Progress in Kaon Physics on the Lattice

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Outline

- Pion, Kaon spectrum
- f_π and f_K
- Indirect CP violation: B_K
- Direct CP violation: $\mathrm{Re}(\epsilon'/\epsilon)$
- K_{l3} decays
- Kaon distribution amplitude

Pion, Kaon Spectrum

Staggered Pion Multiplet Spectrum

- Taste symmetry breaking (staggered χPT):
  $\mathcal{O}(a^2) \approx \mathcal{O}(p^2)$: $SU(4) \rightarrow SO(4)$
  $\mathcal{O}(p^2 a^2) \approx \mathcal{O}(p^4)$: $SO(4) \rightarrow SW_4$

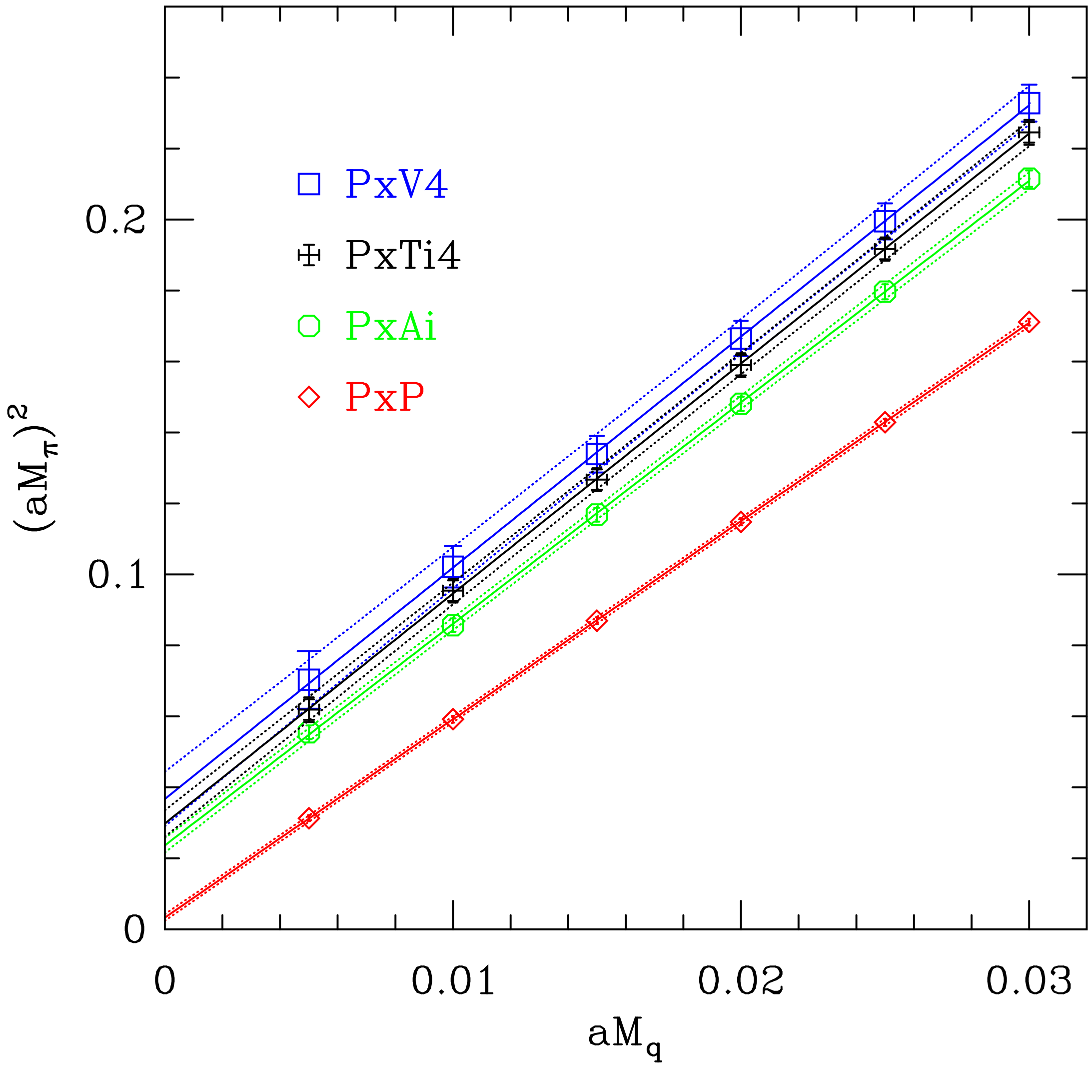
- 1 Goldstone pion: $(\gamma_5 \otimes \xi_5)$

- 15 non-Goldstone pions:

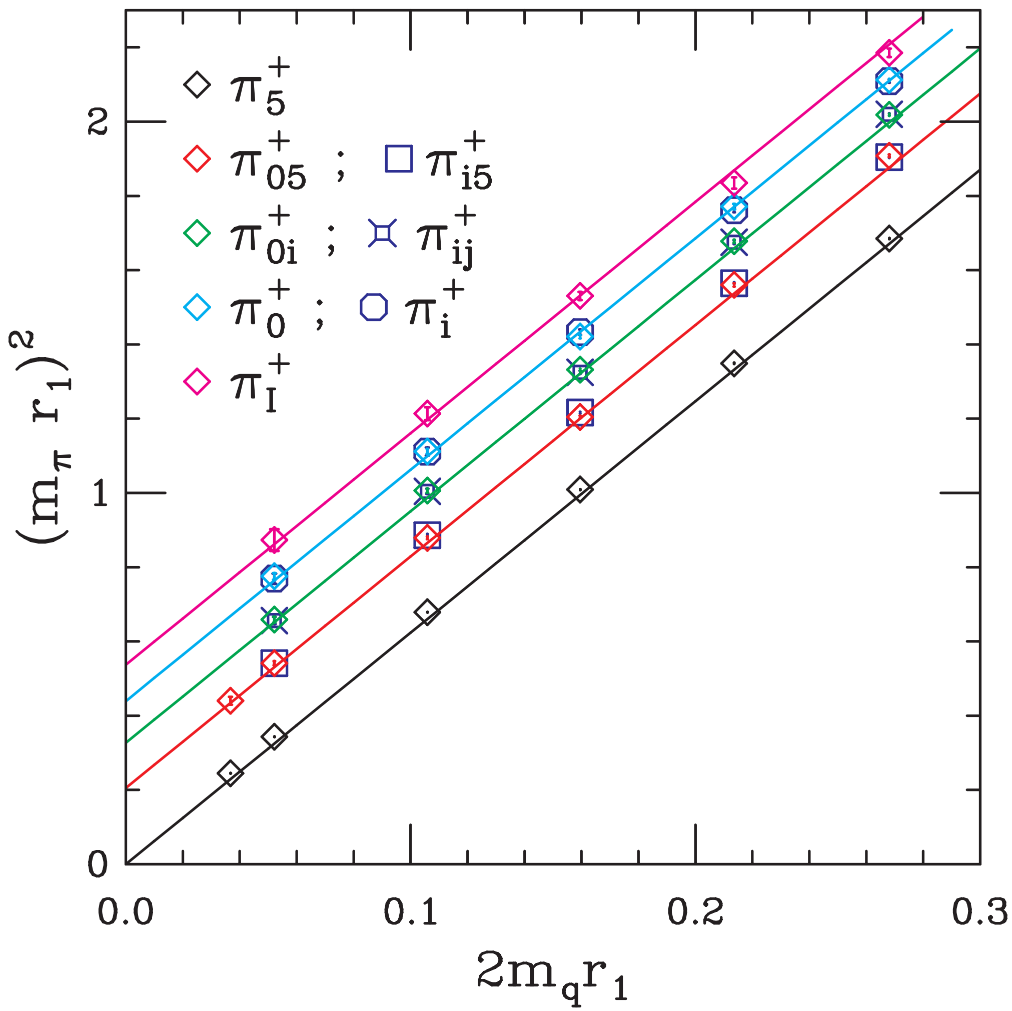
  $(\gamma_5 \otimes 1)$, $(\gamma_5 \otimes \xi_\mu)$, $(\gamma_5 \otimes \xi_{\mu 5})$, $(\gamma_5 \otimes \xi_{\mu\nu})$,

- Splitting between pion multiplets is a non-perturbative measure of taste symmetry breaking.

