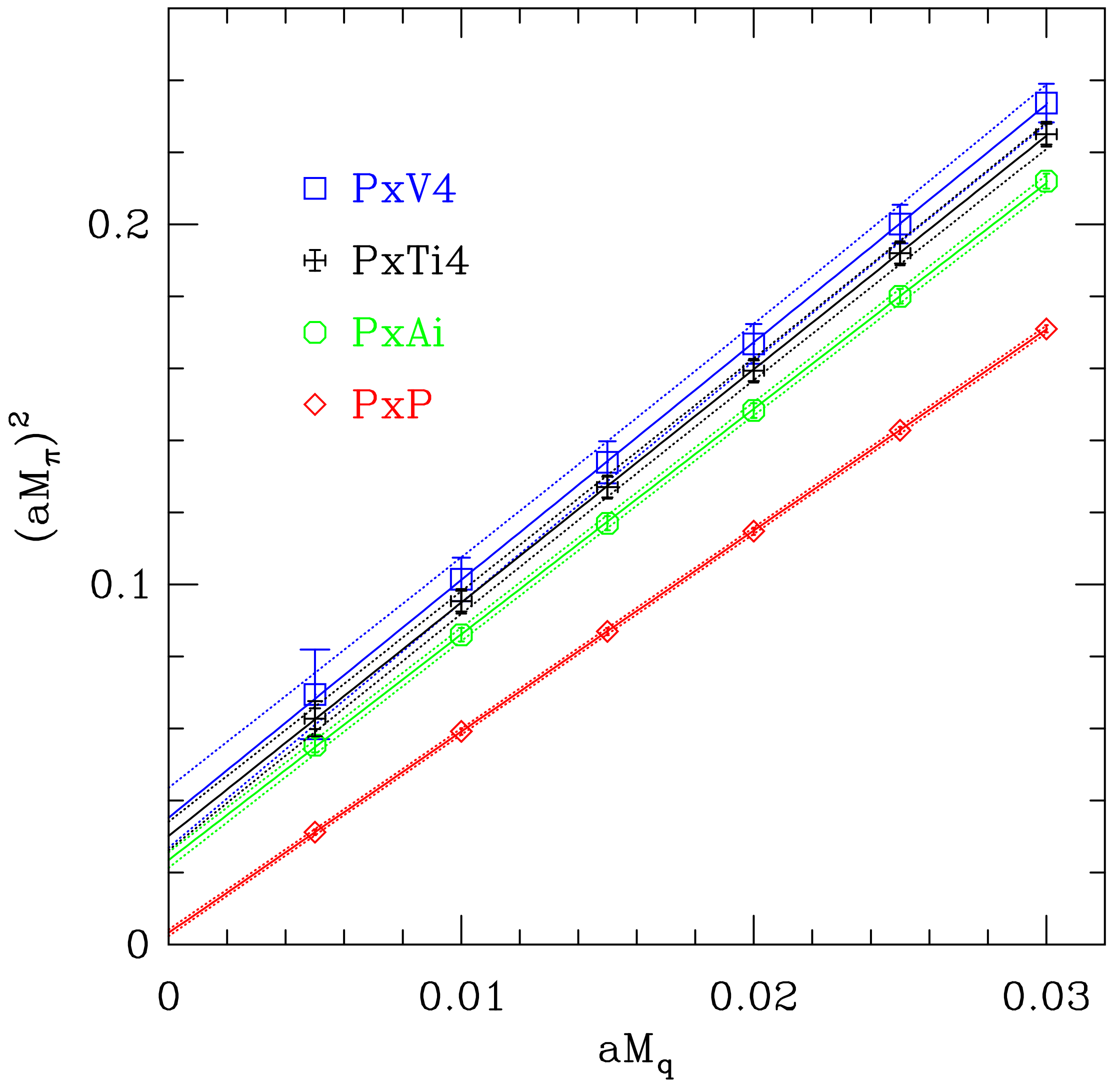
$$M_{S,F}\chi(x) = \prod_{\mu=1,2,3} [(1 - |S_\mu - F_\mu|) + |S_\mu - F_\mu| D_\mu]\chi(x)$$

