

B_K and related matrix elements with unquenched, improved staggered fermions

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Abstract

This is a class A proposal, requesting time on the QCDOC to continue our calculation of kaon matrix elements using HYP-smearred improved staggered quarks. (The project web site is <http://www.phys.washington.edu/users/sharpe/qcdoc/index.html>.) Our major focus will be on a calculation of B_K , which is of particular interest for constraining elements of the CKM matrix. The present (2008-9) allocation should allow completion of work on the MILC fine lattices ($a \approx 0.09$ fm) and the $m_\ell/m_s = 0.2$ superfine ensemble ($a \approx 0.06$ fm). We give a fairly extensive status report on our analysis, showing that $SU(2)$ chiral fitting gives results with 1 – 2% errors. Encouraged by this, we propose to extend this work by adding the lighter sea quark superfine lattice ensemble ($m_\ell/m_s = 0.1$) and by extending the calculation to the $m_\ell/m_s = 0.02$ ultrafine lattices ($a \approx 0.045$ fm).

Graduate students and postdoctoral associates at both the University of Washington and Seoul National University will contribute to this project.

We are requesting 29.5 Mnode-hours on the QCDOC, but are also able to move some of the running to the clusters if needed.

1 Scientific Background

A major goal of lattice QCD is to calculate electroweak matrix elements. This would allow precision tests of the standard model (SM), and is one of the major objectives highlighted in the USQCD white papers and proposals. Here we propose to continue our study of B_K and related hadronic matrix elements using HYP-smearred staggered valence quarks [1] on the existing MILC dynamical “asqtad” lattices. This proposal fits in the “type A” category because it will produce physics results that are listed among the Collaboration’s strategic goals.

The impressive work of our experimental colleagues has made it possible to constrain the angles of the unitarity triangle using hadronic B -decays alone, without theoretical input from the lattice. In particular, one can now use the measured value of CP violation in $\bar{K} - K$ mixing, plus knowledge of the CKM angles, to predict [2]

$$\widehat{B}_K(\text{UT prediction}) = 0.75 \pm 0.07, \quad (1)$$

(This is the “UTangle + V_{ub}/V_{cb} ” fit.) Thus a precise lattice result for B_K will test the SM.¹ Furthermore, if LHC finds evidence for new physics, then a lattice value of B_K constrains the nature of that physics, since it will, in general, contribute to flavor-changing processes such as $\bar{K} - K$ mixing.

There are many ongoing lattice calculations of B_K , all using $2 + 1$ flavors of dynamical fermions, but using a wide variety of fermions. The theoretically most straightforward approaches are those using domain-wall fermions (DWF) which have almost exact chiral symmetry. The most advanced calculation uses valence and sea DWF, and finds, based on results at a single lattice spacing ($a^{-1} = 1.73 \text{ GeV}$) [4]:

$$\widehat{B}_K(\text{DWF}) = 0.720(13)(37). \quad (2)$$

Here the errors are, respectively, statistical and systematic, with the latter dominated by an estimate of the discretization error. The result agrees very well with the “experimental” prediction (1). We note for comparison with our calculation described below, that the lightest sea quark mass is $m_\ell \approx m_s^{\text{phys}}/5$ while the lightest valence quark mass is $\approx m_s^{\text{phys}}/10$. Non-perturbative renormalization (NPR) yields the matching factor with a 1.4% error, and two volumes were used to control finite volume effects.

The remaining uncontrolled systematic, due to discretization errors, is being studied by repeating the calculation a smaller lattice spacing. This very large scale calculation will set the standard for control of errors.

Other calculations underway have been summarized recently by Lellouch [3], who quotes an average of

$$\widehat{B}_K(\text{Lellouch}) = 0.723(11)(35). \quad (3)$$

We note for comparison with our work that there is a (by now quite old) calculation using valence asqtad staggered quarks on the “coarse” ($a = 0.125$) MILC lattices with $am_\ell/am_s = 0.01/0.05$ and $0.02/0.05$. The result is [5]

$$\widehat{B}_K(\text{asqtad}) = 0.83(18), \quad (4)$$

¹Full utilization of a precise result for B_K will require concomitant improvements in the determination of $|V_{cb}|$ and other quantities which enter the Standard Model prediction for ϵ_K [3].

in which the error is dominated by the estimate of the size of the missing two-loop term in the perturbative matching factors. Other errors are estimated to be smaller (3% each from statistics and chiral extrapolation, and 5% from discretization errors). We aim to reduce all these errors using improved staggered fermions.

We think that it is very important for lattice calculations to provide multiple, independent checks of the systematics using, in particular, different fermion discretizations. Thus we think that it is useful to pursue the calculation with staggered fermions for both valence and sea quarks, particularly given the fact that they are relatively computationally inexpensive.

Using USQCD resources allocated to class B proposals (substantially augmented in the present allocation year) we have, for the past 2.7 years, undertaken such a calculation. We use HYP smeared staggered fermions, chosen since they reduce the dominant discretization error—taste-breaking. We describe in some detail below the results for B_K and the pion spectrum based on an extensive analysis of the data taken during the previous allocation year (using MILC coarse and fine lattices). This analysis gives us confidence that systematic errors due to chiral extrapolations can be controlled at the few percent level. We are presently extending this analysis to the additional data we have and are continuing to accumulate on the MILC coarse, fine and superfine lattices.

At the end of this allocation year, we will have completed running on the MILC fine lattices ($a \approx 0.09$ fm) and on the $m_\ell/m_s = 0.2$ superfine ensemble ($a \approx 0.06$ fm). The present proposal aims to extend the calculation to the $m_\ell/m_s = 0.1$ superfine ensemble, and the $m_\ell/m_s = 0.2$ ultrafine ensemble ($a \approx 0.045$ fm).

An important weakness of the approach is that, at present, we rely on perturbative renormalization factors—one-loop factors should be available by this summer. This problem can be alleviated by working at 2-loop or using NPR. Work in both directions is underway, with a separate proposal requesting time for the NPR calculations (Lytle and Sharpe).

Kaon matrix elements of other four-fermion operators are also of considerable interest. We restrict ourselves here to those without “eye” contractions, i.e. both quarks and both antiquarks are contracted with fields in the operators creating the external particles and not with each other. Of particular interest are the $K \rightarrow \pi\pi$ matrix elements of the operators $\mathcal{O}_{7,8}^{I=3/2}$, since these contribute significantly (particularly \mathcal{O}_8) to ϵ'/ϵ through electromagnetic penguin diagrams. Using leading order chiral perturbation theory these can be related to the $K - \bar{\pi}$ matrix elements which we propose to calculate. Beyond the standard model physics can lead to enhanced coefficients for $\Delta S = 2$ four-fermion operators not having the “left-left” chiral structure of the operator appearing in B_K . This provides additional motivation for calculating $K - \bar{K}$ matrix elements of operators having all spin structures.

Since we use the MILC asqtad configurations, we must assume that the use of a rooted fermion determinant leads to the correct continuum limit. This is plausible, based on the extensive numerical results and theoretical work summarized in Ref. [6, 7, 8].