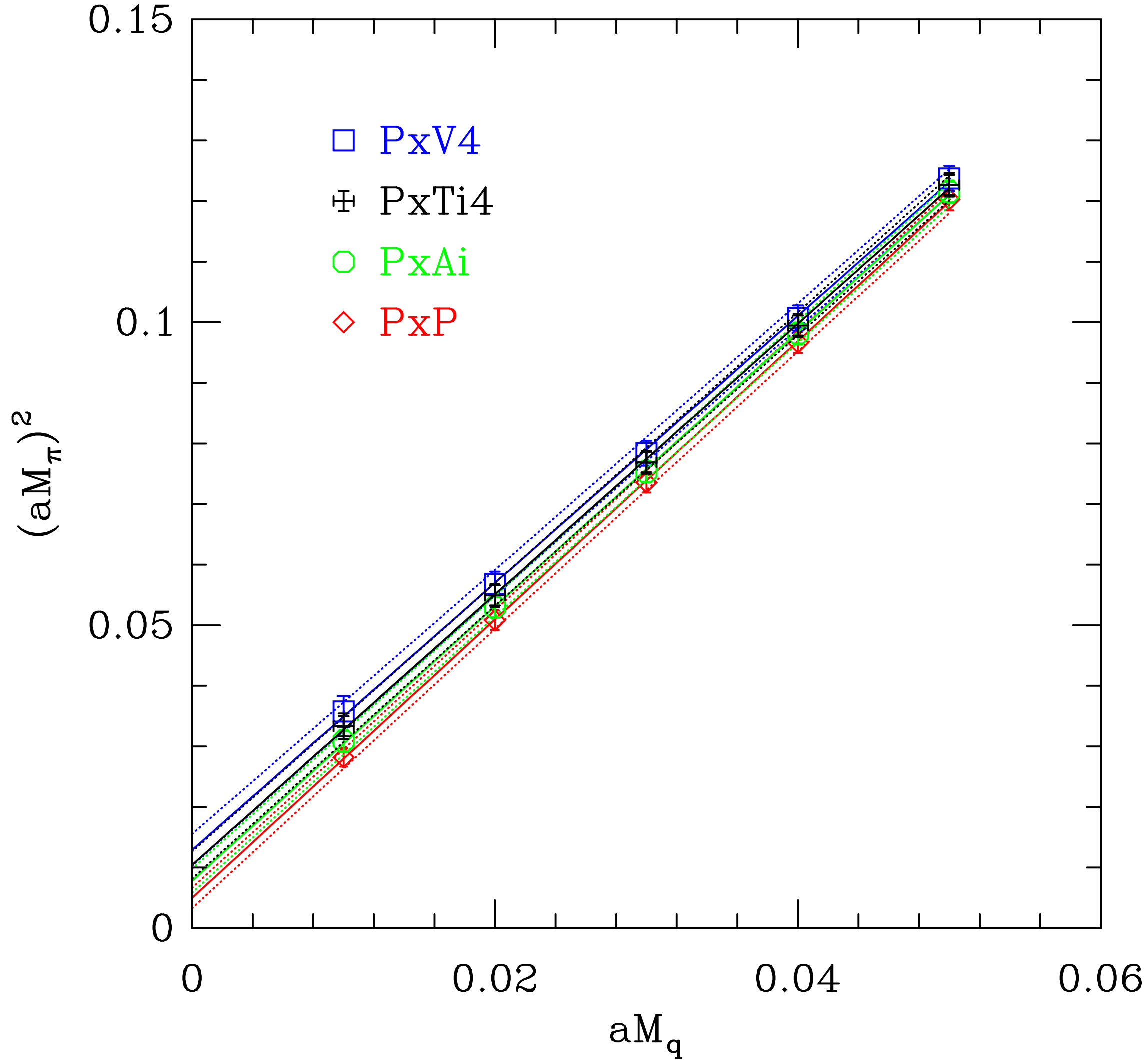
Stag + Wilson glu, Bae



AsqTad + Symanzik glu, MILC 2004



HYP Stag + Wilson glu, Kim



Summary

- For unimproved staggered fermions, we observe different slopes for various pion muliplets, which come from the $\mathcal{O}(a^2p^2)$
- In the case of AsqTad and HYP, the $\mathcal{O}(a^2p^2)$ effects are suppressed so much that they are negligible.
- In the case of AsqTad, the splittings among the pion mutiplets are larger than the light pion masses. $\mathcal{O}(a^2) \approx \mathcal{O}(p^2)$.
- In the case of HYP stag, the splitting among the pion multiplets are smaller than the size of the light pion

masses.
$\mathcal{O}(a^2) \ll \mathcal{O}(p^2)$.

- Improvement efficiency: (taste sym breaking)

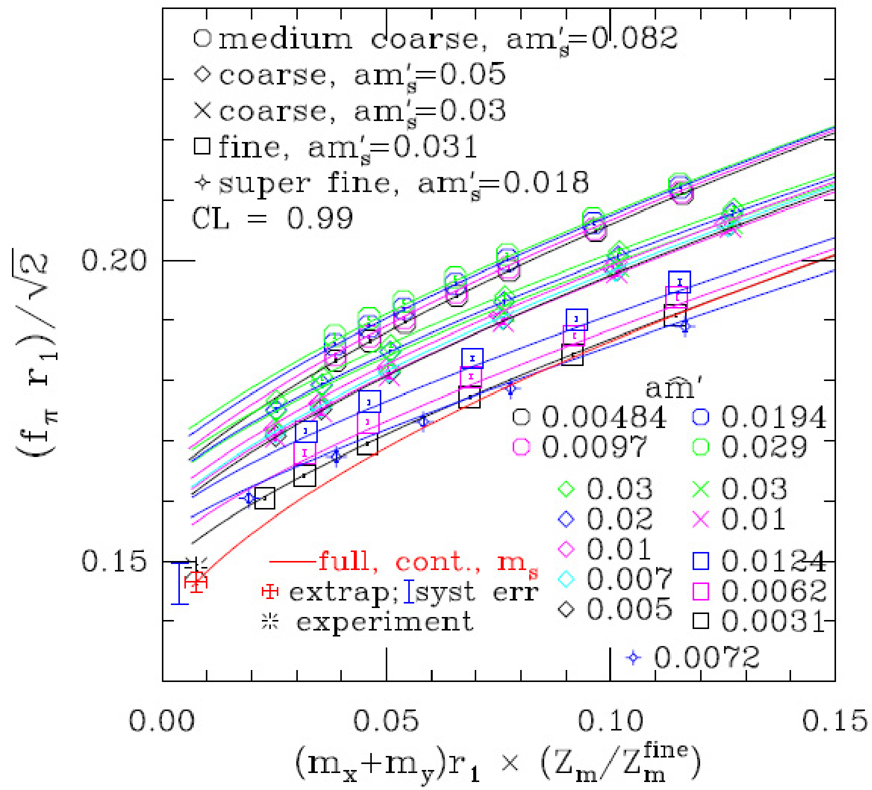
  unimproved stag $<$ AsqTad $<$ HYP stag

