B_K and related matrix elements with unquenched, improved staggered fermions

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Abstract

This is a class B proposal, requesting time on the QCDOC to continue our calculation of kaon matrix elements using HYP-smeared improved staggered quarks. Our major focus will be on a calculation of B_K, which is of particular interest for constraining elements of the CKM matrix. The present (2007-8) allocation should allow completion of work on most of the MILC fine lattices. We give an fairly extensive status report on our results, so as to motivate its continuation. We propose to extend this work in two ways: first, to increase statistics on the MILC fine lattices with the lightest sea quark; and, second, to extend the calculation to superfine lattices.

Graduate students and postdoctoral associates at both the University of Washington and Seoul National University will contribute to this project.

We are requesting 4.1 Mnode-hours on the QCDOC.

1 Scientific Background

A major goal of lattice QCD is to calculate electroweak matrix elements. This would allow precision tests of the standard model (SM), and is one of the major objectives highlighted in the USQCD white papers and proposals. The matrix elements which contribute most directly to constraining the standard model are B_K, $f_{B_d}^2 B_{B_d}$, $f_{B_s}^2 B_{B_s}$, and the $B \to (\rho, \pi, D, D^*)\ell\nu$ semileptonic form factors. Here we propose to continue our study of B_K and related hadronic matrix elements using HYP-smeared staggered valence quarks [1] on the existing MILC dynamical "asqtad" lattices.

The impressive work of our experimental colleagues has made it possible to constrain the angles of the unitarity triangle using hadronic $B-$decays alone, without theoretical input from the lattice. In particular, one can now use the measured value of CP violation in $\bar{K} - K$ mixing, plus knowledge of the CKM angles, to predict [2]

$$\widehat{B}_K(\text{UT prediction}) = 0.75 \pm 0.09\,, \tag{1}$$

Thus a precision calculation of B_K will test the SM. Furthermore, if LHC finds evidence for new physics, then a lattice value of B_K constrains the nature of that physics, since it will, in general, contribute to flavor-changing processes such as $\bar{K} - K$ mixing.

There are many ongoing lattice calculations of B_K, all using $2 + 1$ flavors of dynamical fermions, but using a wide variety of fermions. The theoretically most straightforward approaches are those using domain-wall fermions (DWF) which have almost exact chiral symmetry. The most advanced calculation uses valence and sea DWF, and finds, based on results at a single lattice spacing ($a^{-1} = 1.73\,\text{GeV}$) [3]:

$$\widehat{B}_K(\text{DWF}) = 0.720(13)(37)\,. \tag{2}$$

Here the errors are, respectively, statistical and systematic, with the latter dominated by an estimate of the discretization error. The result agrees very well with the "experimental" prediction. We note for comparison with our calculation described below, that the lightest sea quark mass is $m_\ell \approx m_s^{\text{phys}}/5$ while the lightest valence quark mass is $\approx m_s^{\text{phys}}/10$. Non-perturbative renormalization (NPR) yields the matching factor with a 1.4% error, and two volumes were used to control finite volume effects.

The remaining uncontrolled systematic, due to discretization errors, is being studied by repeating the calculation a smaller lattice spacing. This very large scale calculation will set the standard for control of errors.

Another calculation using valence DWF aims to match this standard [4]. This work uses the existing MILC ensemble with asqtad sea quarks. The systematics are different from the "DWF on DWF" calculation, and it will be very interesting to compare the results.

There is also a calculation using valence asqtad quarks on the "coarse" ($a = 0.125$) MILC lattices with $am_\ell/am_s = 0.01/0.05$ and $0.02/0.05$. The result is [5]

$$\widehat{B}_K(\text{asqtad}) = 0.83(18)\,, \tag{3}$$

in which the error is dominated by the estimate of the size of the missing two-loop term in the perturbative matching factors. Other errors are estimated to be smaller (3% each from statistics and chiral extrapolation, and 5% from discretization errors). We aim to reduce all these errors using improved staggered fermions.

We think that it is very important for lattice calculations to provide multiple, independent checks of the systematics using, in particular, different fermion discretizations. Thus we think that it is useful to pursue the calculation with staggered fermions for both valence and sea quarks, particularly given the fact that they are relatively computationally inexpensive. Using USQCD resources allocated to class B proposals, we have, for the past 1.5 years, undertaken such a calculation. We use HYP smeared staggered fermions, chosen since they reduce the dominant discretization error—taste-breaking. We describe in some detail below the results we have obtained from a partial analysis of our data, and propose extending the calculation to both reduced statistical errors on the fine lattices and move to the superfine lattices.

An important weakness of the approach is that, at present, we rely on perturbative renormalization factors—one-loop factors should be available by this summer. This problem can be alleviated by working at 2-loop or using NPR. Work in both directions is underway, with a separate proposal requesting time for the NPR calculations (Lytle and Sharpe).

Kaon matrix elements of other four-fermion operators are also of considerable interest. We restrict ourselves here to those without "eye" contractions, i.e. both quarks and both antiquarks are contracted with fields in the operators creating the external particles and not with each other. Of most interest are the $K \to \pi\pi$ matrix elements of the operators $\mathcal{O}_{7,8}^{I=3/2}$, since these contribute significantly (particularly $\mathcal{O}_8$) to ϵ'/ϵ through electromagnetic penguin diagrams. Using leading order chiral perturbation theory these can be related to the $K - \bar{\pi}$ matrix elements which we propose to calculate.

Beyond the standard model physics can lead to enhanced coefficients for $\Delta S = 2$ four-fermion operators not having the "left-left" chiral structure of the operator appearing in B_K. This provides additional motivation for calculating $K - \bar{K}$ matrix elements of operators having all spin structures. To our knowledge, only quenched calculations of these matrix elements have been done so far. Even in the quenched approximation, however, there is considerable variation between calculations and results from non-lattice methods [6].

Since we use the MILC asqtad configurations, we must assume that the use of a rooted fermion determinant leads to the correct continuum limit. This is plausible, based on the extensive numerical results and theoretical work summarized in Ref. [7, 8].

2 Calculational method

We propose to continue our calculation of B_K using valence HYP-smeared quarks on MILC lattices.[1] Staggered fermions have the advantage of a remnant $U(1)$ partially conserved axial symmetry, which guarantees that the lattice matrix elements satisfy Ward identities analogous to those in the continuum [9]. We use HYP-smearing (introduced in Ref. [1]) because:

1. It substantially reduces the taste-breaking in the spectrum, the reduction being 2-3 times greater than with asqtad action (which is used for the sea quarks), and comparable to that from the recently introduced HISQ action [10]. Our recent paper provides details [11].