2 Methods

We propose to continue our calculation of B_K using valence HYP-smearred quarks on MILC lattices.² Staggered fermions have the advantage of a remnant $U(1)$ partially conserved axial symmetry, which guarantees that the lattice matrix elements satisfy Ward identities analogous to those in the continuum [9]. We use HYP-smearing (introduced in Ref. [1]) because:

1. It substantially reduces the taste-breaking in the spectrum, the reduction being 2-3 times greater than with asqtad action (which is used for the sea quarks), and comparable to that from the recently introduced HISQ action [10]. Our recent paper provides details [11].
2. It is computationally simpler than both asqtad and HISQ fermions (improvement involving only HYP-smearing of the original lattice and then using the unimproved staggered action).
3. Perturbative corrections to matching factors for bilinears and four-fermion operators are maximally reduced [12].

We have considerable experience with HYP-smearred fermions, and have found that they not only reduce taste breaking [11], which is the dominant discretization error, but also see evidence that they reduce the discretization errors in general [13]. We note that we are using a mixed action, with different versions of staggered fermions for the valence and sea quarks. The impact of this can be parametrized using chiral perturbation theory.

On each (suitably chiral) MILC configuration we use 10 different valence quark masses, running approximately from m_s^{phys} down to $m_s^{\text{phys}}/10$, and calculate B_K and the Goldstone-taste “kaon” mass for all 55 mass combinations. B_K is defined by

$$\frac{8}{3}m_K^2 f_K^2 B_K = \langle \bar{K} | \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d | K \rangle. \quad (5)$$

The methodology for calculating B_K is well established [13]. The lattices are HYP-smearred, using parameters chosen to remove the taste-breaking couplings to gluons with $ak_\nu = \pi$ at tree level (the parameter set denoted HYP(II) in Ref. [12]). We use wall sources having a random $U(1)$ phase on each site and for each color, such that the combination of a quark and antiquark propagator creates only the taste ξ_5 (Goldstone) pion. We use two such wall sources, separated in time by a spacing chosen as a compromise between extending the signal region and reducing the statistical noise. For the $L_t = 64, 96$ and 144 lattices we use $\Delta t = 26, 40$ and 60 , respectively. In order to reduce correlations between configurations, the overall position of the sources is chosen randomly on each configuration. Statistics are improved by using multiple sources on each configuration, maximally spread out in time.

We use gauge-invariant operators having spin-taste structure $(\Gamma \otimes \xi_5)(\Gamma \otimes \xi_5)$, with all 16 choices of Dirac matrices Γ . An important feature is that the links in the operators are HYP-smearred (in distinction to the calculation of Ref. [5] which uses “thin” links). This reduces perturbative corrections and may also reduce the size of discretization errors [17].

Suitable subsets of the resulting data are then fit to forms predicted by staggered chiral perturbation theory [14]. We use both $SU(3)$ [15] and $SU(2)$ versions [16]. Important inputs into

²For much of the following, we use B_K as a shorthand for “ B_K and the related four-fermion kaon matrix elements discussed above”.

these fits are the masses of the valence kaons and pions with non-Goldstone tastes.³ For $SU(3)$ chiral fitting, these are required for both degenerate and non-degenerate valence quarks [15], while only the former are needed for $SU(2)$ fitting. For the fitting presented below, these masses are determined by a direct calculation of the pion spectrum, using the methods described in Ref. [11] and the results summarized in Ref. [18] (with some examples given below).

In this year's running we have calculated B_K and the Goldstone-taste M_K , but not the full spectrum (thus reducing the CPU requirements substantially). As explained below, however, we will be able to use our existing results on taste-splittings to determine those needed for fitting on these new lattices.

3 Code details

Our code is written in C++, and built on top of the Columbia CPS library. The major computation is the calculation of propagators. For the latter we use the level-3 conjugate gradient (CG) inverter (for unimproved fermions since our valence quarks do not use the 3-step Naik term). In this way, our code makes essential and efficient use of the QCD API, both for computations and communications.

In order to test for hardware problems we repeat the calculation on every 10'th configuration; if the check fails, we repeat the calculation on the previous 10 configurations.

The number of nodes needed depends on the lattice size. The determining factor is that the code requires the sites local to each node to have an even number of sites in each dimension. In our running this year the local volumes are $14^3 \times 6$ on the $28^3 \times 96$ lattices (128 nodes), $10^3 \times 12$ on the 40^3 lattices (512 nodes), and $6^2 \times 12 \times 18$, $48^3 \times 144$ lattices (2048 nodes).

In our proposed running on the $64^3 \times 144$ superfine lattices, the local volume would be $8^2 \times 16 \times 18$ on 2048 nodes, while on the $64^3 \times 192$ ultrafine lattices, the local volume would likely be $8^3 \times 24$ on 4096 nodes.

Analysis is being done on workstations at our home institutions.

The times required for the relevant lattices are collected in Table 1. A major issue with this year's running on the larger numbers of nodes (512 and 2048) has been a very significant slowdown of our code due to I/O for propagator staging. We have been adapting our code, with the result that our future running (dominated by the large lattices) will be about twice as slow as we estimated in last year's proposal. We are working on further code improvement to decrease the amount of I/O and exploring possibility of adding more disks on the QCDOC partitions we are using.

We are presently not storing the propagators, so our storage needs are minimal. We would be happy to save the propagators if there was interest in using them.

4 Running this Allocation Year

Our initial allocation for 2008-9 was for 3.96 M QCDOC node-hours. This was increased to 12 Mnode-hours in late October 2008 as part of an overall re-balancing, with the aim of allowing

³Masses of mixed valence-sea pions are not needed at one-loop order. Masses of sea-sea pions are needed and are obtained from MILC collaboration results.

a (fm)	Lattice	Nodes	Proposed Time	Actual Time	Improved Time
0.125	$28^3 \times 64$	128	55 mins	70 mins	--
0.09	$40^3 \times 96$	512	2 hours	10 hours	5 hours
0.06	$48^3 \times 144$	2048	1.6 hours	5 hours	3 hours
0.06	$64^3 \times 144$	2048	--	--	7 hours
0.045	$64^3 \times 192$	4096	--	--	6.5 hours

Table 1: CPU requirements for B_K calculation on different lattices. “Proposed Time” is from last year’s proposal (scaled to the quoted number of nodes), “Actual Time” is from this year’s running to date, while “Improved Time” applies to running starting at roughly the end of March 2009. For the last two lattices, timings are estimated by scaling linearly with the volume and inversely with the lattice quark masses.

us to speed up our calculation. Our running to date during the present allocation period has been as follows:

1. Using four 128-node QCDOC motherboards, we calculated B_K on the large volume ($28^3 \times 64$) coarse lattices with $am_\ell : am_s = 0.01 : 0.05$. We have 8 measurements of B_K on each of 274 lattices. These will be used to test the size of finite volume effects, and for additional statistics on the coarse lattices.
2. We merged the four motherboards into a single 512 node machine, and began calculating B_K on the $40^3 \times 96$ fine lattices with $am_\ell : am_s = 0.0031/0.031$. This provides a second, lighter sea quark mass which will both reduce our statistical errors on the fine lattices and provide a crucial check of our fitting strategies. As of March 9, 2009, we have completed 102 configurations.
3. In November 2008, we obtained a 4096-node partition. This turned out to be unstable due to ethernet failures. We thus divided it into two 2048-node partitions, which have been running stably ($> 70\%$ production) since mid-December 2008. We have been calculating B_K on the $48^3 \times 144$ superfine lattices with $am_\ell : am_s = 0.0036 : 0.018$. As of March 9, 2009, we have completed 457 configurations.

On each of the coarse lattices we use valence masses $am_{\text{val}} = 0.005, 0.01, \dots, 0.045, 0.05$, while on the fine lattices we use $0.003, 0.006, \dots, 0.03$, and on the superfine we use $0.0018, 0.0036, \dots, 0.018$.