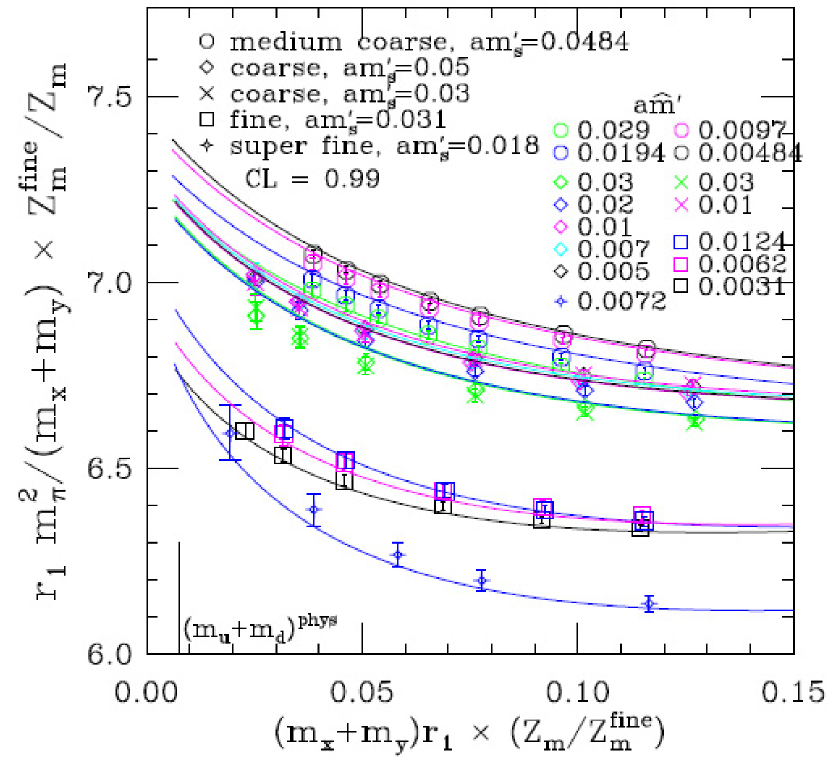
- For more details:

  go to POSTER by Bae, Kim, Lee and Sharpe.

f_π and f_K

MILC: $N_F = 2 + 1$, 978 d.p., Bernard

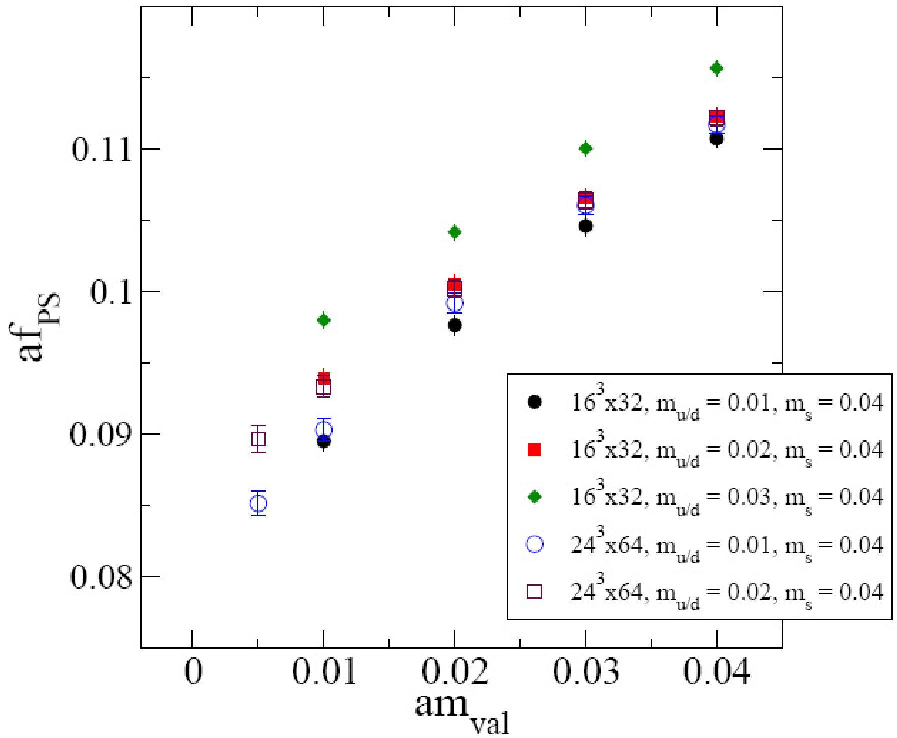


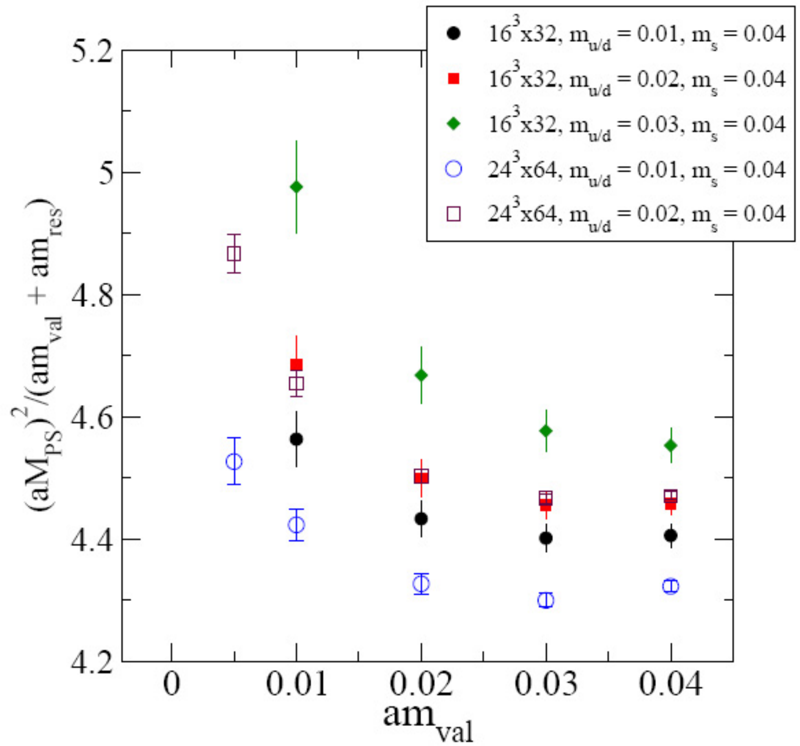


$$f_\pi = 128.6 \pm 0.4 \pm 3.0 \text{ MeV}$$

$$f_K = 155.3 \pm 0.4 \pm 3.1 \text{ MeV}$$

$N_F = 2 + 1$, DWF, RBC+UKQCD, Lin





$f_\pi = 125 \pm 4$ MeV (Preliminary)

$f_K = 148 \pm 4$ MeV (Preliminary)

f_K/f_π and L_5

- Chiral Perturbation Theory (NLO):