$$D_\mu \chi(x) = \frac{1}{2}(\chi(x+\mu) + \chi(x-\mu))$$

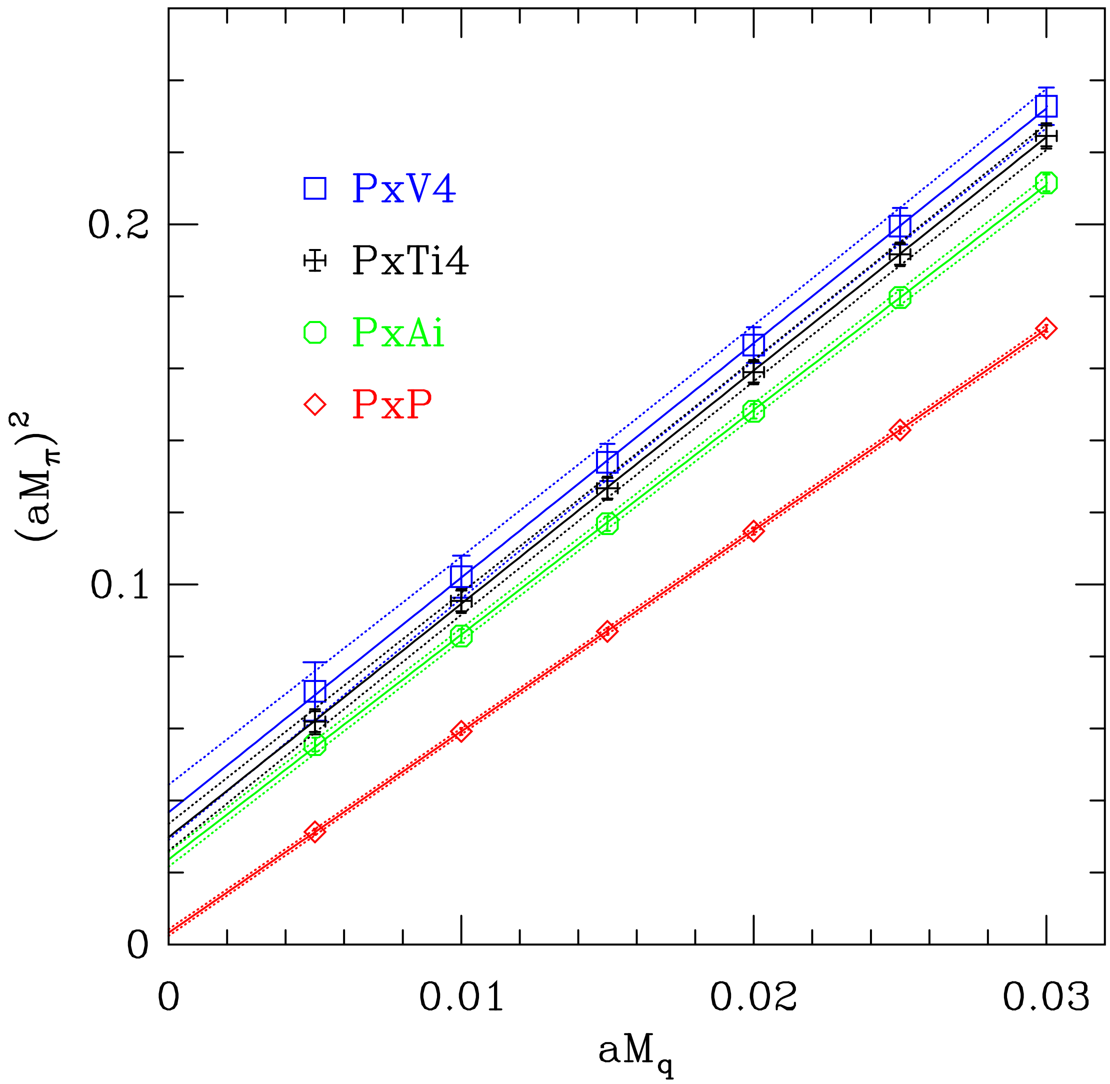
Simulation Parameters for Unimproved Staggered Fermions

parameter	value
β	6.00 (quenched)
lattice geometry	$16^3 \times 64$
# of configurations	187
$1/a$	1.95 GeV
gauge fixing	Coulomb
quark mass	$0.005, 0.01, 0.015, 0.02, 0.025, 0.03$
Z_m	≈ 2.5

Kluberg-Stern Method (Unimproved)



Golterman Method (Unimproved)



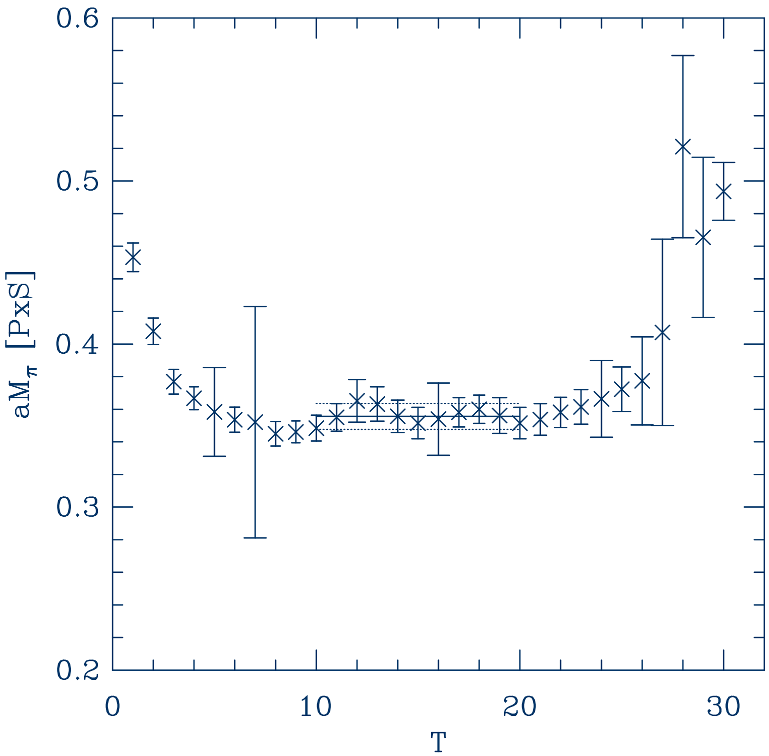
Summary (unimproved stag)