The CPU time used so far is hard to determine exactly because of the hardware problems noted above. In terms of effective throughput, using the timings given above and including a 10% overhead, as of March, 9th 2009 we have used 0.3, 0.6 and 5.2 Mnode-hours on the $28^3 \times 64$, $40^3 \times 96$ and $48^3 \times 144$ lattices, respectively, for a total of about 6.1 Mnode-hours. If we were to run for 20 hours/day on the 512 and two 2048-node machines, our remaining 5.9 Mnode-hour allocation would be completed in 64 days. As an example, and assuming the improved times from above, and a 10% overhead, this will allow us to calculate B_K on an additional 600 each of the $40^3 \times 96$ and $48^3 \times 144$ lattices. Thus we should complete our calculations on these two sets of lattices during this allocation year, possibly with a second set of measurements on a subset of lattices. Table 2 includes a summary of the expected status at the end of this allocation year.

ID	a (fm)	am_l/am_s	Size	Confgs.	spectrum	B_K anal.
C1	0.12	0.03/0.05	$20^3 \times 64$	564	Y	tree
C2	0.12	0.02/0.05	$20^3 \times 64$	486	Y	tree
C3	0.12	0.01/0.05	$20^3 \times 64$	671	Y	tree
C4	0.12	0.007/0.05	$20^3 \times 64$	651	Y	tree
C5	0.12	0.005/0.05	$24^3 \times 64$	509	Y	tree
C6	0.12	0.01/0.03	$20^3 \times 64$	312	Y	In progress
C7	0.12	0.01/0.05	$28^3 \times 64$	8×274	N	In progress
F1	0.09	0.0062/0.031	$28^3 \times 96$	995	Y	tree
F2	0.09	0.0031/0.031	$40^3 \times 96$	~ 700	N	In progress
S1	0.06	0.0036/0.018	$48^3 \times 144$	$\sim 1.6 \times 650$	N	In progress
S2	0.06	0.0018/0.018	$64^3 \times 144$	650	N	Proposed
U1	0.045	0.0030/0.015	$64^3 \times 192$	650	N	Proposed

Table 2: Expected status of calculations on MILC lattices at end of allocation year, and future plans. The spectrum calculation has been done on ensembles labeled “Y”. The “ B_K anal.” column lists configurations on which a nearly complete analysis with tree-level matching has been done (as described in the text), as well as those which will be analyzed after this allocation year’s running (labeled “In progress”). Proposed runs are also shown. Additional calculations, not listed, are underway on the coarse lattices using cluster at SNU so as to reduce the statistical error.

5 Analysis and results

In this section we summarize the status of the analysis of the B_K and spectrum data. As noted above, this has been complete on most of the coarse lattices and one of the fine lattices—those with a label “Y” in the spectrum column and “tree” in the B_K column of Table 2. Once our methods are finalized, their application to the “In progress” ensembles should be straightforward.

We have focused our analysis efforts on understanding the systematic errors introduced by chiral extrapolation, discretization errors and finite volume effects, in order to understand the limitations on precision caused by these systematics. For staggered fermions, these errors are intertwined, and include the unphysical effects due to rooting. Furthermore, for staggered fermions the discretization errors introduce many new low-energy coefficients (LECs) into the chiral representation [15]. Thus we felt that it was essential to study them first and determine the extent that we could control them.

This effort has been at the expense of the one-loop perturbative matching calculation, so that our results presented here are with tree-level matching. The one-loop matching calculation is, however, nearing completion (with two independent calculations), and, with the successful first pass at fitting behind us, we expect to produce results with all errors estimated by this summer.

5.1 Taste-breaking in the pion spectrum

We begin by showing two relevant results from our study of the spectrum. In Fig. 1 we show the scaling of the taste-splittings

$$\Delta(T) = \lim_{m_V \rightarrow 0} \left[M_\pi^2(T) - M_\pi^2(\xi_5) \right]. \quad (6)$$

between MILC coarse and fine lattices. The expected reduction by $a^2\alpha^2 \approx 0.3$ is observed, with the pattern of taste-splittings preserved. We recall also that these splittings are 2.5 – 3 times smaller than those obtained using valence asqtad quarks.

In staggered chiral perturbation theory (SChPT), the standard power counting is $(a\Lambda_{\text{QCD}})^2 \sim m/\Lambda_{\text{QCD}}$. On the fine lattices, we find this ratio to be significantly smaller than unity even for our lightest valence pion. A quantitative measure is $\Delta(\xi_{\mu\nu})/M_\pi^2(\xi_{\mu\nu})$ which is ≈ 0.25 for our lightest valence pion. This implies that one might be able to treat discretization effects, and in particular taste-breaking, as a NLO rather than leading order (LO) contribution. In practice, we have not reordered the power counting in our expressions, but we expect that the smallness of taste-breaking on the fine lattices should lead to simpler fitting to SChPT forms.

The second result concerns the light sea-quark mass dependence of the taste splittings, which is shown for the coarse lattices in Fig. 2. The values of lattice spacing are taken from Ref. [19]. We see that, with the exception of the lightest sea quark mass (where there are likely to be more significant finite volume effects in on sea-quark pions), the taste splittings are independent of the light sea-quark mass. Such dependence is a NLO effect, and should be present at some level, but is apparently very small.

We will need these two results when we analyze the $m_\ell/m_s = 0.1$ fine and superfine results for B_K . For the former, we will take the taste splittings from the $m_\ell/m_s = 0.2$ fine lattices, while for the latter we will use Fig. 1 to estimate taste-splittings. We stress that the taste splittings will be very small on the superfine lattices, and certainly a NLO effect in SChPT, so that this somewhat imprecise determination should be sufficient. In this way we can avoid the costly calculations of the full pion spectrum on these new lattices.

5.2 $SU(3)$ PQSChPT fits to B_K

We now discuss our results for B_K . We begin with fits to $SU(3)$ PQSChPT, which was the method we originally proposed for fitting. We use the theoretical analysis of Ref. [15], extended to include the impact of our use of a mixed action. This results of this extension have been summarized in Ref. [20] (full details will be given in Ref. [16]). There we argue that, due to the smallness of taste-breaking with HYP-smearred quarks, the extension does not, to good approximation, introduce any new fitting parameters.

Nevertheless, at NLO there remain 12 parameters in a fit at fixed lattice spacing and fixed sea quark masses. We denote the two valence “flavors” in the valence kaon as x and \bar{y} , with mass of the former to be extrapolated to m_d^{phys} and that of the latter to m_s^{phys} . The inventory of parameters is:

- The LO value, B_0 , which is multiplied by $1 + \text{chiral logs}$, with the chiral logs predicted in terms of the masses of valence pions of all tastes and the taste-singlet sea pions, all of which are available from our and MILC’s data. (We use the physical f_π as a fixed input.)

- Two LECs proportional to $m_x + m_y$ and $(m_x - m_y)^2/(m_x + m_y)$.
- Seven LECs introduced by discretization and matching errors, and contributing through non-hairpin diagrams in ChPT. Each multiplies a different function of the valence masses $X = M_{xx}^2$, $Y = M_{yy}^2$ and $K = M_{xy}^2$, with one for each of the tastes I, P, V, A, T and an additional one each for $V + A$ and T tastes. The latter two functions are absent for $m_x = m_y$.
- Two further LECs introduced by discretization and matching errors which arise from “hairpin” diagrams in ChPT, and involve the taste V and A hairpins. These are absent for $m_x = m_y$.
- In addition, to obtain good fits to all our data we need to add a NNLO analytic term proportional to $(m_x + m_y)^2$.