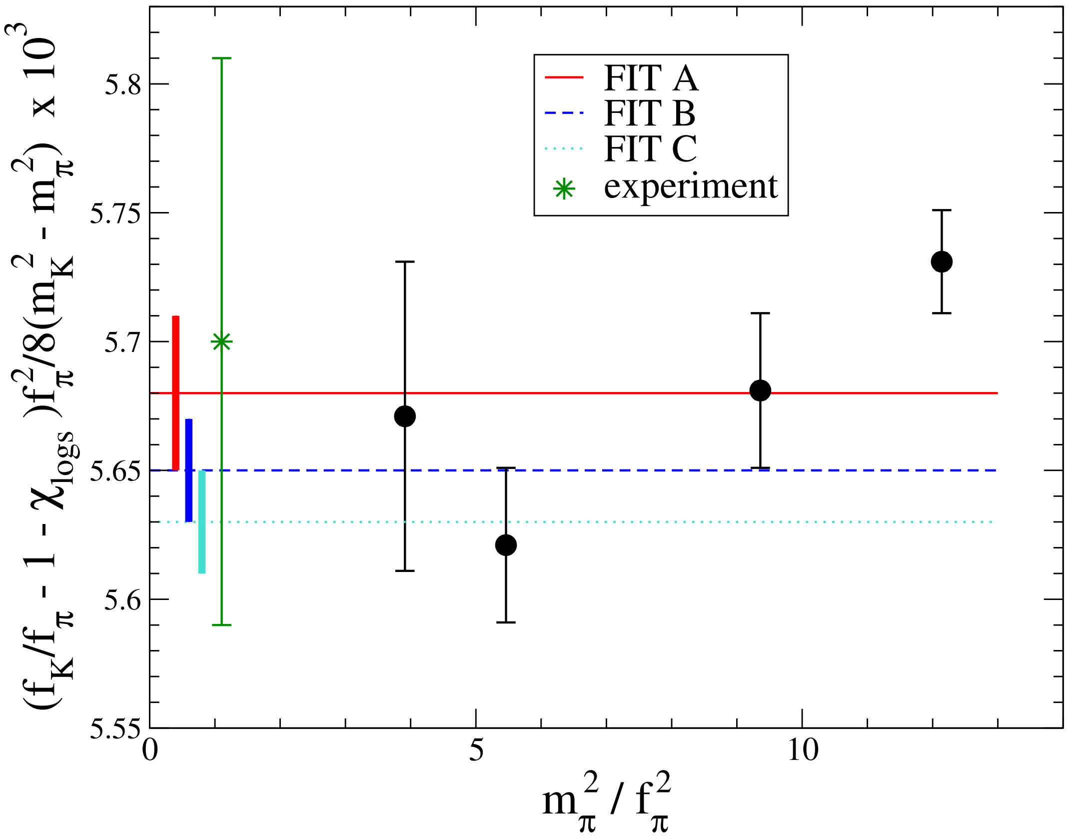
$$\frac{f_k}{f_\pi} = 1 + \chi_{log} + (8y)L_5$$

$$y = \frac{m_K^2 - m_\pi^2}{f_\pi^2}$$

- L_5 at NLO:

$$L_5 = \left(\frac{f_k}{f_\pi} - 1 - \chi_{log}\right)\frac{1}{8y}$$

L_5 (DWF/AsqTad, Beane, et al)



$$f_K/f_\pi = 1.218(2)^{+0.011}_{-0.024} \qquad L_5 = 5.65(2)^{+0.18}_{-0.54} \times 10^{-3}$$

Summary

- $f_\pi^{quenched}/f_\pi^{N_F=2+1} \approx 1.28$ with the r_1 scale. This issue is completely resolved.

- The NPLQCD calculation with a mixed action (DWF/AsqTad) gives a reasonable value for f_K/f_π consistent with MILC.

- Puzzle: MILC has used stag χPT and NPLQCD did not use stag χPT.

B_K

Definition

- Experiment:

$$\epsilon = (2.280 \pm 0.013) \times 10^{-3} \times \exp(i\pi/4)$$

- Standard model: ϵ

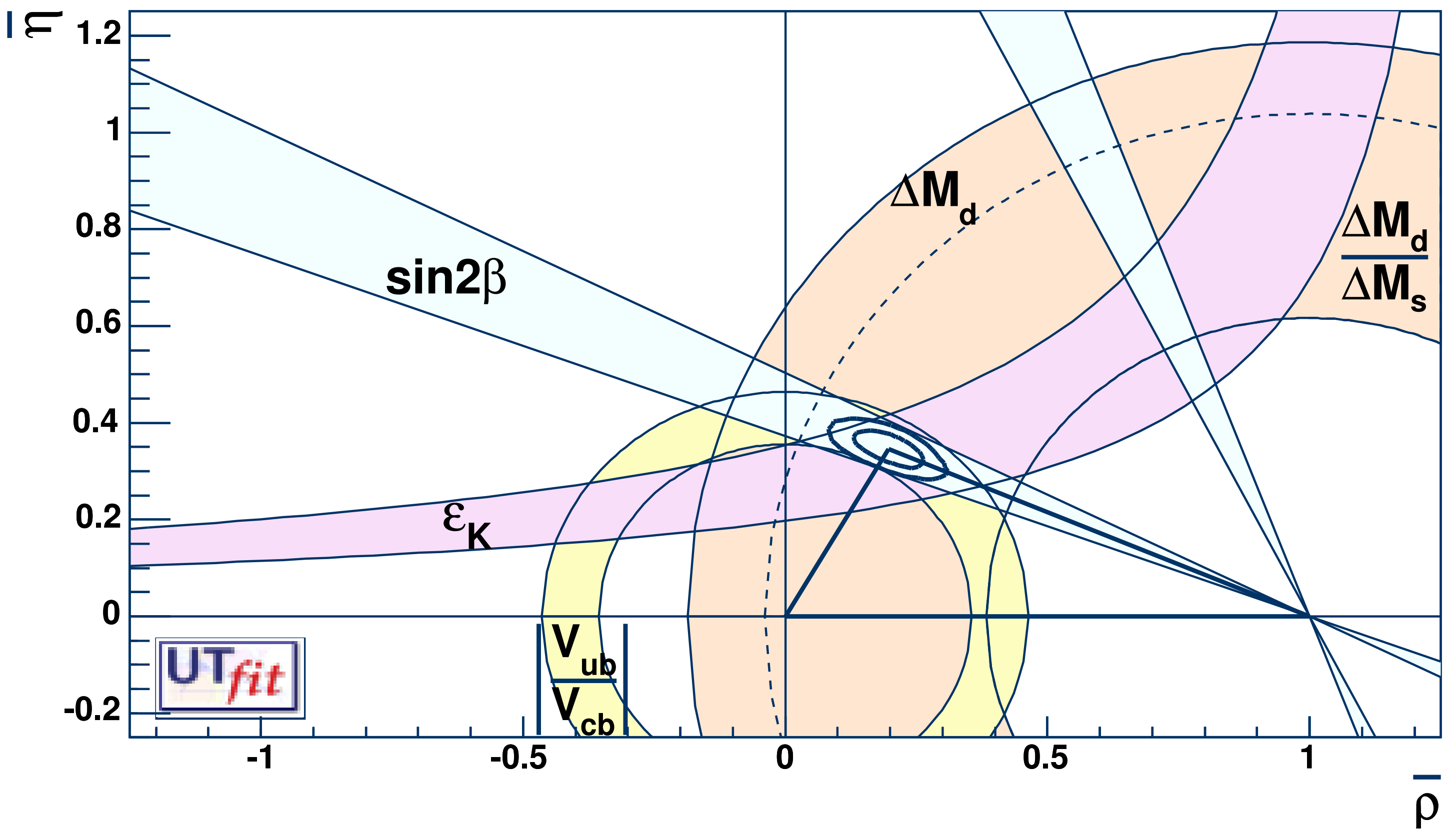
$$\begin{aligned}
\epsilon &= C_\epsilon \ \exp(i\pi/4)\ \mathrm{Im}\lambda_t \ X \ \hat{B}_K \\
X &= \mathrm{Re}\lambda_c[\eta_1 S_0(x_c) - \eta_3 S_3(x_c, x_t)] - \mathrm{Re}\lambda_t \eta_2 S_0(x_t) \\
\lambda_i &= V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2 \\
C_\epsilon &= \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}
\end{aligned}$$