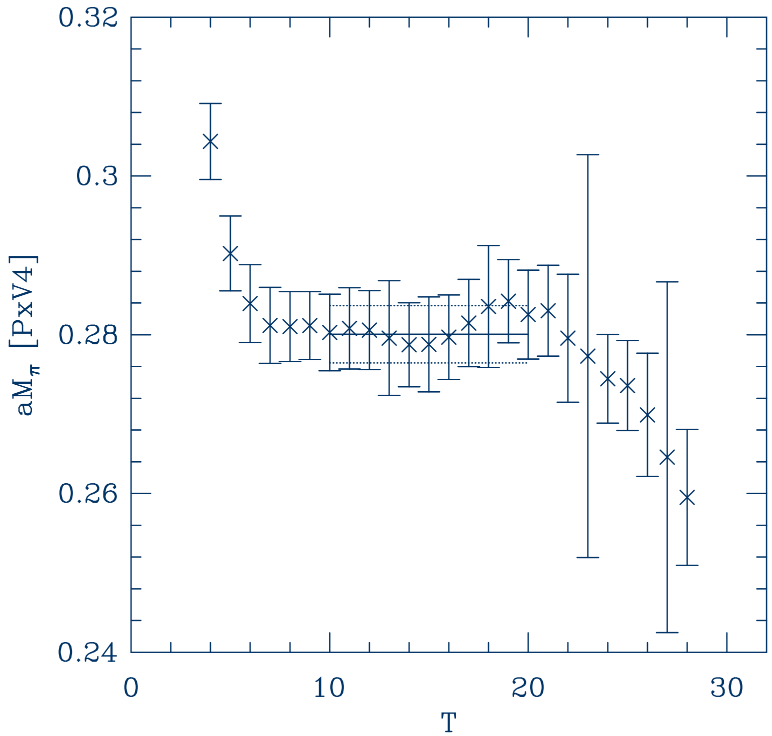
- No difference between Kluberg-Stern and Golterman methods.
- No difference between cubic wall and cubic U(1) sources.
- The results are consistent with those of Aoki, et al.
- We observe a noticeable effect of $\mathcal{O}(a^2p^2)$.

Simulation Parameters for HYP Stag

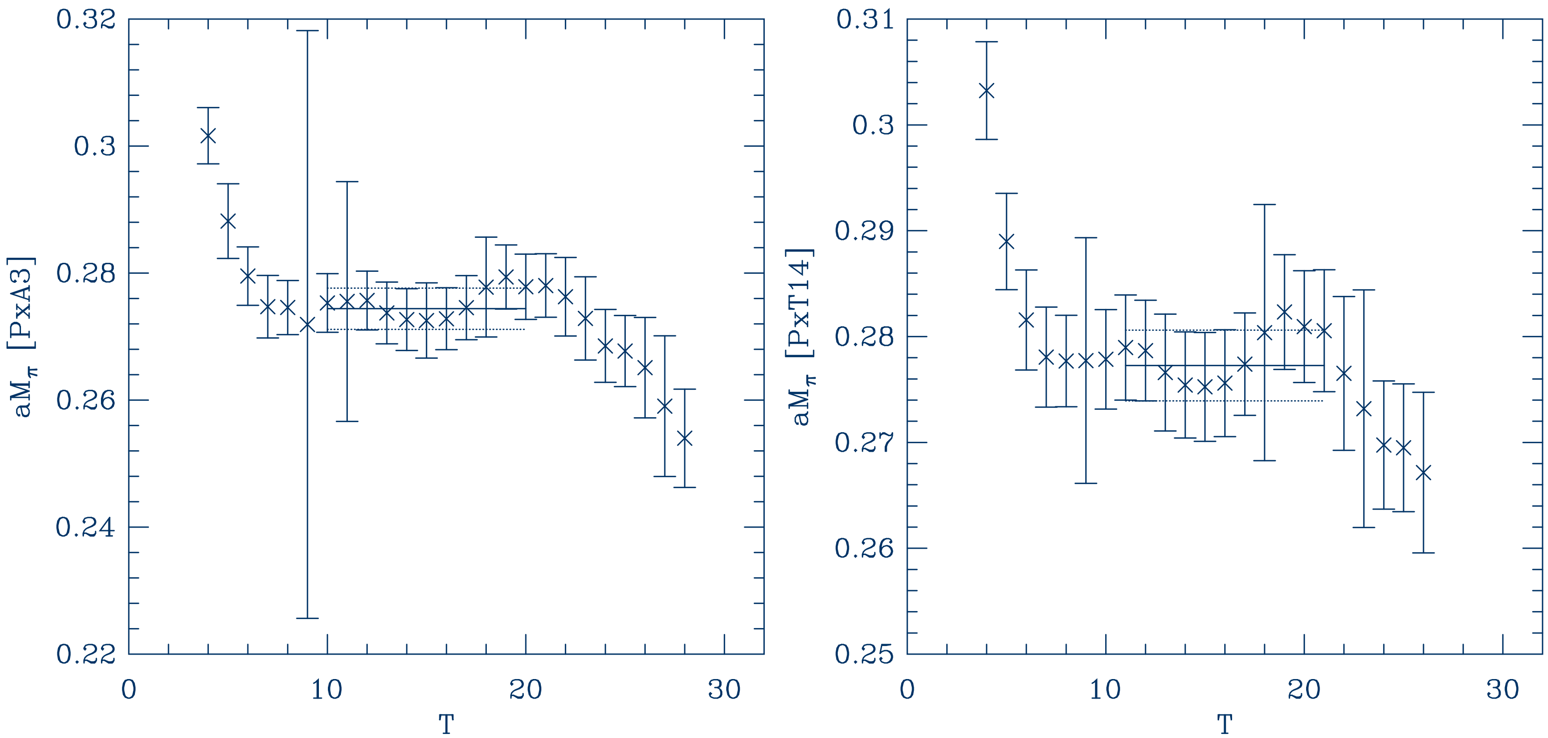
parameter	value
β	6.00 (quenched)
lattice geometry	$16^3 \times 64$
# of configurations	218
$1/a$	1.95 GeV
gauge fixing	Coulomb
quark mass	$0.01, 0.02, 0.03, 0.04, 0.05$
Z_m	≈ 1