Full details of the necessary functions can be reconstructed from Refs. [15, 20] and will be presented in Ref. [16]. We note that when combining different sea-quark masses we must introduce another LEC multiplying $2m_\ell^{\text{sea}} + m_s^{\text{sea}}$, and that B_0 has an implicit dependence on a^2 and $\alpha(a)$.

Even though we have 55 data points on each ensemble, a direct 13 parameter fit is not possible, especially because there are a number of “flat directions” since several of the LECs multiply very similar functions. By investigating the form of these functions for our quark masses, we have determined that we can, to good approximation, collapse the fits down to 7 parameters:

- Four terms present in the continuum limit: B_0 , and the three mass-dependent terms (two at NLO, one at NNLO).
- Two “latticey” LECs multiplying representative functions from non-hairpin diagrams (those involving the tensor tastes). Only one is present for $m_x = m_y$.
- One latticey LEC from hairpin diagrams (axial taste), present only for $m_x \neq m_y$.

For the degenerate subset ($m_x = m_y$) there are thus three continuum and one latticey LEC. In addition, we constrain the latticey LEC’s to have magnitudes no larger than their expected sizes [$O(a^2)$ and/or $O(\alpha^2)$].

Having done a fit we then extract our most continuum-like estimate for B_K for the given lattice by setting all latticey LECs and pion taste-splittings to zero, and setting m_x and m_y to their physical values. This leaves sea-quark mass and non-taste breaking discretization errors to be removed by extrapolations using multiple lattice ensembles.

Within this framework we have tried many fitting approaches, e.g. first fitting to the degenerate subset and then the entire data set or directly fitting to the entire set, and varying the tightness of the constraints. An example of a fit is shown in Fig. 3, which serves also to illustrate the statistical errors in “last year’s” data. We show the residuals from this fit in Fig. 4. The largest deviations are at the 1.5% level, and are for when both m_x and m_y are small. We note that using staggered PQChPT is essential; the residuals in a fit to continuum PQChPT, shown in Fig. 5, are much larger.

Overall, we find reasonable fits, with reasonable convergence of the chiral expansion, and with errors in the resulting “physical” B_K from individual fits of $\approx 2\%$ (on the coarse lattices).

The variation *between* the results from different fits is, however, closer to $\approx 5\%$. We are also concerned about finite volume effects—these are not included in the fits at present, but can be estimated from ChPT and are comparable for the lightest $m_{x,y}$ to the residuals.

Thus more work is needed to determine whether we can reduce the overall systematic on a single lattice ensemble below the 5% level from $SU(3)$ fitting.

5.3 $SU(2)$ PQSChPT fits to B_K

We have found that $SU(2)$ fitting, pioneered by the RBC/UKQCD group, is more promising. Here we fit only to the subset of our data with the smallest m_x (typically the lightest 4 values) and largest m_y (typically the heaviest 3 values). Only the $x\bar{x}$ and $\ell\bar{\ell}$ pions are treated as light, which is clearly a better approximation than that used in $SU(3)$ fitting. On the other hand, one uses only a subset of the data (12 out of 55 points), and expects somewhat enhanced chiral corrections (m_π^2/m_K^2 rather than $m_\pi^2/(1 \text{ GeV})^2$).

In our case, however, there turns out to be an additional important simplification which favors $SU(2)$ ChPT: *all* the LECs introduced by discretization errors and incomplete matching contribute at NNLO in the $SU(2)$ power counting, rather than at NLO for $SU(3)$ power counting. This is because all such terms carry a factor of M_π^2/M_K^2 . This can be seen from the NLO $SU(3)$ expression, and we have shown that it holds to all orders in $SU(3)$ ChPT [16]. The net result is that $SU(2)$ fits can be made to the schematic form

$$B_K = B_0(m_y, m_s, a^2, \alpha^2)(1 + \text{chiral logs} + d_1 m_x + d_2 m_\ell + d_3 m_x^2), \quad (7)$$

in which we have included the analytic NNLO term as above. Taste-breaking enters only in the pion masses that should be used in the chiral logarithms—this is the only way in which the fitting differs from that with DWF. In this form we have made explicit that the constant term B_0 depends now on the strange quark masses (in an *a priori* unknown way) as well as upon a^2 and α^2 .

An example of the fit versus $X = M_{xx}^2$ for fixed a , m_y , m_s and m_ℓ , on both coarse and fine lattices, is shown in Fig. 6. Since this is a 3 parameter fit to four points, it is not surprising that a good fit can be obtained. What is important to note, however, is that the coefficients have the expected magnitudes, and that the expected curvature due to the chiral logarithm is observed.

These fits allow us to determine a “physical” B_K (the red points in Fig. 6), by setting taste-splittings to zero, and setting m_x and m_ℓ both equal to the value needed to attain $M_{xy} = M_K^{\text{phys}}$ for the given m_y . The second stage is then to extrapolate these values to $m_y = m_s^{\text{phys}}$. This is illustrated in Fig. 7. A linear extrapolation works well, with a quadratic “fit” giving essentially the same result.

The resulting B_K has errors at about the 2% level, but is also stable at this level to changes in fitting strategy. Furthermore, the expected size of finite volume effects, as estimated by ChPT at NLO is well below 1% (which is another, serendipitous, advantage of $SU(2)$ over $SU(3)$ fitting).

The final two stages are to extrapolate in m_ℓ and to the continuum limit.⁴ Concerning the former, we find that the result is independent of m_ℓ on the coarse lattices. As for the latter, we quote only our preliminary numbers (from $SU(2)$ fits like those shown in the Figures) to give a sense of the size of the effect: $B_K(\text{coarse, tree}) = 0.68(1)$ and $B_K(\text{fine, tree}) = 0.64(1)$. Since

⁴Strictly speaking, we also need to extrapolate m_s , the sea strange quark mass, to its physical value. We have not yet done this, but expect it to lead to a very small change in B_K .

we use tree-level matching, the difference is due to a mix of a^2 , α , etc.. Clearly we have more work to do to produce a full error budget, but we are encouraged by the success and simplicity of the $SU(2)$ analysis.

6 Proposed running in 2009-10

The analysis of our results from the present year's running will improve the statistics and control of finite-volume effects on the coarse lattices, anchor our fine lattice result with a second sea-quark mass, and add a superfine data point. This will provide crucial checks of our present analysis, and reduce the systematic errors.

We propose here to push the calculation of B_K one further step towards the continuum limit:

1. Calculate B_K on ensemble S2, the $m_\ell/m_s = 0.1$ superfine lattices. A calculation on the 650 of the existing 750 such lattices at 7 hours/lattice on 2048 nodes with 10% overhead will require **10.5 Mnode-hours on the QCDOC**.
2. Calculate B_K on ensemble U1, the $m_\ell/m_s = 0.2$ ultrafine lattices. A calculation on the existing 650 such lattices at 13 hours/lattice on 2048 nodes with 10% overhead will require **19 Mnode-hours on the QCDOC**.

Thus in total we are requesting **29.5 Mnode-hrs**. This corresponds to having a dedicated 4096-node block running for the 7200 hour year.

We note that we would be able to run some or all of this project on clusters if needed.