- Definition of B_K and $\hat{B}_K$

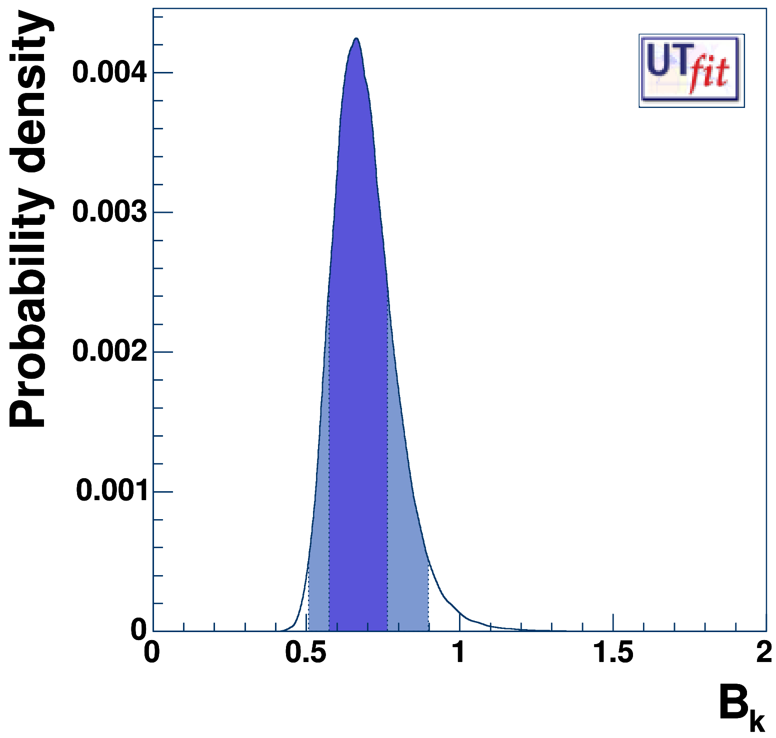
$$
\begin{aligned}
B_K &= \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle} \\
\hat{B}_K &= C(\mu) B_K(\mu) \\
C(\mu) &= \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]
\end{aligned}
$$

- Key Point: Can SM explain the experiment?

CKM fit

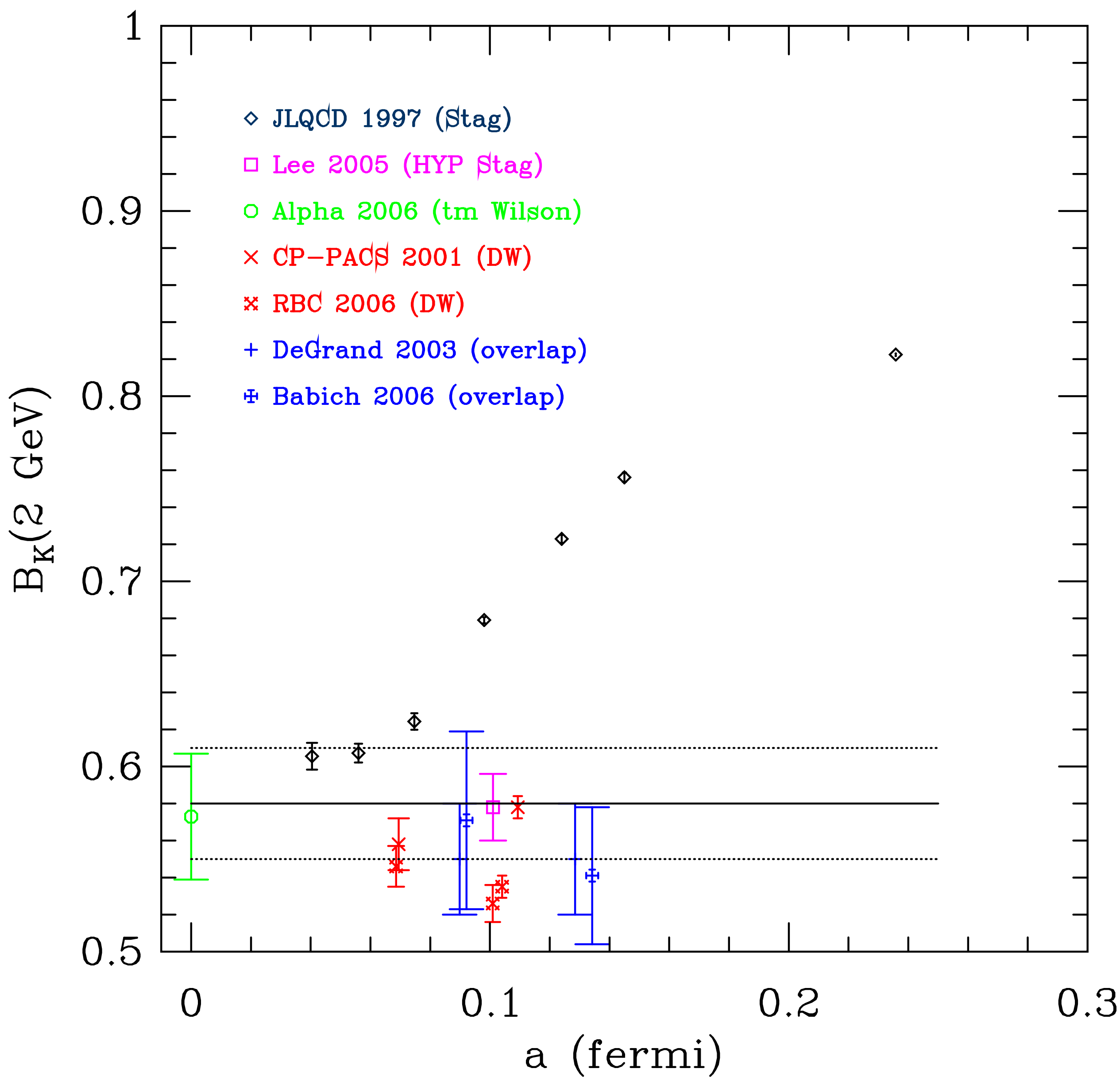

$\bar{\eta}$
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-1
-0.5
0
0.5
1
$\bar{\rho}$
sin2β
ε_K
ΔM_d
$\frac{\Delta M_d}{\Delta M_s}$
$\frac{V_{ub}}{V_{cb}}$
UTfit