Eff. Mass Plot ($[P \times S]$, $[P \times V_4]$)

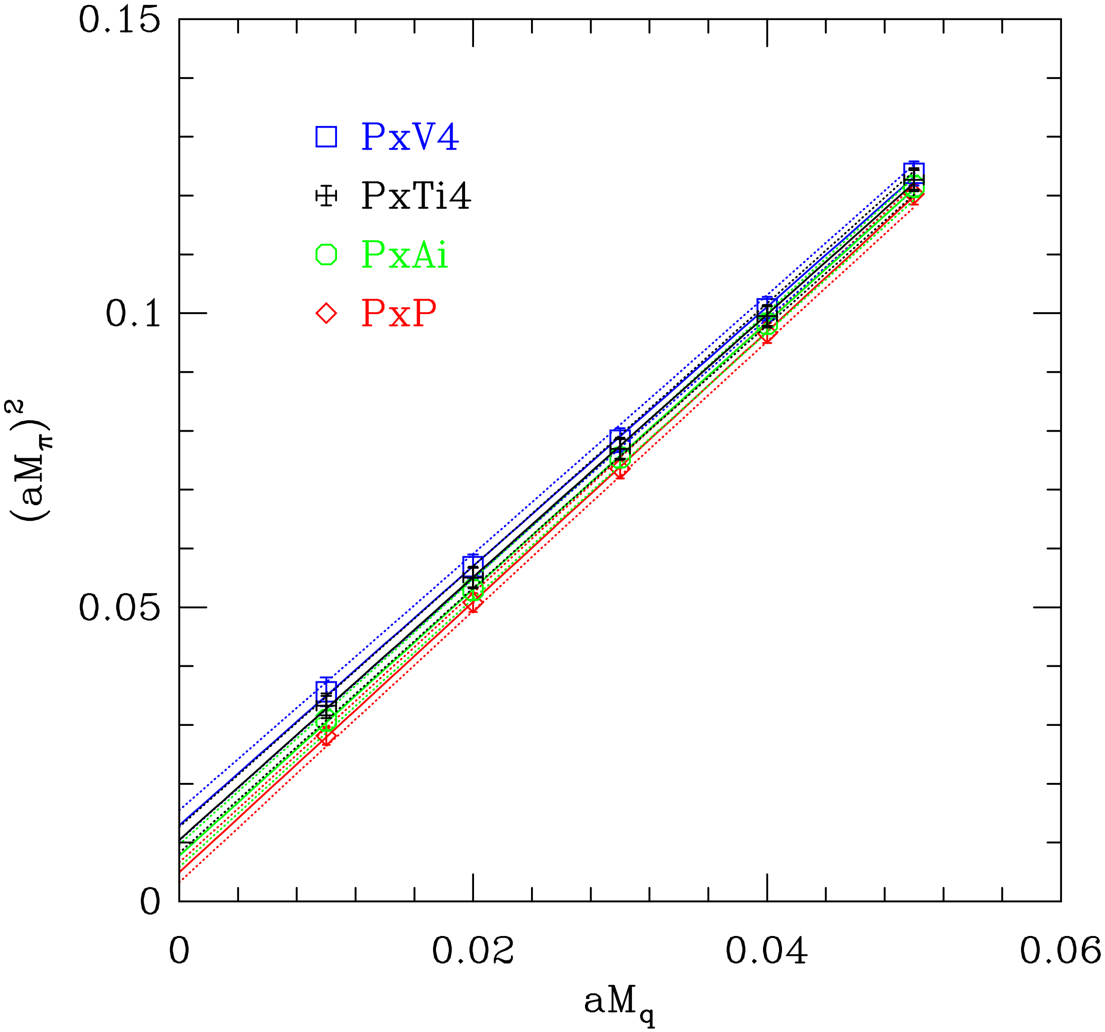




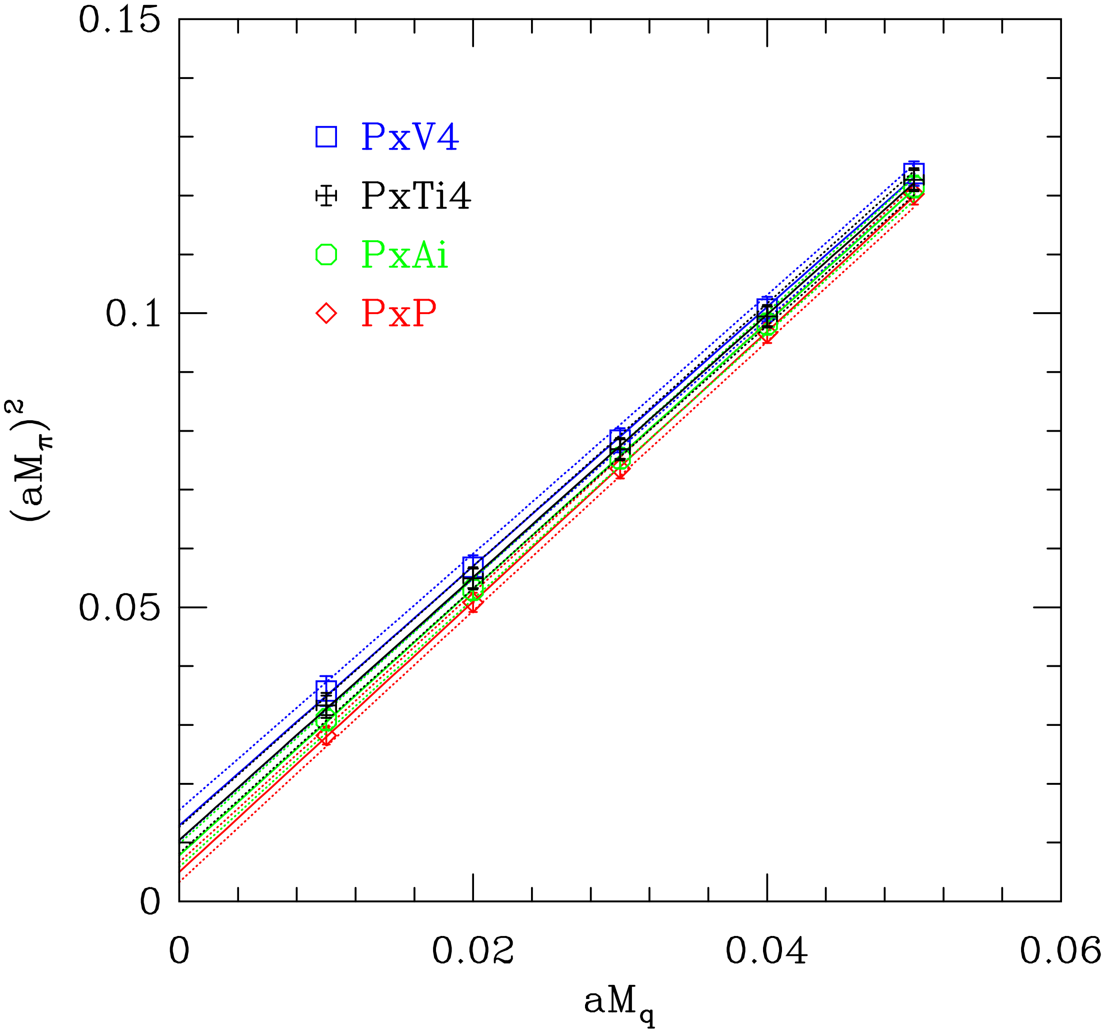
Eff. Mass Plot ($[P \times A_3]$, $[P \times T_{14}]$)



Local Kluberg-Stern CU1 (HYP)



Local Golterman CU1 (HYP)