7 Summary

The kaon B -parameter is a potential “gold-plated” prediction from lattice QCD. If errors can be controlled and reduced below the 5% level, then there will be important implications for both the SM and for new physics.

We propose to continue our calculation using HYP-improved staggered valence and asqtad sea fermions, so as to improve our control of the systematics due to taste-breaking and continuum extrapolation. Our aim is to bring all errors except those due to matching down to the few percent level. If successful, a non-perturbative or 2-loop matching calculation should bring the matching error down to 5% or smaller. This will provide an important check on results using other types of fermion.

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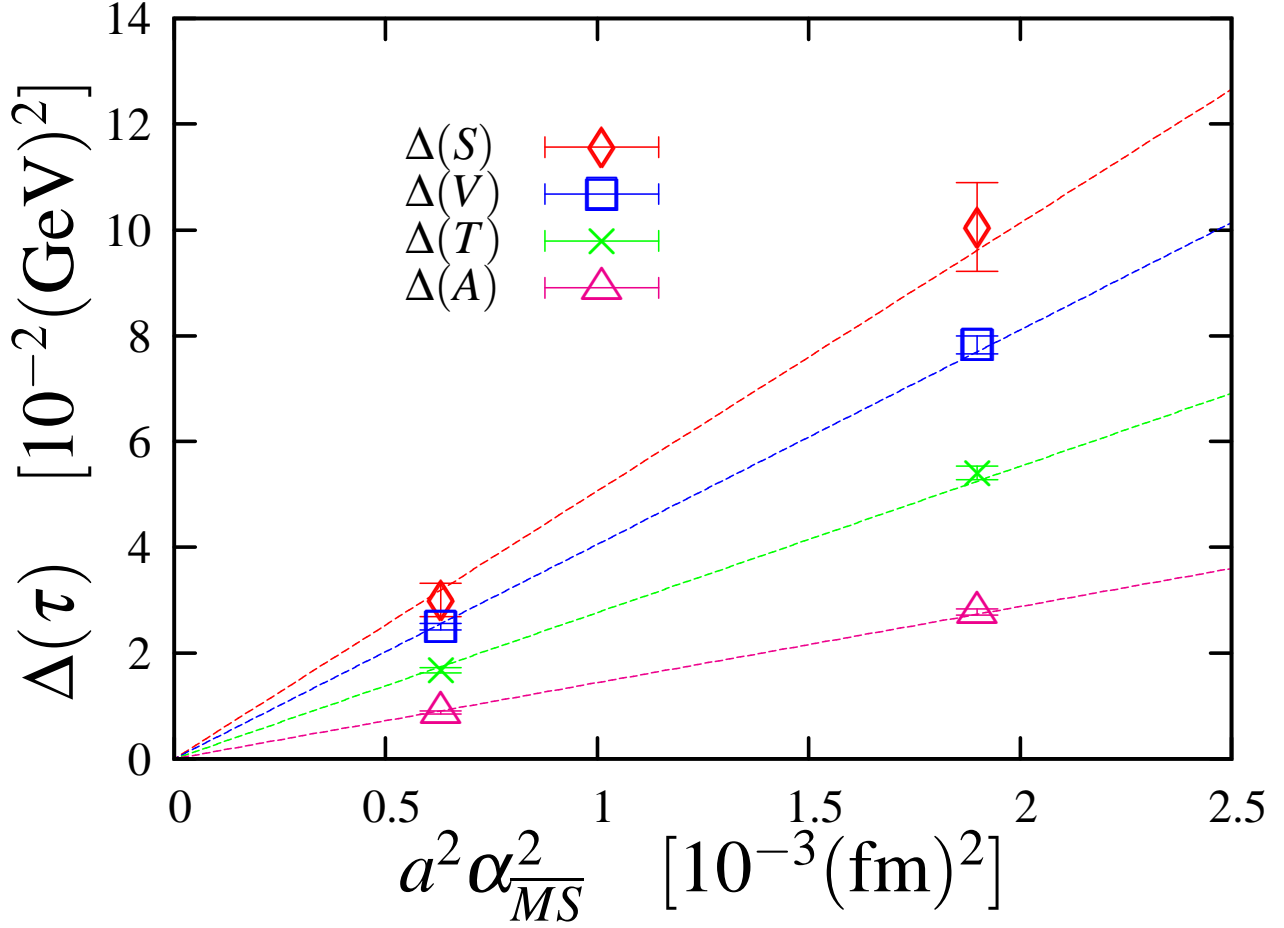


Figure 1: Taste-splittings (defined in Ref. 6) with valence HYP-smearred quarks on the coarse and fine MILC lattices having $m_\ell/m_s = 0.2$. The lines show linear fits vs $a^2\alpha(1/a)^2$ constrained to pass through the origin. Tastes τ are labelled S= $\mathbf{1}$, V= ξ_μ , T= $\xi_\mu\xi_\nu$ and A= $\xi_5\xi_\mu$. From Ref. [18].