Unitarity Triangle

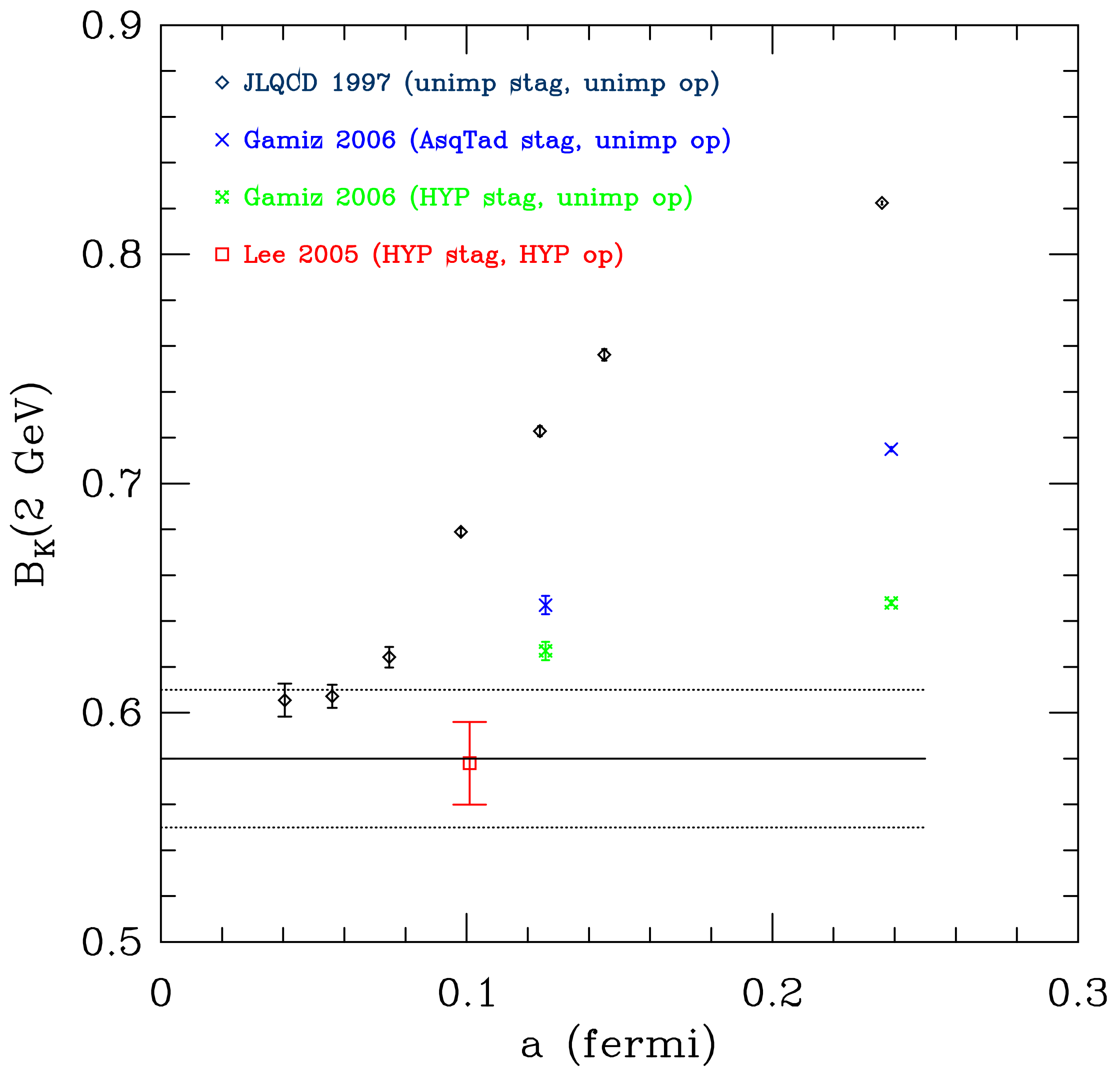


$$\hat{B}_K = 0.68 \pm 0.10$$

B_K in quenched QCD


JLQCD 1997 (Stag)
Lee 2005 (HYP Stag)
Alpha 2006 (tm Wilson)
CP−PACS 2001 (DW)
RBC 2006 (DW)
DeGrand 2003 (overlap)
Babich 2006 (overlap)
$B_K(2\ GeV)$
1
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3
a (fermi)

Improvement eff. for Stag



Quenched B_K

- World Average:

$$B_K(\mathrm{NDR}, 2GeV) = 0.58 \pm 0.03 \text{ (statistical)}$$

$$\hat{B}_K = 0.81 \pm 0.04 \text{ (statistical)}$$

- Systematic Errors: = degeneracy + quenching $\approx$ 10%

$$B_K(\mathrm{NDR}, 2GeV) = 0.58 \pm 0.03 \pm 0.06$$

$$\hat{B}_K = 0.81 \pm 0.04 \pm 0.08$$

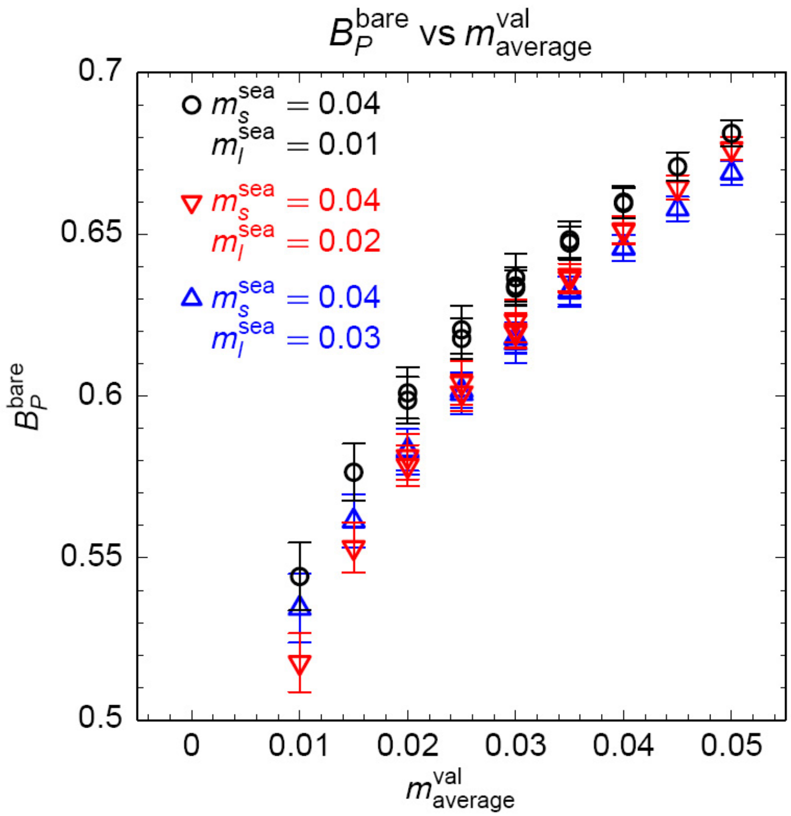
Summary

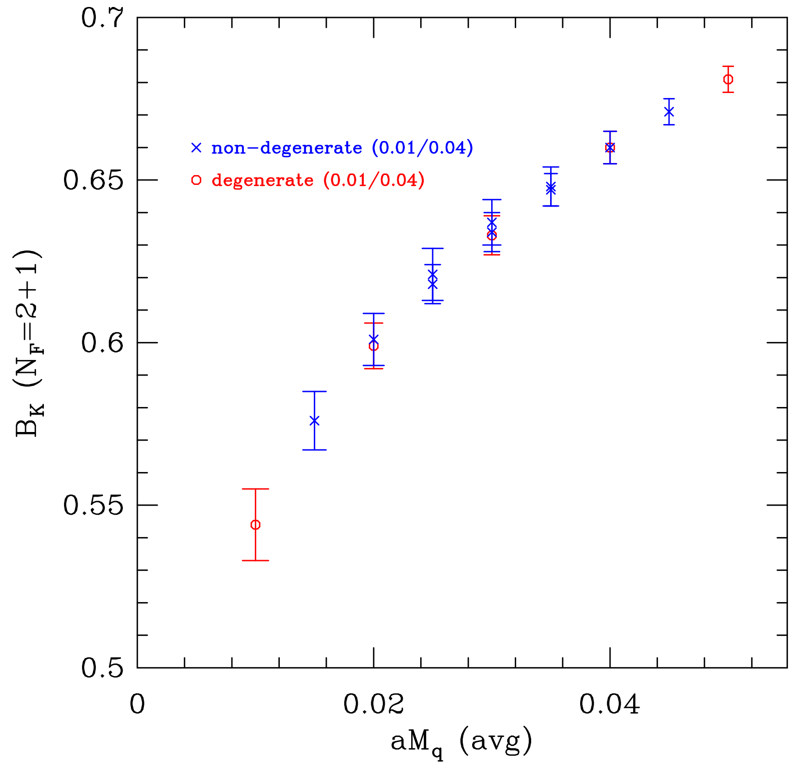
- In the end, the B_K result of TM Wilson agrees with those of HYP stag, Domain Wall, and Overlap (Plenary talk by Carlos Pena).
- It turns out that HYP stag has much smaller scaling violation in B_K than AsqTad.
- Improvement Efficiency: (scaling violation)