2. It is computationally simpler than both asqtad and HISQ fermions (improvement involving only HYP-smearing of the original lattice and then using the unimproved staggered action).

[1] For much of the following, we use B_K as a shorthand for "B_K and the related four-fermion kaon matrix elements discussed above".

3. Perturbative corrections to matching factors for bilinears and four-fermion operators are maximally reduced [12].

We have considerable experience with HYP-smeared fermions, and have found that they not only reduce taste breaking [11], which is the dominant discretization error, but also see evidence that they reduce the discretization errors in general [13]. We note that we are using a mixed action, with different versions of staggered fermions for the valence and sea quarks. The impact of this can be parameterized using chiral perturbation theory.

Our overall aim is to calculate B_K on as many of the MILC lattices which are suitably chiral as is feasible. We have made substantial progress towards this goal: the present status is summarized in Table 2. On each configuration we use 10 different valence quark masses, running approximately from $m_s^{\rm phys}$ down to $m_s^{\rm phys}/10$, and calculate B_K for all 55 mass combinations. This is then to be fit to the form predicted by staggered chiral perturbation theory [14] as applied to B_K [15]. One of the most important inputs into these fits are the masses of the pions with non-Goldstone tastes, and we calculate these (along with the entire spectrum) as well.

B_K is defined by

$$\frac{8}{3} m_K^2 f_K^2 B_K = \langle \bar{K} | \bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|K\rangle \,. \tag{4}$$

The calculational methodology is well established [13]. The lattices are HYP-smeared, using parameters chosen to remove the taste-breaking couplings to gluons with $ak_\nu = \pi$ at tree level (the parameter set denoted HYP(II) in Ref. [12]). We use wall sources having a random $U(1)$ phase on each site and for each color, such that the combination of a quark and antiquark propagator creates only the taste ξ_5 (Goldstone) pion. We use two such wall sources, separated in time by a spacing chosen as a compromise between extending the signal region and reducing the statistical noise. For the $L_t = 64$, the separation is $\Delta t = 26$ timeslices, while for $L_t = 96$, we use $\Delta t = 40$. In order to reduce correlations between configurations, the overall position of the sources is chosen randomly on each configuration. Statistics can be improved by using multiple sources on each configuration, maximally spread out in time, and we plan to use this possibility in our proposed running (described below).

We use gauge-invariant operators having spin-taste structure $(\Gamma \otimes \xi_5)(\Gamma \otimes \xi_5)$, with all 16 choices of Dirac matrices Γ. An important feature is that the links in the operators are HYP-smeared (in distinction to the calculation of Ref. [5] which uses "thin" links). This reduces perturbative corrections and may also reduce the size of discretization errors [16].

As noted above, for the chiral fits we need the masses of the pions of all tastes composed of valence HYP fermions. These are required for both degenerate and non-degenerate valence quarks [15]. Thus we determine the complete spectrum for all tastes. This requires different sources from the B_K calculation. Having fixed to Coulomb gauge, we divide a timeslice into cubes, and place random $U(1)$ phases (one for each color) on only one point in the cubes. We have tested two variants: one with the same $U(1)$ sources on each cube ("cubic wall"), the other with different sources on each cube ("cubic $U(1)$"). Both lead to consistent results, but we have found that the errors are somewhat smaller with the cubic wall source. Thus, starting in January 2008, we have used only this source. Note that there are eight sources, one for each position in a 2^3 cube, from which one can construct combinations that project onto two of the sixteen tastes. The sink operators can then pick out one of these two. More details are given in Ref. [11].

Using this set up, we calculate the masses of pseudoscalar, vector, axial vector and scalar mesons for the same choices of quark masses as for the B_K calculation.

3 Code details

Our code is written in C++, and built on top of the Columbia CPS library. The major computations are Coulomb gauge-fixing and the calculation of propagators. For the latter we use the level-3 conjugate gradient (CG) inverter (for unimproved fermions since our valence quarks do not use the 3-step Naik term). In this way, our code makes essential and efficient use of the QCD API, both for computations and communications.

In order to test for hardware problems we repeat the calculation on every 10'th configuration; if the check fails, we repeat the calculation on the previous 10 configurations.

The number of nodes needed depends on the lattice size. The determining factor is that the code requires the sites local to each node to have an even number of sites in each dimension. In our present running on 128 nodes, the local volumes are, respectively, $10^3 \times 4$, $6^3 \times 32$ and $14^3 \times 6$ on the $20^3 \times 64$, $24^3 \times 64$ and $28^3 \times 96$ lattices. Memory limitations force us to more nodes for the 40^3 lattices, and our present plan is to use a local volume of $10^3 \times 12$ with 512 nodes, although we note that the code would run somewhat more efficiently on a larger number of nodes. On the $48^3 \times 144$ superfine lattices the smallest viable partition is 1024 nodes (with the local volume being $12^2 \times 6 \times 18$), although a larger number of nodes would improve the code's speed by reducing the local memory requirement, and also reduce the I/O overhead.

Analysis is done on workstations at our home institutions.

The estimates of time required given in last year's proposal proved accurate, except that we have sped up the gauge fixing code by 20%. Since our proposal this year is for calculating B_K alone, we recall the timings for this code. Using the $20^3 \times 64$ lattices for concrete estimates, we attain 145 MFlop/s/node when running inversions using CG, while the computations for B_K run at only 23 MFlop/s/node, with I/O overhead due to propagator staging at about 50%. The CG portion takes somewhat less than half the time required for the B_K calculation (this rises to close to half on the fine lattices) so we sustain about 75 MFlop/s/node (i.e. slightly less than 10% of peak).

The times required for calculations on various size lattices are collected in Table 1. For convenience all times are given for 128 nodes. For the $28^3 \times 64$, $40^3 \times 96$ and $48^3 \times 144$ lattices the times are estimates based on scaling gauge fixing, B_K and spectrum calculations linearly with the volume, and inversions both linearly with the volume and as $1/(am)$.

We are presently storing only the gauge-fixed configurations, but not the propagators. Thus our storage needs are minimal. We would be happy to save the propagators if there was interest in using them.