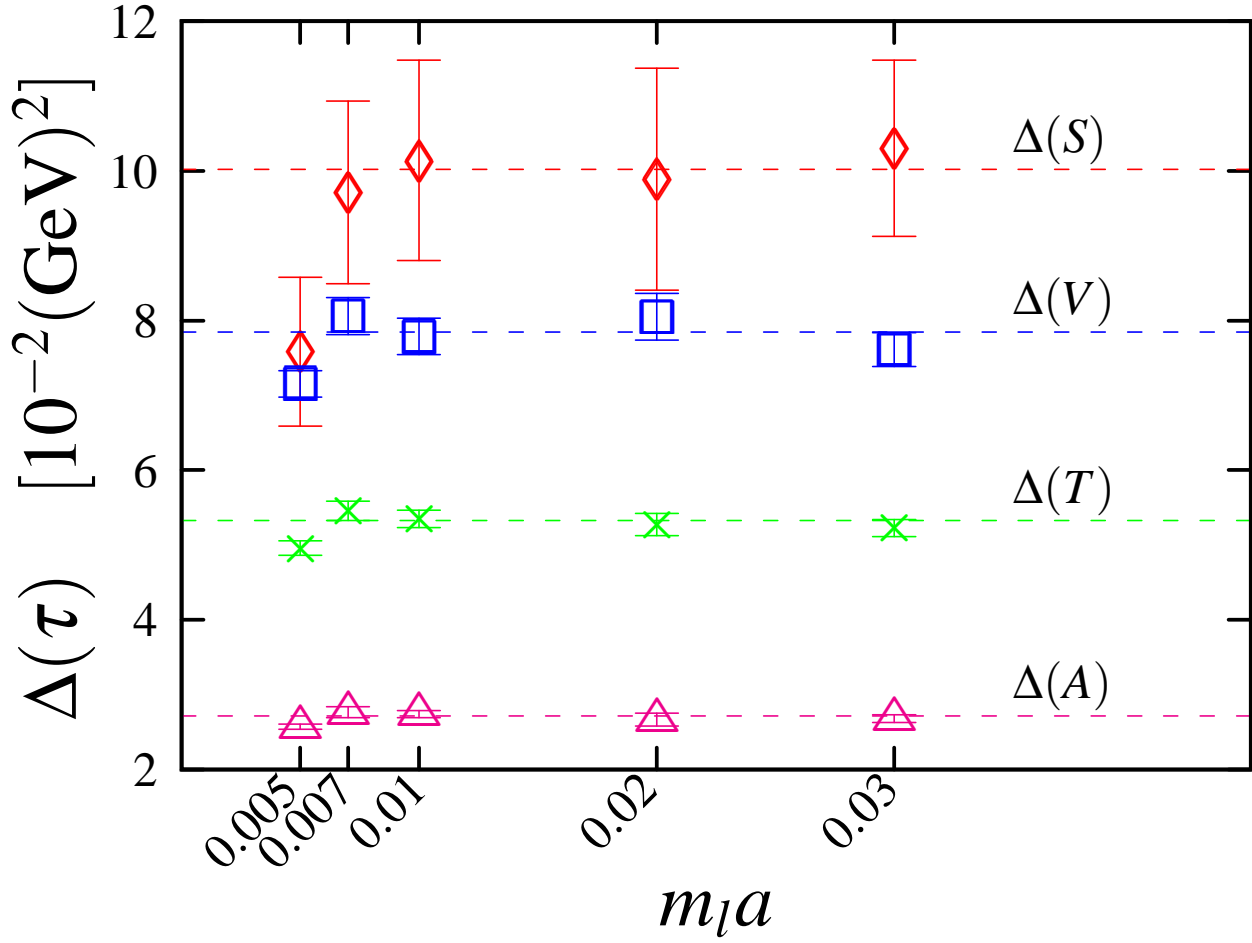


Figure 2: Taste-splittings on the coarse MILC lattices as a function of the light sea-quark mass. Notation for tastes is as in the previous figure. From Ref. [18].

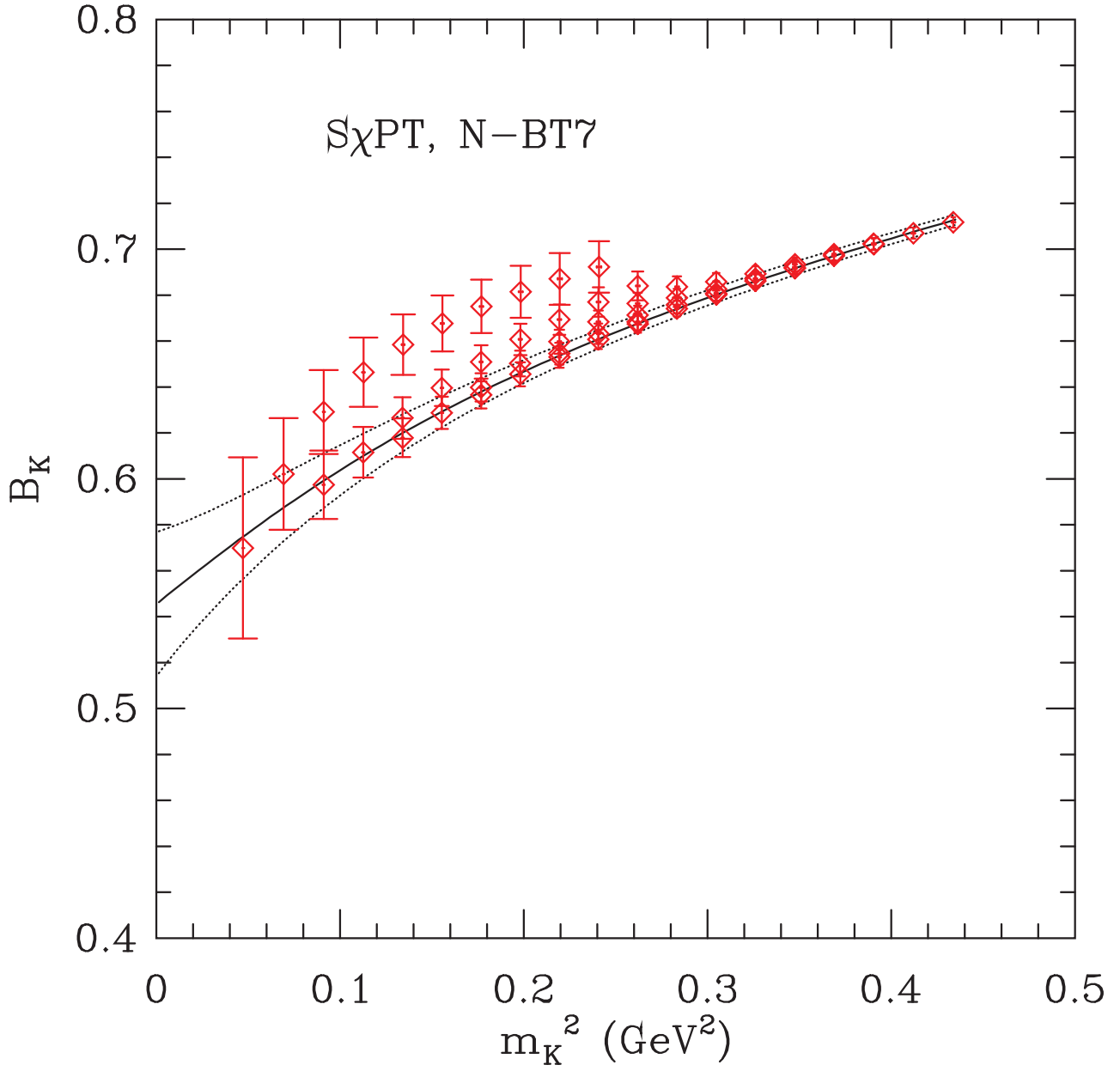


Figure 3: Example of a partially constrained PQSchPT 7-parameter fit to tree-level B_K . The fit is to all 55 points, but only the fit function Data is from the C3 ensemble (see Table 2). Results are preliminary.