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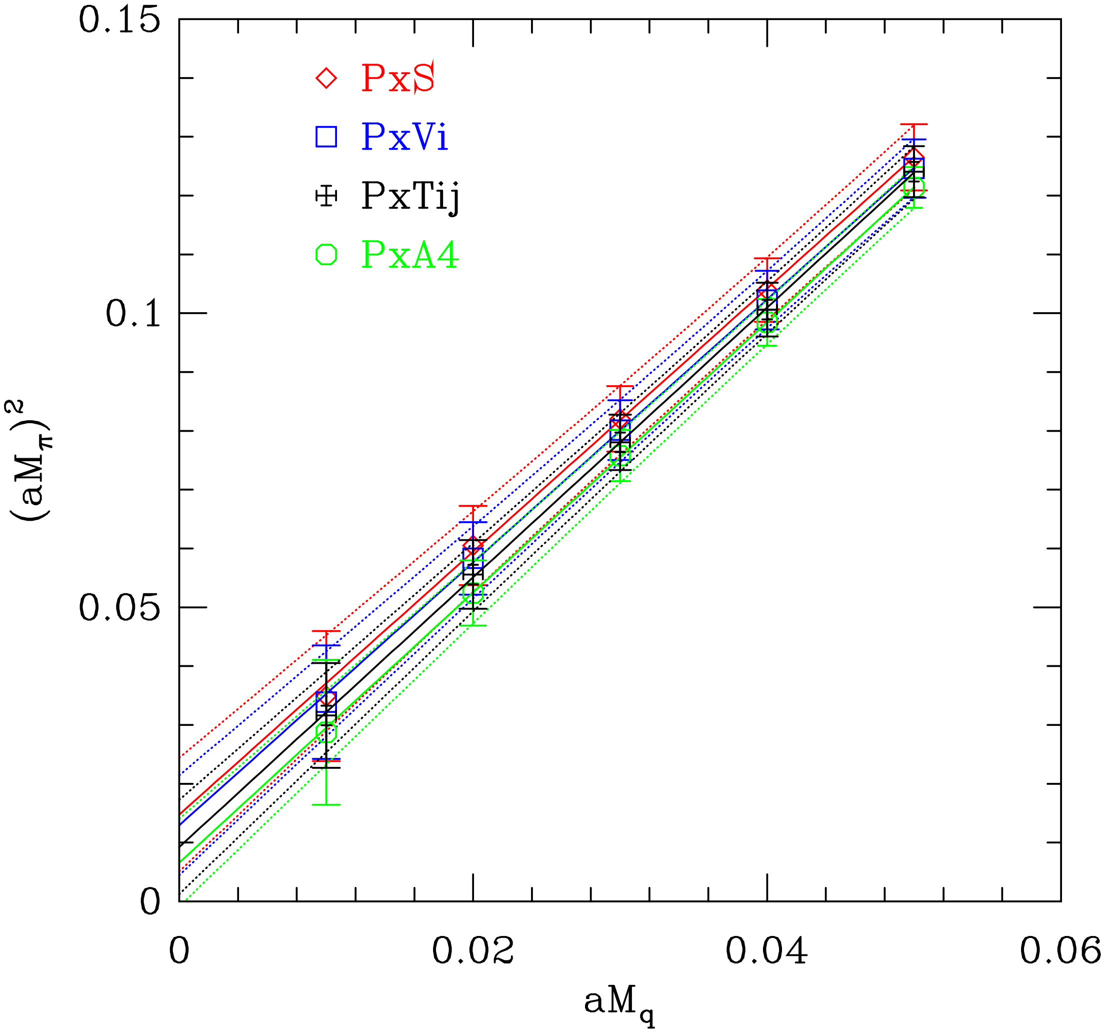