4 Status and Results

Our calculations have been done on four or five 128-node QCDOC motherboards We began our production running in February 2007, first running on the 0.01/0.05 coarse lattices in order to test our code by comparing to results obtained on the SNU cluster. Since then we have steadily worked through the coarse and now the fine MILC ensembles. The present status is

a (fm)	Lattice	Gauge fix	Spectrum + B_K	B_K alone
0.125	$20^3 \times 64$	60 mins	60 mins	20 mins
0.125	$24^3 \times 64$	100 mins	100 mins	35 mins
0.125	$28^3 \times 64$	165 mins	165 mins	55 mins
0.09	$28^3 \times 96$	5 hours	7 hours	2.7 hours
0.09	$40^3 \times 96$	15 hours	20 hours	8 hours
0.06	$48^3 \times 144$	38 hours	66 hours	25 hours

Table 1: CPU requirements on different lattices. All times are quoted for 128 nodes, although the larger lattices require more nodes. The times in the columns "spectrum + B_K" and "B_K" include both the time for the needed propagator calculations and the contractions. Note that Coulomb gauge-fixing is not needed for calculations of B_K alone.

a (fm)	am_l/am_s	Size	Configs.	spectrum anal.	B_K anal.
0.12	0.03/0.05	$20^3 \times 64$	564	CU1	In progress
0.12	0.02/0.05	$20^3 \times 64$	486	CU1	tree
0.12	0.01/0.05	$20^3 \times 64$	671	CU1	tree
0.12	0.007/0.05	$20^3 \times 64$	655	In progress	In progress
0.12	0.005/0.05	$24^3 \times 64$	509	In progress	In progress
0.12	0.01/0.03	$20^3 \times 64$	312	In progress	In progress
0.09	0.0062/0.031	$28^3 \times 96$	615	CW/CU1 (on 399)	tree (on 399)
0.09	0.0124/0.031	$28^3 \times 96$	192	In progress	In progress

Table 2: Status of calculations on MILC lattices (as of 2/15/2008). "Configs." lists the number of configurations on which spectrum and B_K calculations have been completed. The last two columns indicated the status of the analysis, with "CW" and "CU1" refer to cubic-wall and cubic-U(1) sources, respectively, while "tree" indicates tree-level B_K analysis. CW calculations on some of the coarse lattices are being undertaken on the SNU cluster.

summarized in table 2. On each of the coarse MILC lattices we use valence masses $am_{\text{val}} = 0.005, 0.01, \ldots 0.045, 0.05$ while on the fine MILC lattices we use $0.003, 0.006, \ldots, 0.03$. We do not save the quark propagators, but we do save the gauge fixed lattices, allowing later calculation with time-shifted sources.

Fitting and final analysis lag behind the calculation of propagators, mainly due to the time-consuming nature of checking fits to the spectrum data. Nevertheless, we have completed enough analysis to give a sense of the quality of the results we should obtain. We show in Fig. 1 the masses (squared) of four tastes of pions, for three coarse lattice sets ($am_\ell = 0.01, 0.02, 0.03$, $am_s = 0.05$). The other four independent tastes have larger errors, and are not shown for clarity. We also do not show non-degenerate combinations, which clutter the plot and lie very close to the linear fits. We observe that the valence pion masses are very weakly dependent on the light sea-quark mass, particularly for the smaller quark masses. Thus it appears that, once one has tuned the lattice spacings to be close to equal, as for these three sets, further dependence on the sea quark mass is very weak. This is consistent with results using asqtad and DW valence quarks [17, 4]. The practical consequence is that we need only evaluate the taste splittings on a single lattice set (say $am_\ell = 0.01$) and then add this to the mass-squared of the Goldstone pion on whatever lattice set one is working (as long as the lattice spacing is the same). We plan to use this strategy on the fine lattices.

We show in Fig. 2 the taste splittings on coarse and fine lattices with $m_\ell^{\rm sea}/m_s^{\rm sea} = 1/5$. We now set the scale using r_1, rather than $1/a$, to allow a direct comparison. As expected, the taste splittings are reduced substantially. This is made quantitative in Table 3, which quotes results for

$$\Delta(T) = m_\pi(T)^2 - m_\pi(\xi_5)^2 \,, \tag{5}$$

(T being the taste) after extrapolation to the chiral limit. The ratio of the splittings, also quoted, is $\approx 1/3$. We recall also that, on the coarse lattices, the splittings with HYP-smeared valence quarks are smaller than those with asqtad valence quarks by a factor of 2.5-3, as shown in last year's proposal and in more detail in Ref. [11]. Our new results show that this holds true also on the fine MILC lattices.

Taste	$\Delta(T, a = 0.125fm)$ (GeV^2)	$\Delta(T, a = 0.09fm)$ (GeV^2)	$r(T)$
ξ_{i5}	$[0.162(2)]^2$	$[0.094(2)]^2$	0.336(13)
ξ_{i4}	$[0.227(2)]^2$	$[0.131(2)]^2$	0.333(14)
ξ_4	$[0.274(4)]^2$	$[0.160(3)]^2$	0.341(17)
ξ_{45}	$[0.152(15)]^2$	$[0.099(23)]^2$	0.42(21)
ξ_{ij}	$[0.216(13)]^2$	$[0.135(17)]^2$	0.39(11)
ξ_i	$[0.273(18)]^2$	$[0.163(17)]^2$	0.36(9)
I	$[0.31(2)]^2$	$[0.187(17)]^2$	0.36(8)

Table 3: Taste-splittings $\Delta(T)$ on coarse and fine MILC lattices with valence HYP-fermions and their ratio $r(T) = \Delta(T, a = 0.0.09)/\Delta(T, a = 0.125fm)$. Results are for lattices with sea quarks having $m_\ell/m_s = 1/5$, with only 399 fine lattices analyzed. Results in the upper panel are from tastes requiring operators local in time, for which the errors are smaller.

Our conclusion from these results is that it is reasonable to to treat taste-breaking effects as of NLO rather than LO in staggered ChPT on the fine lattices, which would simplify the functional form needed for fitting. To make this concrete, we note that from the Table that the average taste splitting on these lattices is $\Delta(\xi_i\xi_4) = (0.13 \text{ GeV})^2 \approx (m_\pi^{\rm phys})^2$, so that the taste-breaking effects are comparable in size to the chiral symmetry breaking due to the *physical light quark masses.* Thus the ratio of taste breaking to continuum LO contributions for a kaon is $\sim \Delta(\xi_i\xi_4)/(am_K^{\rm phys})^2 \approx 0.07$, smaller than the corresponding ratio for generic NLO effects, $\sim m_K^2/\Lambda_\chi^2 \approx 0.2$.

We also have tree-level results for B_K on two coarse and one fine lattice ensemble (the latter only for 399 lattices). Results are at tree-level since one-loop or non-perturbative computations of matching factors are not complete. These results can be used to gauge the statistical quality of the data, but not yet to extract a useful value for phenomenology. Figure 3 shows the results for degenerate quarks on all three lattices. We note that the two coarse lattices, for which the renormalization factors will be equal, yield consistent results, indicating a weak sea-quark mass dependence of B_K. There is some evidence for the downward curvature expected in ChPT at NLO, although the results at the lightest quark masses have large errors. There is a 10% difference between the results at the two lattice spacings, some of which is likely to remain after renormalization factors are included.