  unimproved stag $<$ AsqTad $<$ HYP stag

B_K in full QCD

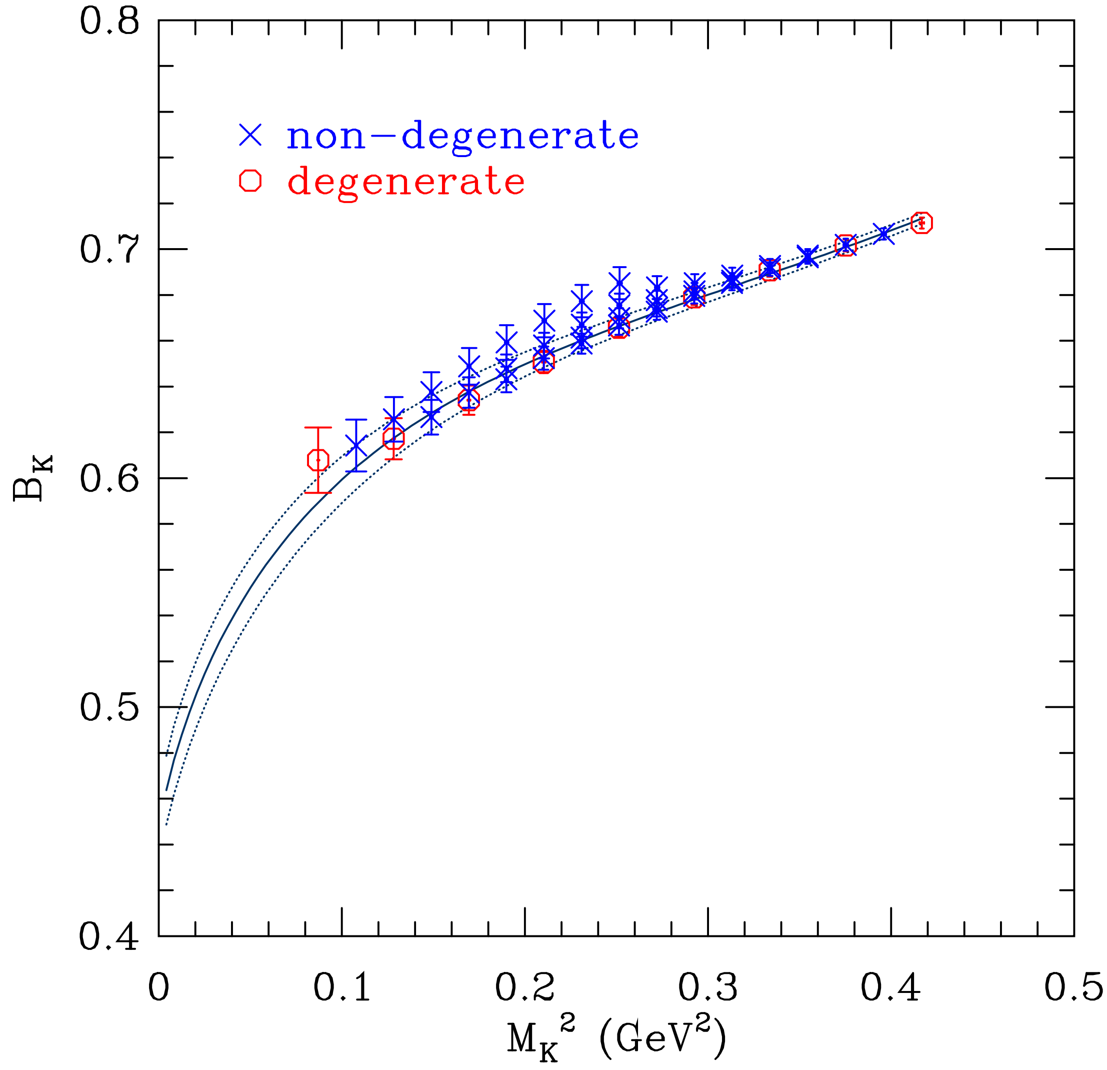
B_K ($N_F = 2 + 1$, DWF, RBC+UKQCD)



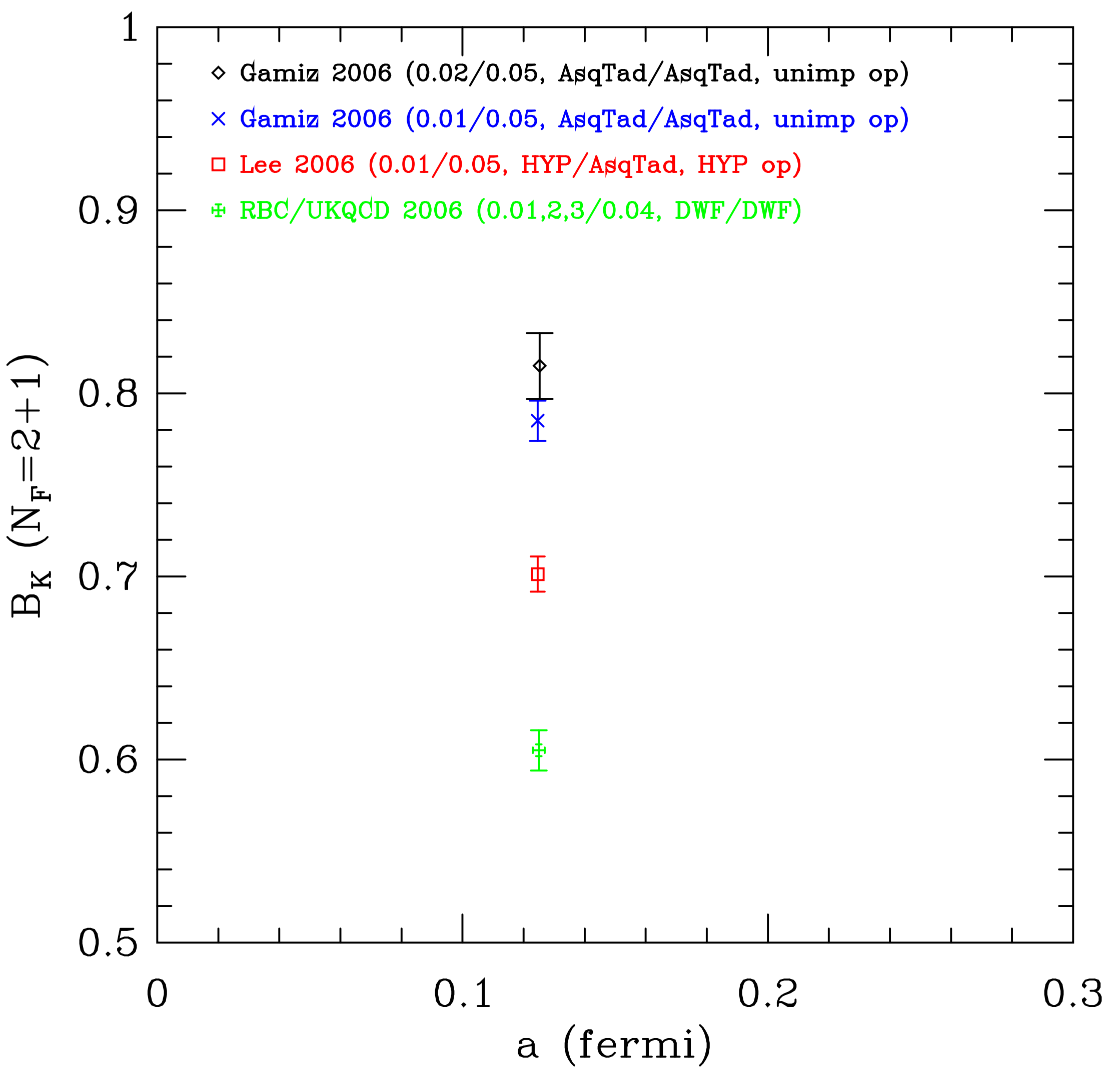


$B_K(\mathrm{NDR}, \mu = 2GeV) = 0.545(22)$ (Preliminary)

B_K ($N_F = 2+1$, HYP/AsqTad, Lee)



Bare B_K ($N_F = 2 + 1$)


Gamiz 2006 (0.02/0.05, AsqTad/AsqTad, unimp op)
Gamiz 2006 (0.01/0.05, AsqTad/AsqTad, unimp op)
Lee 2006 (0.01/0.05, HYP/AsqTad, HYP op)
RBC/UKQCD 2006 (0.01,2,3/0.04, DWF/DWF)
B_K (N_F=2+1)
a (fermi)
1
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3

Summary

- It will be possible to do full fitting to the functional form suggested by staggered χPT (in 2007).
- It will be possible to probe the dependence on the sea quark masses (in 2007).
- It is possible to pin down the effect of non-degenerate quark masses on B_K systematically (in 2006, 2007).