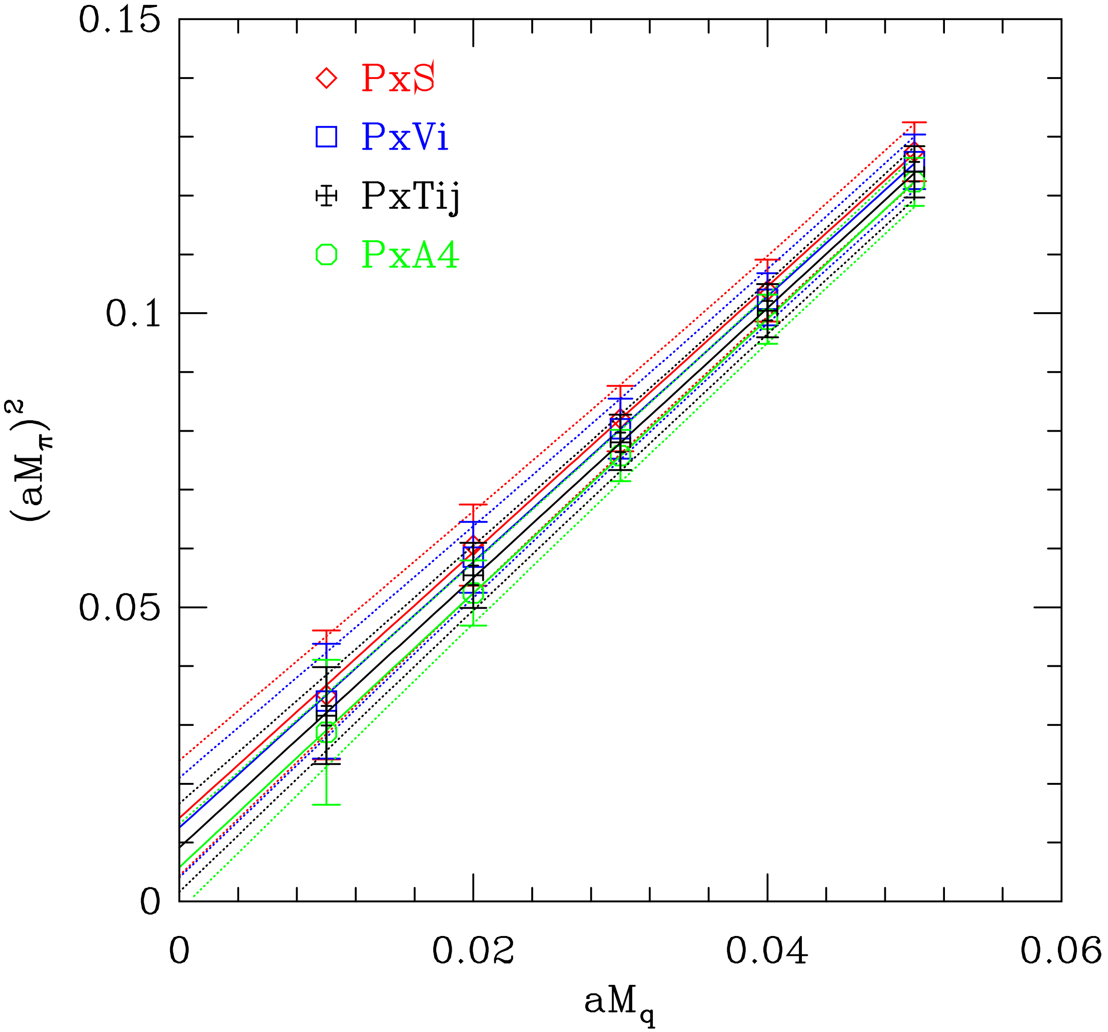
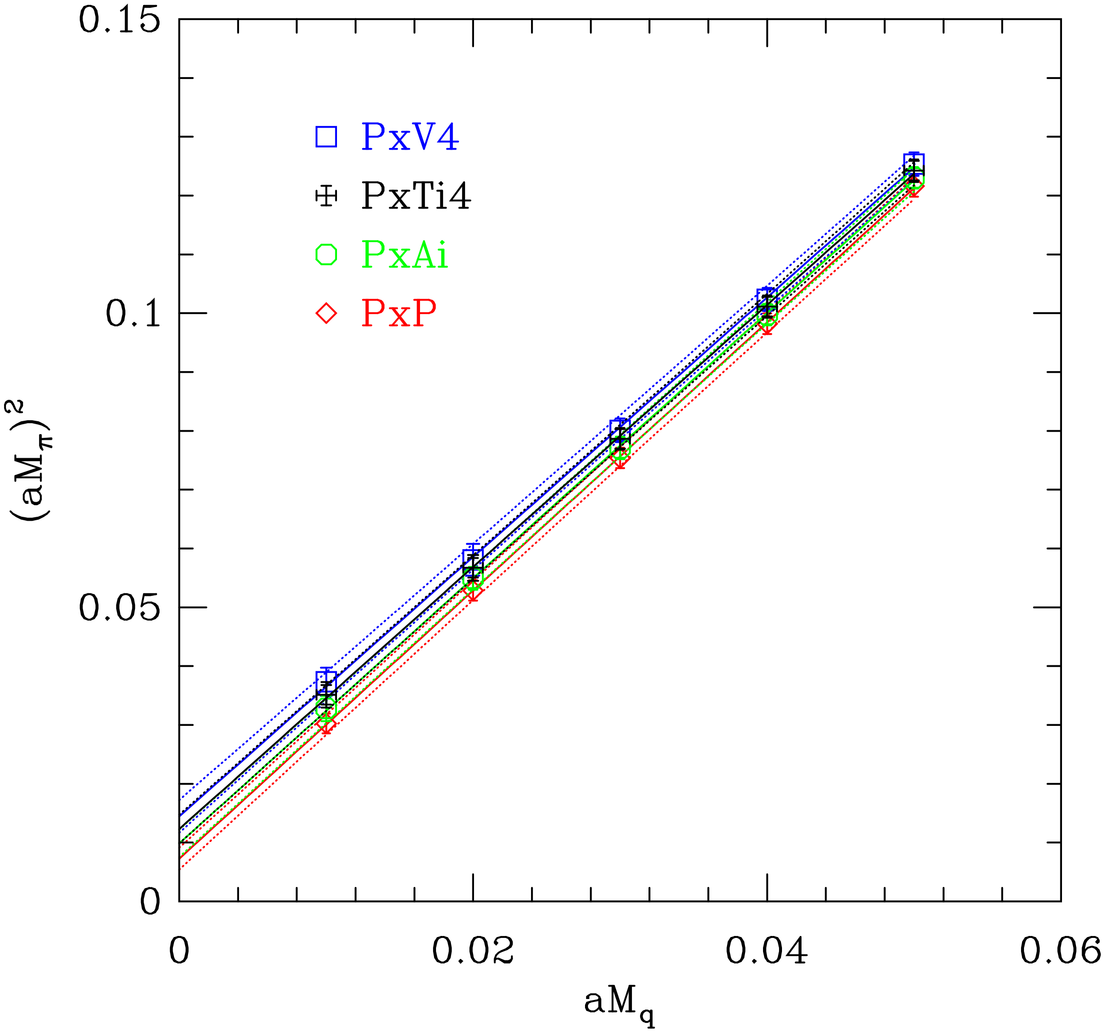
Non-local Kluberg-Stern CU1 (HYP)