The statistical errors in B_K for a "kaon" composed of degenerate quarks at the physical kaon mass are about 0.6% on the coarse lattices, and 0.8% on the fine lattices (though with only

a (fm)	am_l/am_s	Size	Configs.	Programs	Days/(5×128) nodes
0.12	0.01/0.05	$28^3 \times 64$	275	B_K (8 sources)	20
0.09	0.0062/0.031	$28^3 \times 96$	381	CW + B_K	38
0.09	0.0031/0.031	$40^3 \times 96$	500	B_K (1.25 sources)	42

Table 4: Proposed running for the remainder of our present allocation (2/16/2008-6/30/2008). Time estimates are given in days required on our present allocation of five 128-node partitions.

399 configurations). These errors clearly grow rapidly as m_q decreases, however, making a fit to ChPT poorly constrained. We are also concerned about finite volume effects at the smallest quark mass, where $m_\pi L = 2.7$ and 2.5 on the coarse and fine lattices, respectively. Our running plans for the rest of this year are mainly aimed at addressing these concerns.

A precision determination of B_K requires an extrapolation in the mass difference between the s and d quarks. Thus it is the errors in B_K for the non-degenerate case that are of most importance. We show in Fig. 4 and 5 the "non-degenerate part"

$$\Delta B_K = B_K(m_s, m_d) - B_K(m_{\rm av}, m_{\rm av}) \,, \qquad m_{\rm av} = (m_s + m_d)/2 \,, \tag{6}$$

where we have used a PQChPT fit to the degenerate-mass data for the subtraction. (This provides an approximate interpolation and is used only to show the nature of our results.) We have plotted these against the form of the analytic term in ChPT, $(m_s - m_d)^2$.

Each approximately horizontal sequence of points in these two plots corresponds to fixed m_d, with varying m_s. Each sequence must vanish as $(m_s - m_d)^2$, up to statistical errors, and systematics associated with our subtraction. It is clear that we have some work to do understanding these plots. That the analytic term does not describe the data (this would lead to a linear dependence) is reasonable because we know there are loop terms. At this stage, we note only that the extrapolation to a physical kaon holding with $m_s \approx m_s^{\rm phys}$ fixed is made by using the right-most points of each sequence, and that we need take one more "step" in the light-quark mass to reach the physical $m_d \approx 0$. Thus it is plausible that our present data will yield an extrapolated ΔB_K with a statistical error of ± 0.02 ($\approx 3\%$) or less. In fact, these errors are somewhat inflated due to the subtraction used to display the non-degenerate data. This can be seen from the smaller errors in the non-degenerate results themselves, an example of which is shown in Fig. 6. Nevertheless, it is clear that it is important to reduce the statistical errors.

In light of the preliminary results presented above, we have developed a tentative plan for the rest of this year's running. This is summarized in Table 4, based on timing information given in Table 1 above. We assume continued access to 5×128 nodes for about 90% of the time from 2/15/08-6/31/08. Our aims with this running are

- Complete the full set of 0.0062/0.031 fine lattices, calculating both the spectrum and B_K, so as to provide accurate pion masses on the fine lattices for fitting, and to improve the accuracy of our B_K results.
- Calculate B_K (but not the spectrum) on the larger volume 0.01/0.05 coarse lattices so as to test for finite volume effects, particularly at the smallest quark masses. Note that, to obtain good statistics, we propose to calculate with 8 different positions of the sources on each lattice (with the precise number depending on continued availability of the nodes and on the errors we are finding).

- Calculate B_K alone on the most chiral fine lattices. Due to memory limitations, in order to run on the $40^3 \times 96$ lattices we will need four of the 128-node motherboards to be wired into a 512-node partition. Based on our experience on the coarse lattices, as outlined above, we do not think it is necessary to recalculate the full pion spectrum on these lattices. We will obtain the Goldstone (taste ξ_5) pion mass from the B_K calculation, and we plan to use this, together with the splittings obtained on the 0.0062/0.031 lattices, in order to determine the non-Goldstone pion masses to use in our fitting function.

We have decided not to extend the running on the 0.0124/0.031 lattices, since the light sea quark may not be in the chiral regime [3, 4, 17]. Instead, we have chosen to direct more resources to the more chiral lattice sets.

If this running goes as planned, the net result will be that, at the end of this year's allocation, we will have completed the B_K calculation on more lattices, and with higher statistics, than we proposed last year.

5 Proposed running in 2008-9

We think that the preliminary results given above show that HYP-smeared staggered fermions have the potential to give a competitive result for B_K. We are particularly encouraged by the smallness of the taste splittings on the fine lattices. Nevertheless, we need to complete the detailed analysis of our data, and in particular of the chiral, continuum and infinite volume extrapolations, to really know our systematic errors. We also need to include matching factors beyond one-loop order, results from which will not be available for about a year. Thus we think that it is appropriate to continue this project, for now, at the class B level. We do note, however, that we could make good use of additional resources, were they to become available.

At the completion of our present allocation the major weaknesses of our numerical results will likely be (a) the size of statistical errors on the fine lattices, particularly at the smallest quark masses; and (b) the continuum extrapolation errors. Based on this, we propose in the 2008-9 allocation period to

1. Increase our statistical power for B_K alone on the $m_\ell/m_s = 0.1$, $40^3 \times 96$ fine lattices, by adding another 2.75 sources per configuration. This will require 92 days on 5×128 nodes, or **1.4 Mnode-hrs**. As noted above, these calculations require a partition of at least 512 nodes.

2. Calculate B_K alone on the $48^3 \times 144$ superfine lattices with $m_\ell/m_s = 0.2$. Note that taste-breaking should be reduced by a further factor of about 3 compared to the fine lattices, and other discretization errors will also be reduced, so that one may well be able to fit the data with continuum PQChPT. In any case, the results from a third lattice spacing will provide a strong check on the results from the other two.

   Assuming 500 lattices, and using 1.7 sources on each, requires 175 days on 5×128 nodes, or **2.7 Mnode-hrs**. As noted above, these calculations require a partition of at least 1024 nodes.

3. In practice, on the basis of ongoing analysis, we may change the relative allocations for these two sub-projects, or decide to add statistics on previously used lattices.

In total we are requesting **4.1 Mnode-hrs**.

Ideally, we would like to calculate the full spectrum as well on the superfine lattices. This would allow us to check that taste-breaking is of the expected size. To calculate with a single source on 500 lattices, including gauge fixing, would require an additional 6.6 Mnode-hrs. This would be one possible option if additional time were available. An alternative would be to extend the B_K calculation to the superfine lattices with $m_\ell/m_s = 0.1$, if the generation of these is completed.