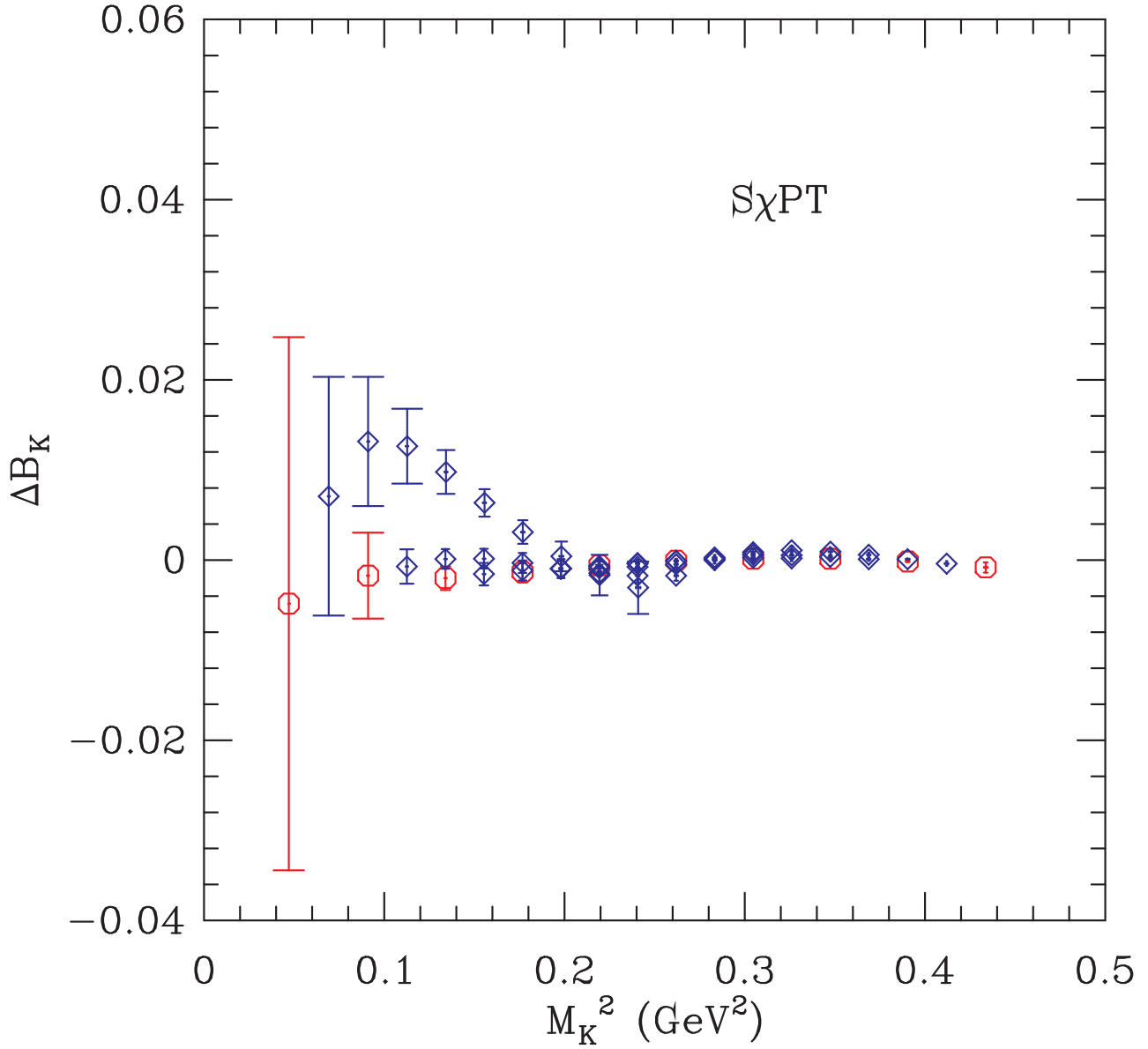


Figure 4: Residuals for fit shown in previous figure plotted versus m_K^2 in physical units. Degenerate “kaons” are shown in red, non-degenerate are in blue. Data is for the ensemble C3. Results are preliminary.

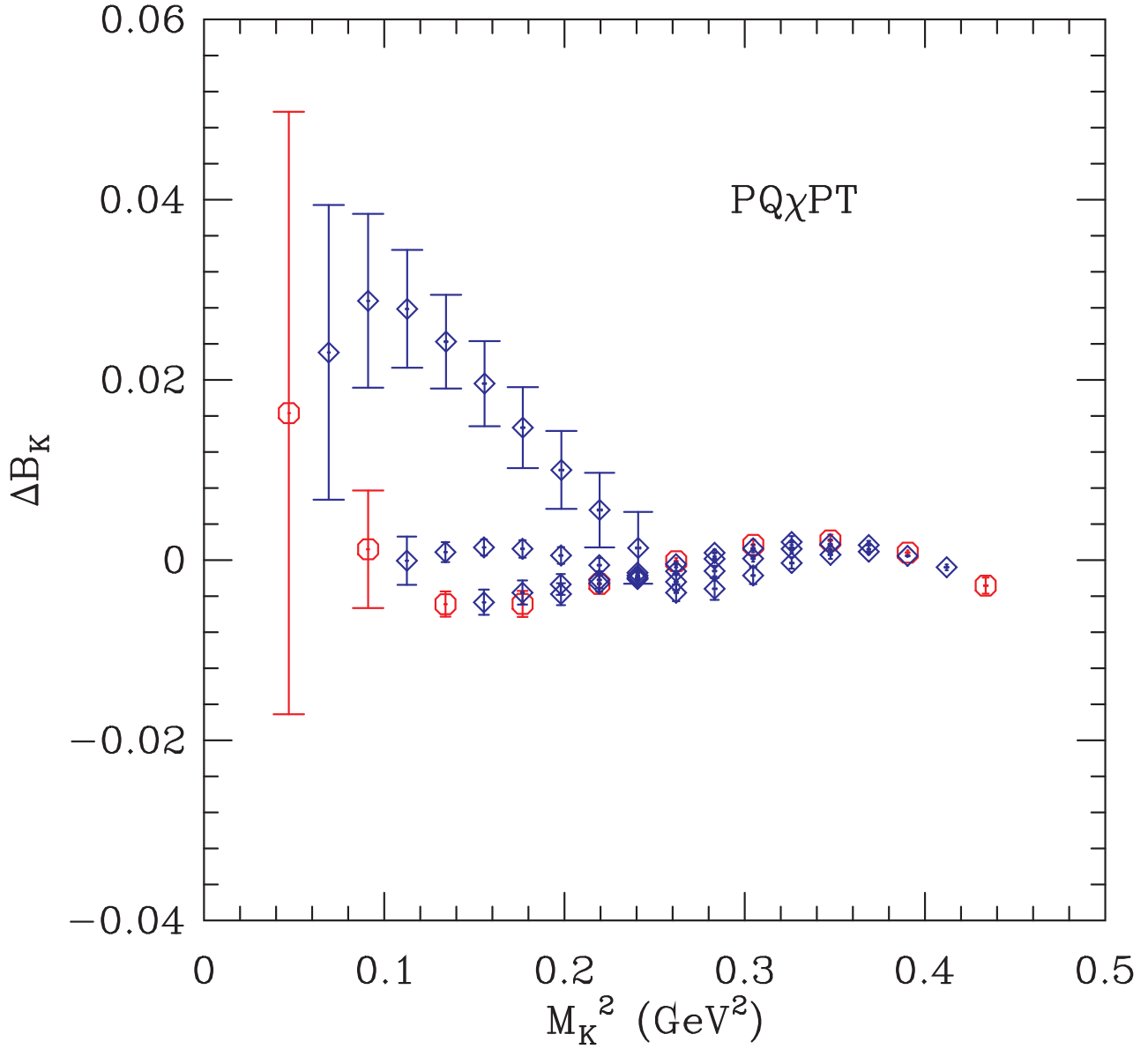


Figure 5: Residuals for a fit of the data in Fig. 3 to continuum PQChPT. (Tree-level) B_K is plotted versus m_K^2 in physical units. Degenerate “kaons” are shown in red, non-degenerate are in blue. Results are preliminary.