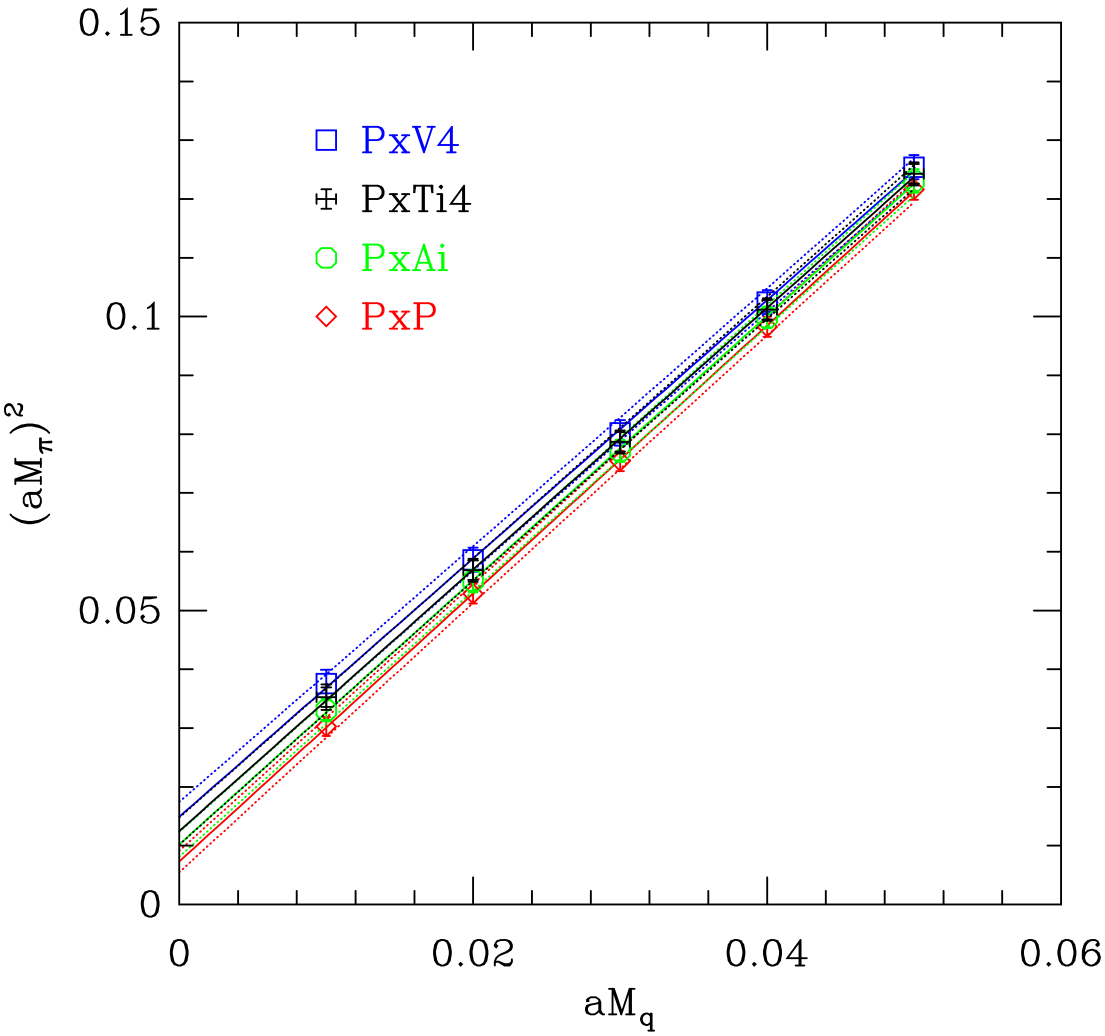
Non-local Golterman CU1 (HYP)