6 Ancillary theoretical work

Three theoretical calculations are needed for this project. The first, which has been completed, is the generalization of the staggered ChPT analysis for B_K due to our use of a mixed action. Four additional hairpin parameters are introduced, two each for axial and for vector tastes, corresponding the valence-valence and valence-sea hairpins. The sea-sea hairpins have been determined with reasonable accuracy by the MILC collaboration. These extra parameters only enter for non-degenerate s and d quarks.

These extra parameters will certainly complicate the fitting, although, since they involve taste breaking by HYP-smeared quarks they are likely to be very small (and can be roughly estimated by scaling from the approximately known asqtad sea-sea hairpins).

We realize that the complicated fitting forms of SChPT will be challenging to use [15], in light of the large number of parameters. It is plausible, however, that some of these parameters are smaller than used in the power-counting of Ref. [15], and our strategy will be to start with simple fits with all terms known to be significant, and then test for the presence of further terms. We will also test both three and two flavor ChPT (the latter having been found more useful in the DWF analysis [3]).

The second theoretical calculation is that of the one-loop perturbative matching factors for the improved gauge action that is used to generate the MILC configurations. Our previous work assumed a Wilson gauge action [12]. The change is not trivial because the symmetry of the gluon propagator is reduced; calculations are underway independently in Seattle and Seoul, and should be completed in the next few months.

The third calculation is the extension of the staggered ChPT theory analysis to $B_{7,8}^{3/2}$. This is a larger project which we will likely undertake once we have numerical data in hand.

In addition, if we are successful in controlling all lattice systematics at the few percent level, then it will be essential to reduce the 2-loop truncation error in the matching factors. One approach is to calculate the 2-loop matching, and this is being undertaken by one of us (WL) in collaboration with students at Seoul National University. Another approach is to apply NPR to HYP-smeared staggered fermions. This is being undertaken by one of us (SS) with University of Washington student Andrew Lytle and is the subject of a separate proposal.

7 Summary

The kaon $B-$parameter is a potential "gold-plated" prediction from lattice QCD. If errors can be controlled and reduced to the 5-10% level, then there will be important implications for both the SM and for new physics. We propose to continue our calculation using HYP-improved staggered valence and asqtad sea fermions, so as to improve our statistical errors and the systematics due to taste-breaking and continuum extrapolation. Our aim is to bring all errors except those due to matching down to the few percent level. If successful, a non-perturbative or 2-loop matching calculation should bring the matching error down 5% or smaller. This will provide an important check on results using other types of fermion.

References

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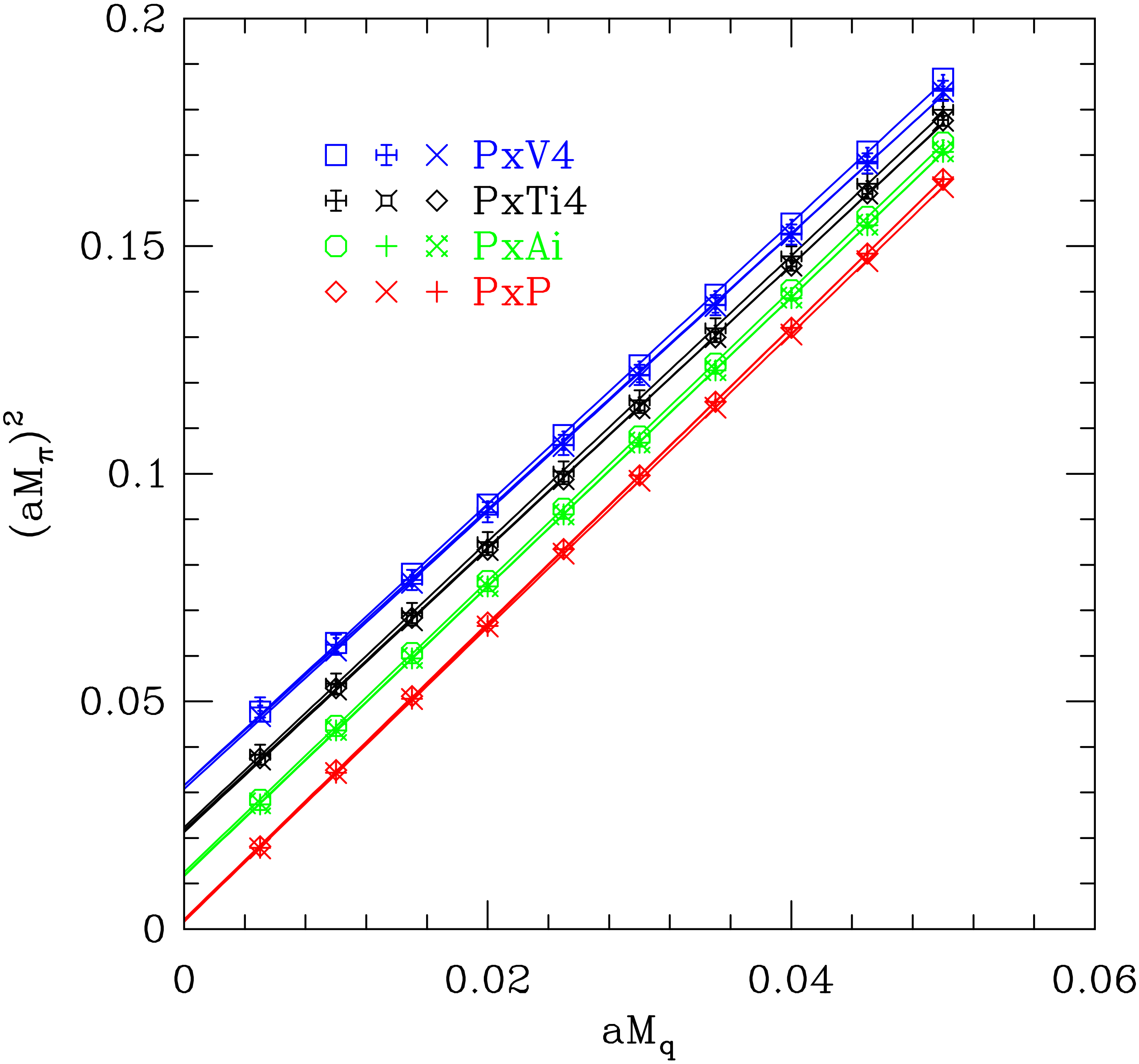

Figure 1: $(am_\pi)^2$ vs. am_q for pions composed of degenerate quarks on the $m_\ell/m_s = (0.01, 0.02, 0.03)/0.05$ coarse MILC configurations, using HYP-smeared valence quarks. The four tastes with the smallest errors are shown, with the tastes as labelled (ξ_4, $\xi_i\xi_4$, $\xi_5\xi_i$ and ξ_5). The three columns of symbols correspond to $am_\ell = 0.01$ 0.02 and 0.03, moving from left to right. The lines are linear fits to each taste. Results on the $am_\ell = 0.02$ and 0.03 lattices are preliminary.