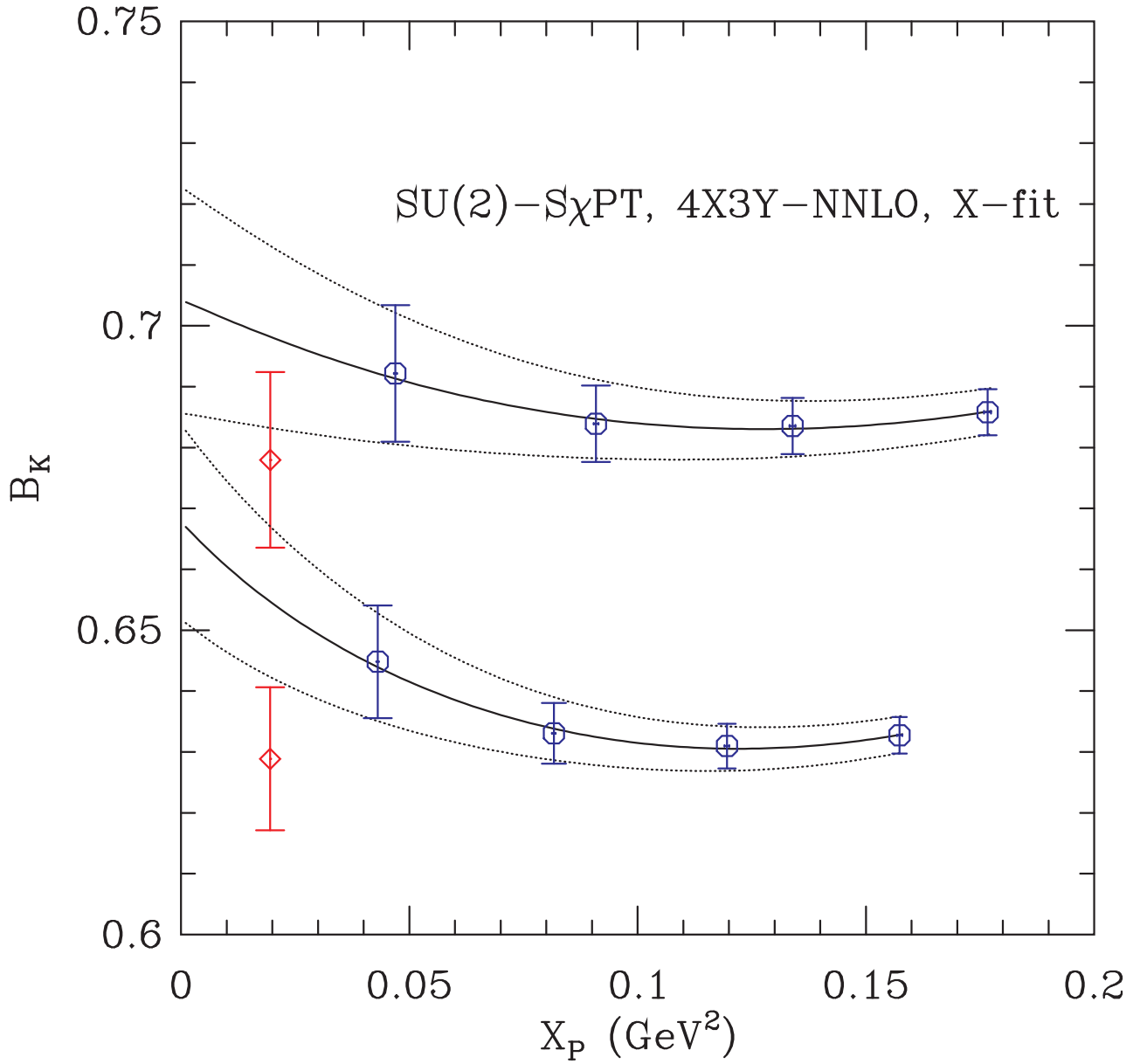


Figure 6: Example of $SU(2)$ PQSChPT NNLO fit to B_K , for a valence kaon of composition $\bar{y}x$, plotted against $X = M_{xx}^2$ in physical units, for fixed m_y . Upper/lower data sets are from coarse/fine MILC ensembles C3/F1 (both having $m_\ell/m_s = 0.2$). Red points show resulting value of B_K at the physical kaon mass, with taste-splittings set to zero. Results are preliminary.

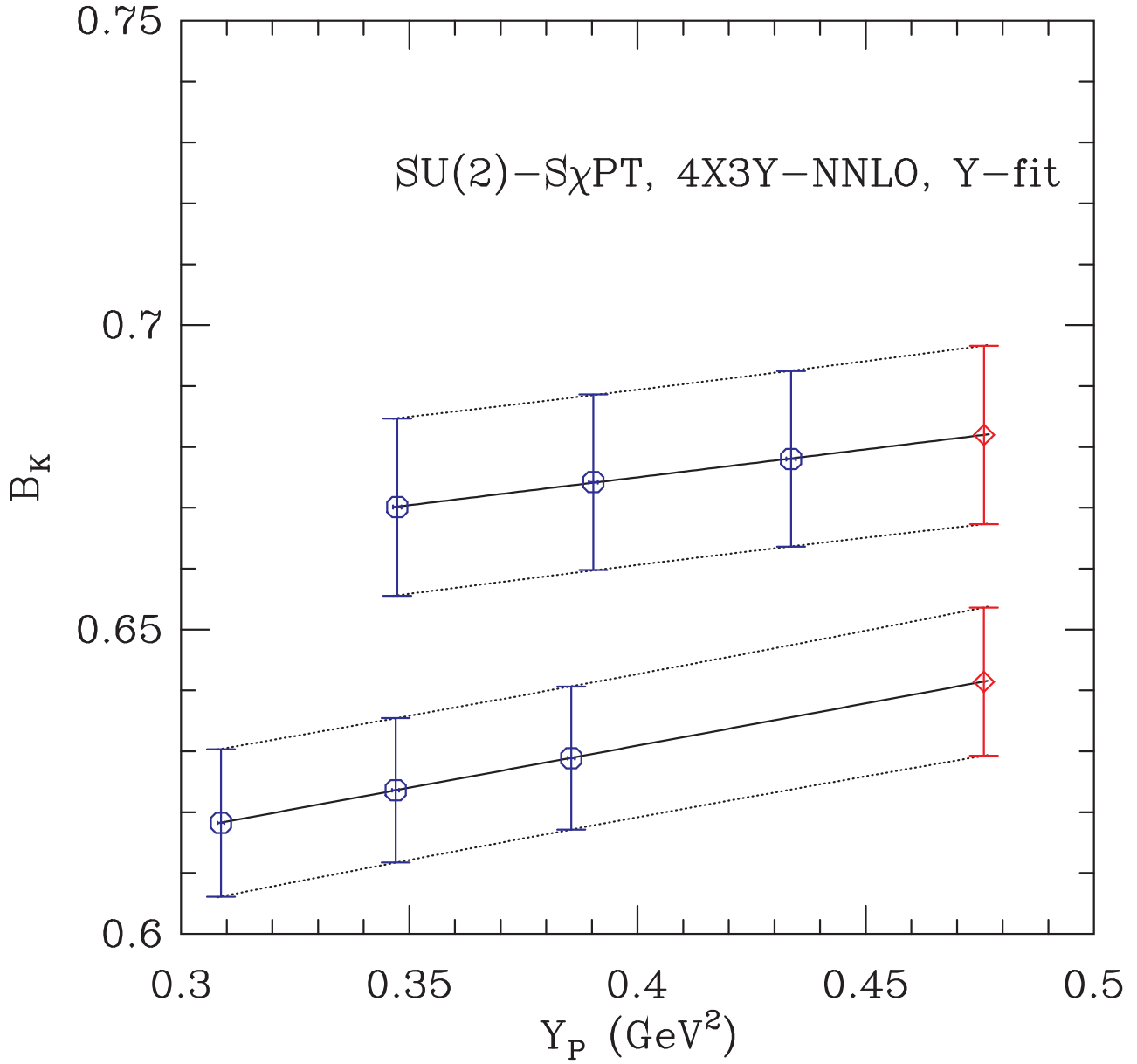


Figure 7: Linear extrapolation of results from previous figure versus $Y = M_{yy}^2$ to the “physical” value (at which $m_y = m_s^{\text{phys}}$). Final result is shown in red. Upper/lower curves for MILC coarse/fine ensembles C3/F1. Results are preliminary.