Local Kluberg-Stern CW (HYP)



Local Golterman CW (HYP)



Simulation Parameters for MILC AsqTad

parameter	value
β	6.76
lattice geometry	$20^3 \times 64$
# of configurations	640
$1/a$	1.588(19) GeV

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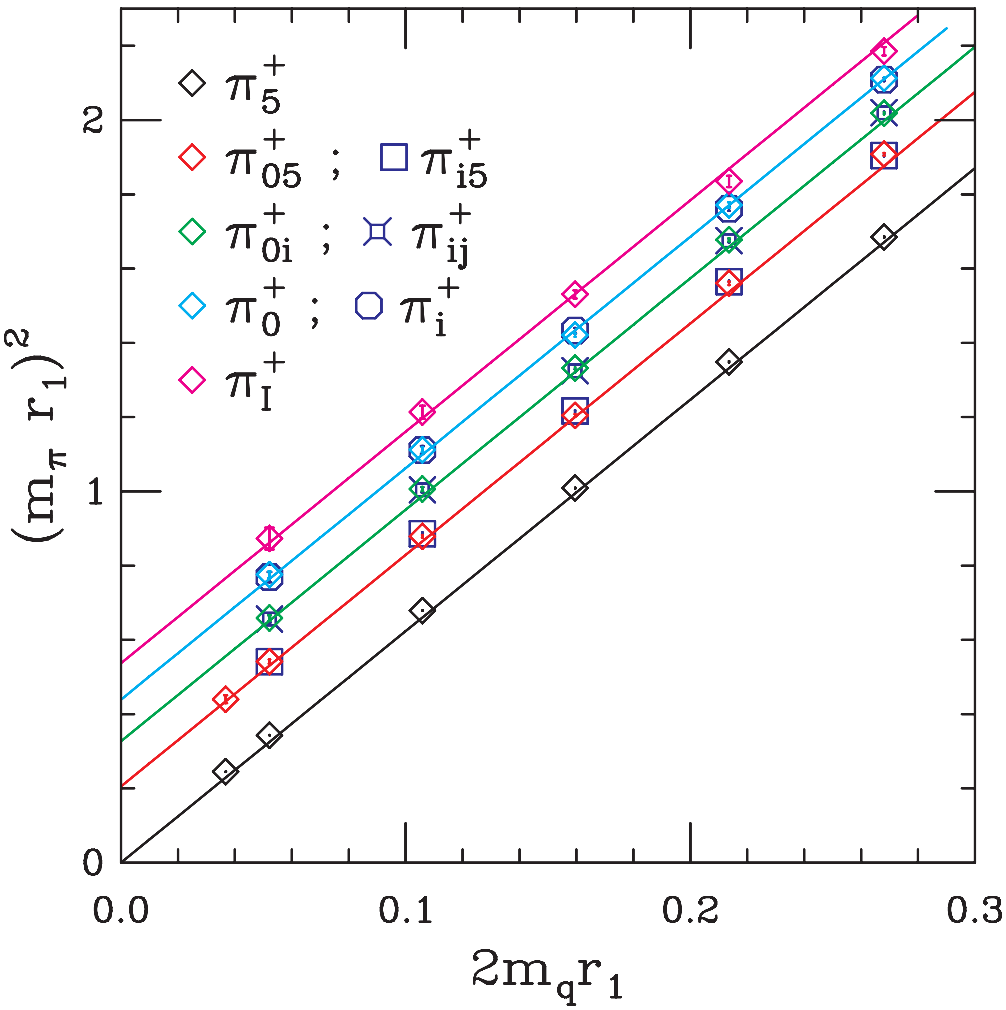
MILC coarse lattice (AsqTad)



Summary and Conclusion

- We need to calculate pion mutiplet spectrum in order to use full fitting function for B_K suggested by staggered chiral perturbation theory.
- When we use unimproved staggered fermions, we observe different slopes which comes from the $\mathcal{O}(a^2p^2)$ effect (consistent with Aoki, et al).
- In the case of AsqTad, the $\mathcal{O}(a^2)$ splitting among the pion multiplets are so noticeable that the size is larger than the light pion masses.
- In the case of HYP, the $\mathcal{O}(a^2)$ splitting among the pion multiplets are so small that the size is much smaller than that of the light pion masses.

- In the case of AsqTad and HYP, the $\mathcal{O}(a^2 p^2)$ effect are suppressed so much that this effect is negligible.
- From this study, we conclude that the HYP improvement scheme is more efficient to reduce the taste symmetry breaking effect and reduce other discretization errors.
- We plan to measure pion multiplet spectrum and use them to fit B_K data to the form suggested by staggered χPT.