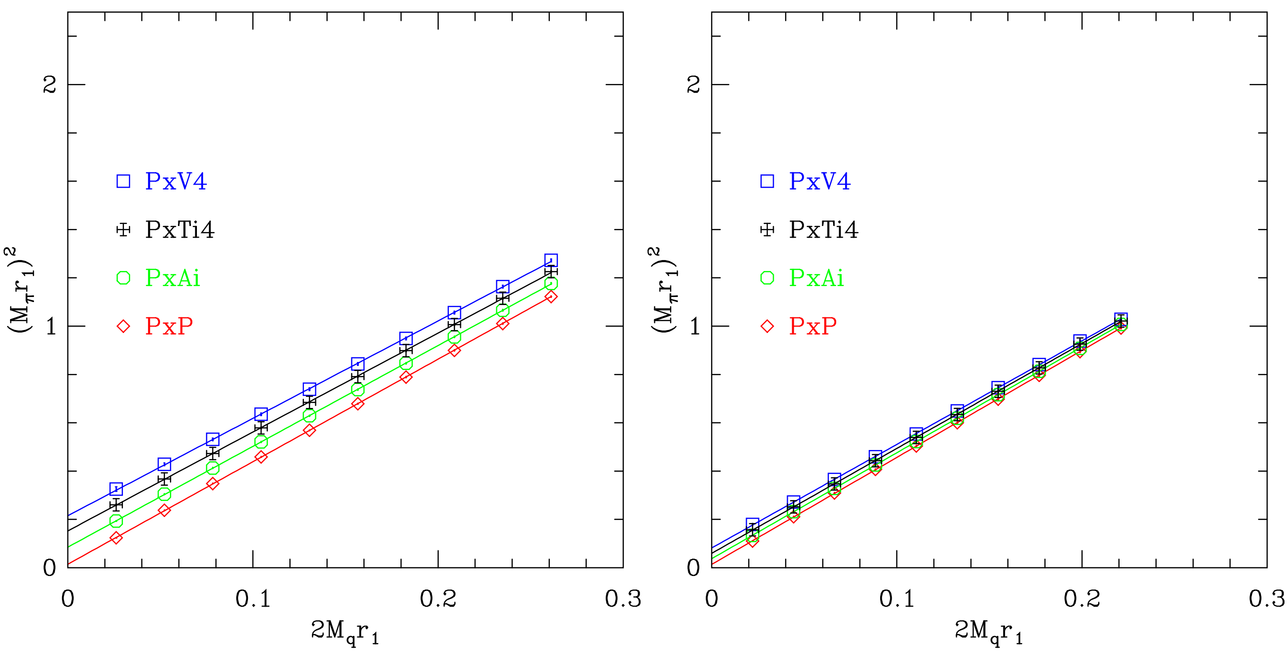

Figure 2: Pion mass-squareds on coarse ($am_\ell/am_s = 0.01/0.05$) and fine (0.0062/0.031) lattices. Same tastes are shown as in Fig. 1, except that scale is set using r_1 rather than a^{-1}. Results on the fine lattices are from 399 lattices and are preliminary.

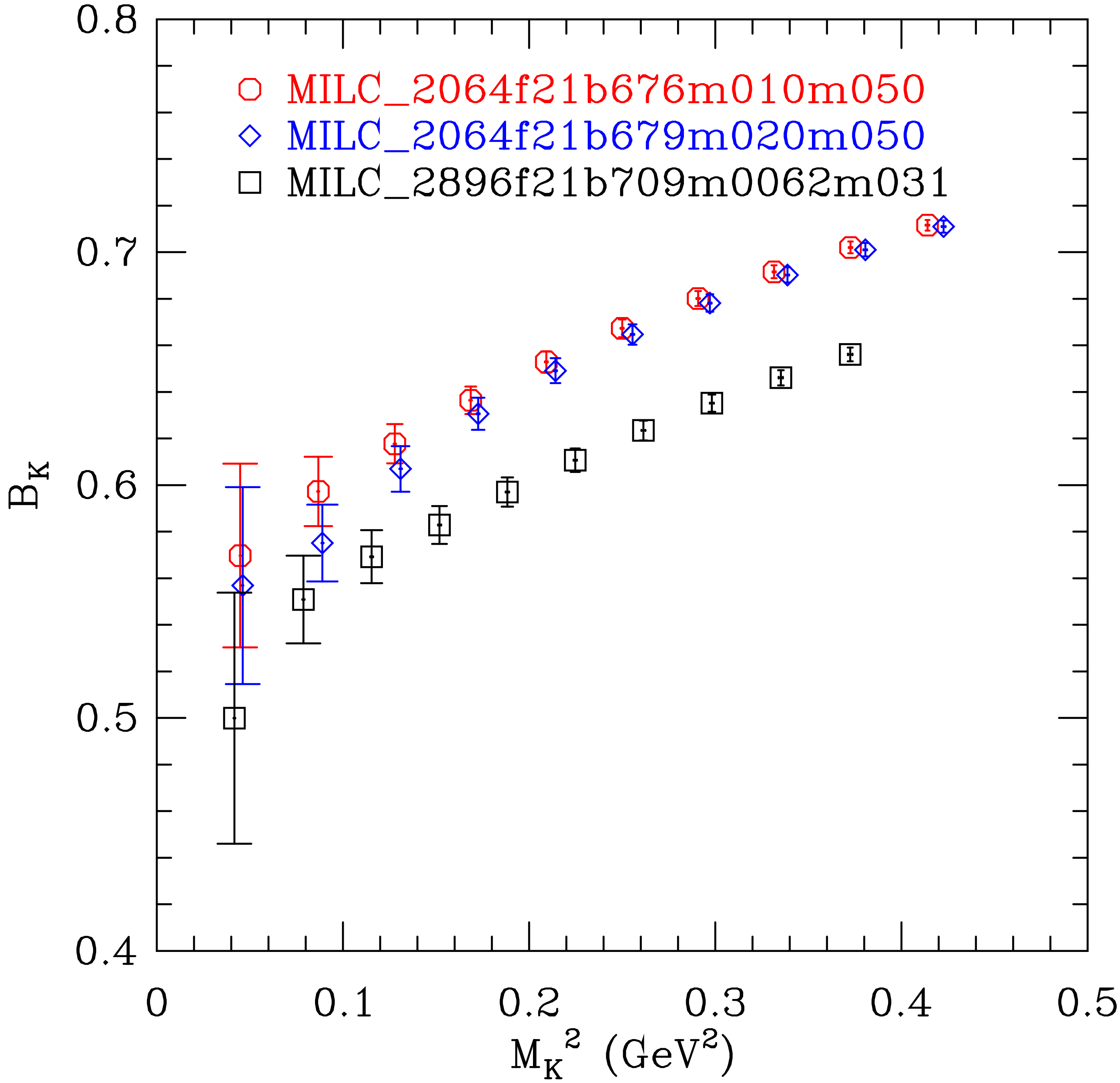

Figure 3: Tree-level B_K vs. m_K^2 in physical units, for two coarse ($am_\ell/am_s = 0.01/0.05$ and 0.02/0.05) and one fine (0.0062/0.031) lattice. Only results for “kaons” composed of degenerate quarks are shown. All results are preliminary.

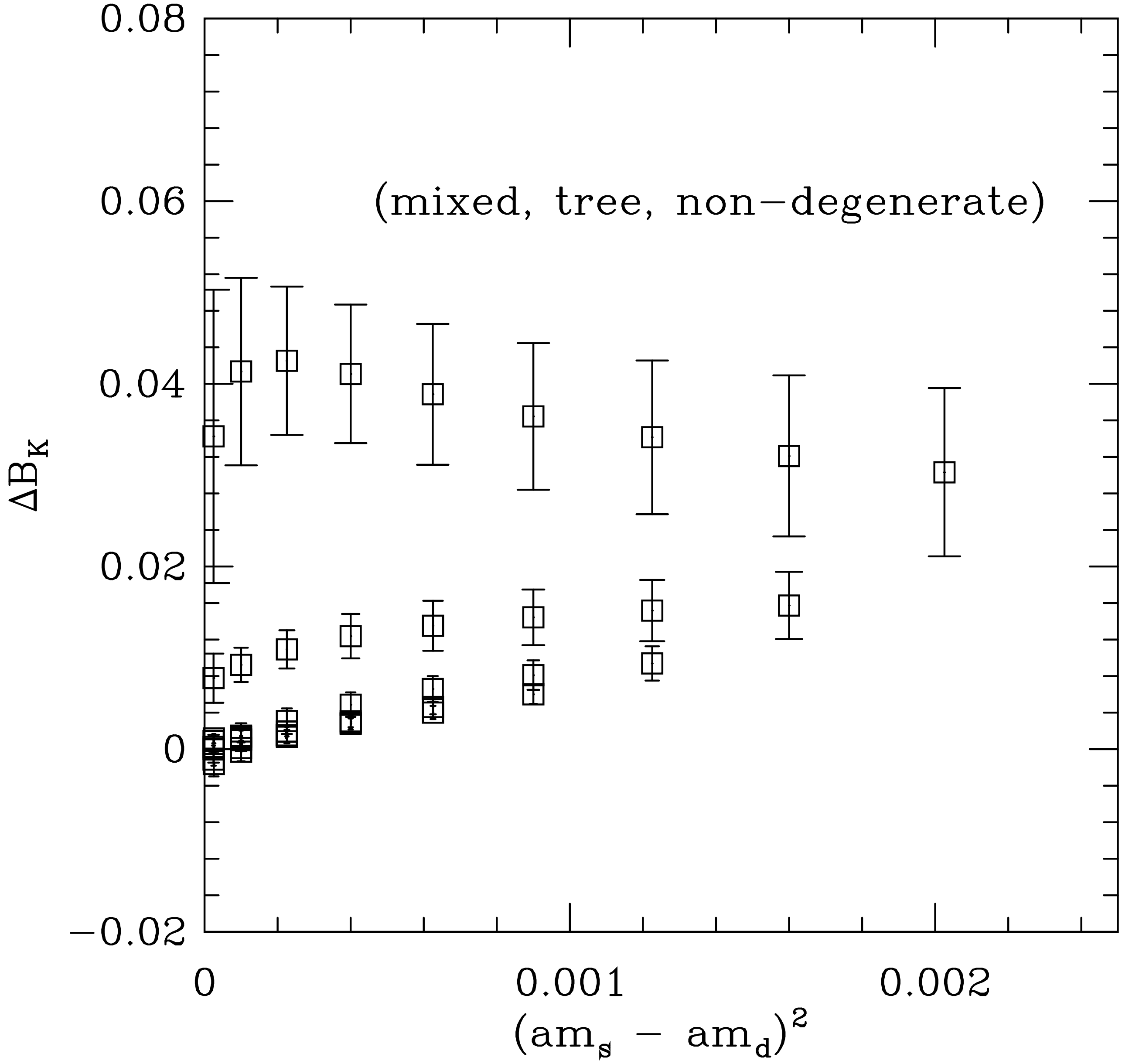

Figure 4: Non-degenerate part of tree-level B_K, ΔB_K (defined in text), vs. squared bare quark mass difference, on the coarse $am_\ell/am_s = 0.01/0.05$ lattices. Results are preliminary.

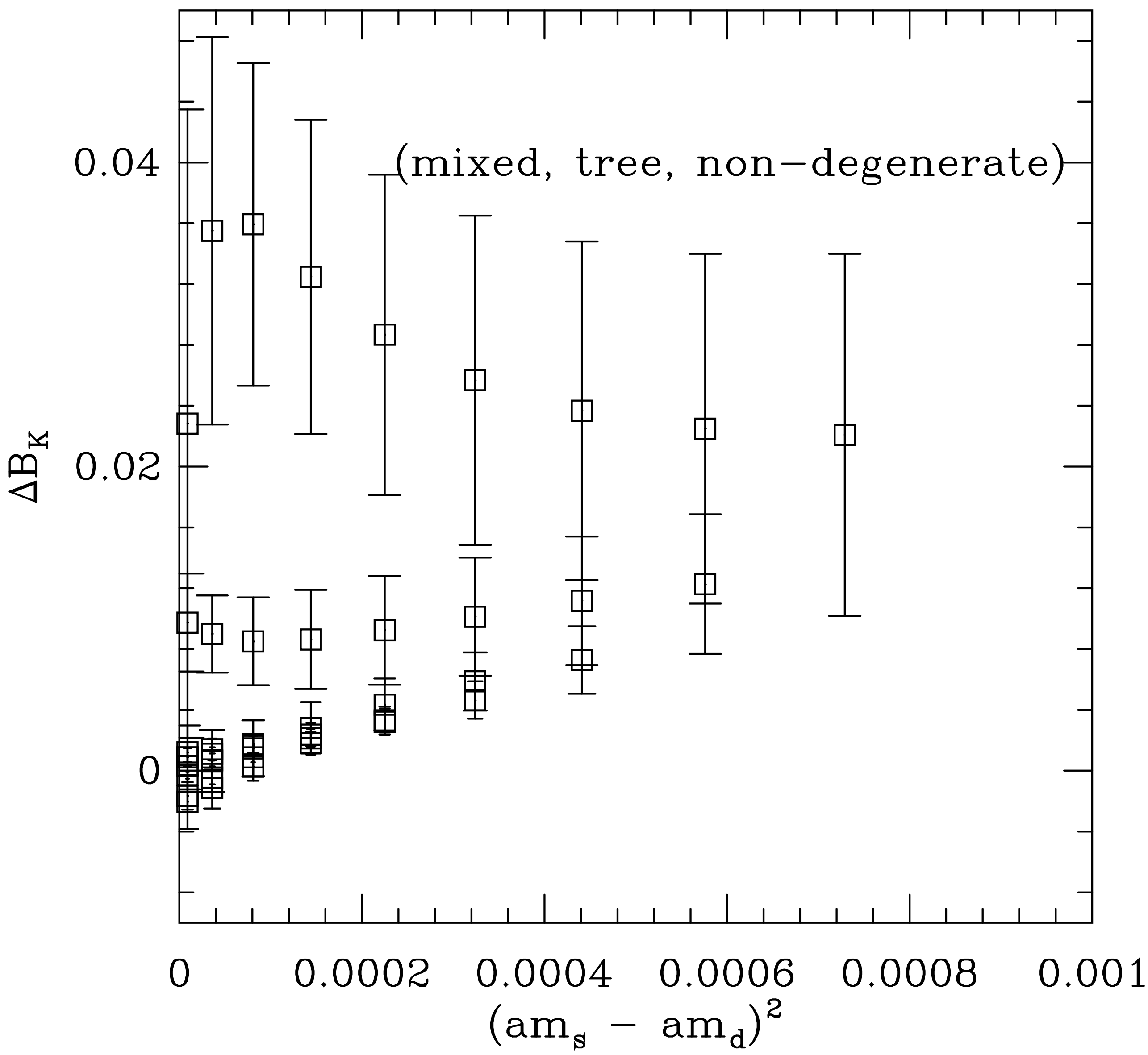

Figure 5: As for Fig. 4 but on the fine $am_\ell/am_s = 0.0062/0.031$ lattices.

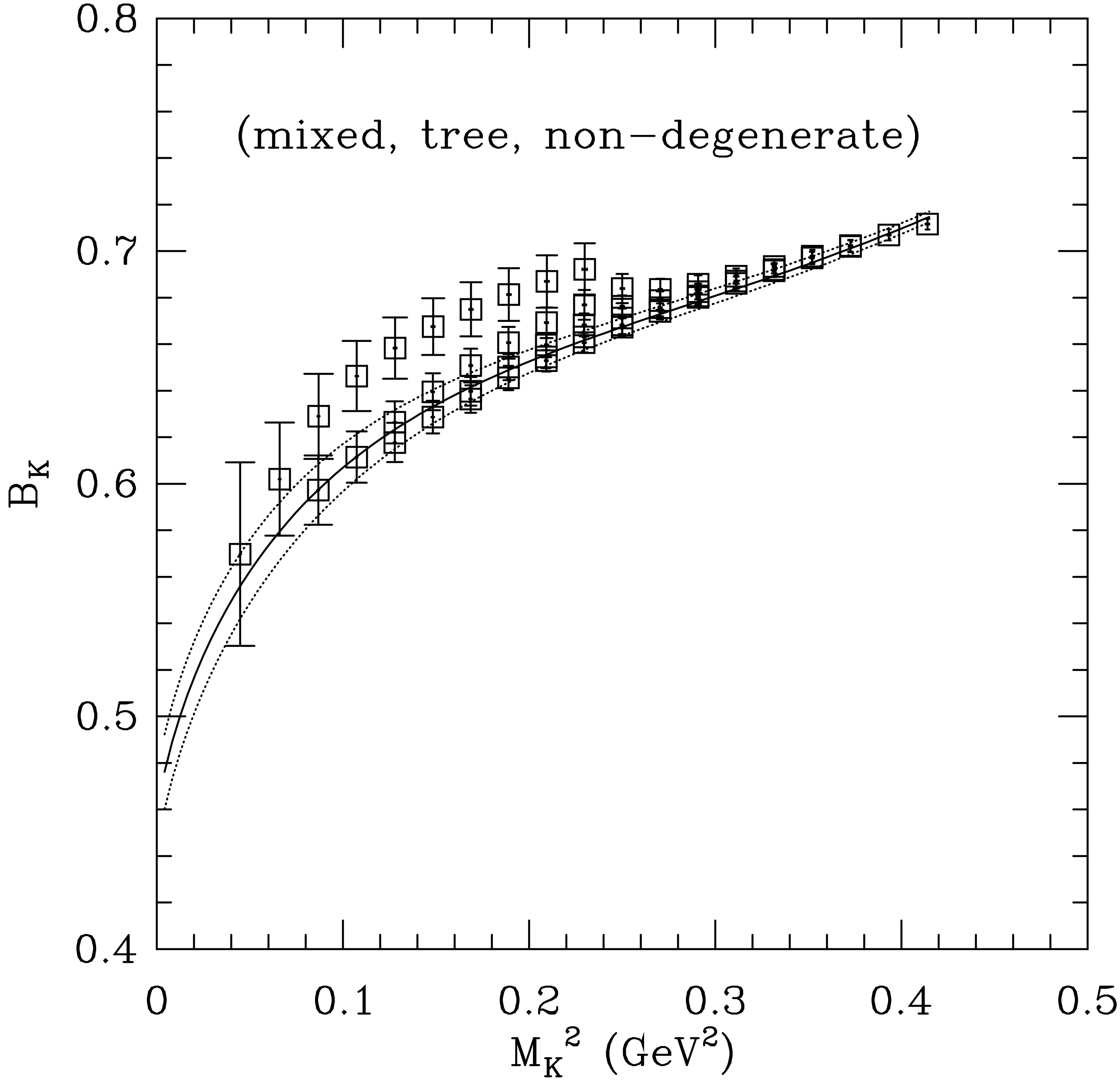

Figure 6: Results with degenerate and non-degenerate quarks for B_K at tree-level on the coarse $am_\ell/am_s = 0.01/.05$ lattices. The fit is to the continuum PQChPT form, and is for illustration only (it is *not* the fit used to calculate ΔB_K). Results are preliminary.