Future Perspective

- Precision goal = 10% (Good!)

  (1) One-loop matching

  (2) Partially quenched χPT

- Precision goal = 5% (Excellent!)

  (1) Two-loop matching

  (2) Non-perturbative renomalization

  (3) Staggered χPT

ϵ'/ϵ

ϵ'/ϵ (Theory)

- Standard model:

$$\epsilon'/\epsilon = Im(V_{ts}^* V_{td})\left[P^{(1/2)} - P^{(3/2)}\right]$$

$$P^{(1/2)} = r\sum_{i=3}^{10} y_i(\mu)\langle Q_i\rangle_0(1-\Omega_{\eta+\eta'})$$

$$P^{(3/2)} = \frac{r}{\omega}\sum_{i=3}^{10} y_i(\mu)\langle Q_i\rangle_2$$

$$\langle Q_i\rangle_I = \langle\pi\pi(I)|Q_i|K\rangle$$

$$r = \frac{G_F\omega}{2|\epsilon|ReA_0}$$

ϵ'/ϵ (Theory)

- No contribution from $\langle Q_1\rangle_I$ and $\langle Q_2\rangle_I$.
- $P^{(1/2)}$ is dominated by $\langle Q_6\rangle_0$.
- $P^{(3/2)}$ is dominated by $\langle Q_8\rangle_2$.

Ambiguity in QCD Penguin Op.

- Example: Left-Right QCD Penguin Operator:

$$Q_6 \;=\; \bar{s}\gamma_\mu(1-\gamma_5)d\sum_q \bar{q}\gamma_\mu(1+\gamma_5)q$$

- $Q_6 \in (8,1)$ of $SU(3)_L \otimes SU(3)_R$
- Unfortunately, it mixes with $Q_3^*, \ldots, Q_6^*$ in (partially) quenched QCD.

$$Q_6^* \;=\; \bar{s}\gamma_\mu(1-\gamma_5)d\sum_{\tilde{q}} \bar{\tilde{q}}\gamma_\mu(1+\gamma_5)\tilde{q}$$

Group Theory for Q_6.

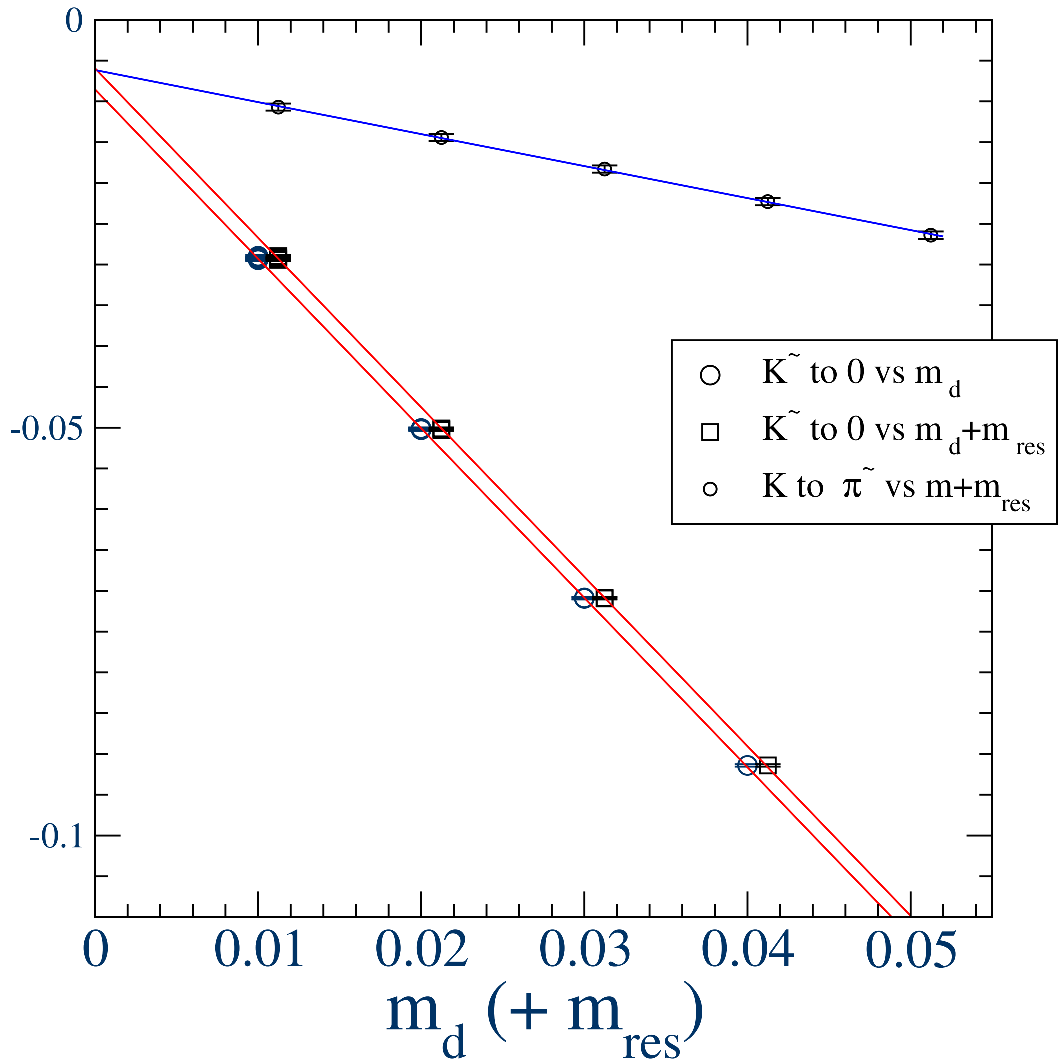
- Golterman and Pallante (2001):

$$
\begin{aligned}
Q_6 &= Q_6^S + Q_6^{NS} \\
Q_6^S &\in \text{ singlet rep of } SU(3+N_F,3)_R \\
Q_6^{NS} &\in \text{ adjoint rep of } SU(3+N_F,3)_R
\end{aligned}
$$

- We need additional low energy constant: α_{NS}. Golterman & Peris predicted that $\alpha_{NS} \approx 60\alpha_{(8,1)}$.

- This implies that we can not obtain $\langle Q_6 \rangle_0$ in quenched QCD reliably.

α_{NS} (quenched, RBC)



Group Theory for $Q_1, \cdots, Q_4$

- Golterman and Pallante (2006): for $SU(3+N_F, 3)_L$,

$$
\begin{aligned}
Q^{S}_{\pm} &= (\bar{s}_\alpha d_\alpha)_L (\bar{q}_\beta q_\beta)_L \pm (\bar{s}_\alpha d_\beta)_L (\bar{q}_\beta q_\alpha)_L \\
Q^{NS}_{\pm} &= (\bar{s}_\alpha d_\alpha)_L (\bar{q}_\beta A q_\beta)_L \pm (\bar{s}_\alpha d_\beta)_L (\bar{q}_\beta A q_\alpha)_L \\
A &= \mathrm{diag}(1 - \frac{3}{N_F}, 1 - \frac{3}{N_F}, 1 - \frac{3}{N_F}, -\frac{3}{N_F}, \cdots, -\frac{3}{N_F})
\end{aligned}
$$

- We need two sets of LEC for $\pm$.
- We need additional LEC (lattice artifacts): $\{\alpha^{NS\pm}_{1a}, \alpha^{NS\pm}_{1b}, \alpha^{NS\pm}_{2}\}$
- This changes $\mathrm{Re}A_0$ and ϵ'/ϵ through Q_1, Q_2 and Q_4 respectively.

Summary

- Old Testament: in quenched QCD,

$$\begin{aligned} Re(\epsilon'/\epsilon) &= (-7.7 \pm 2.0) \times 10^{-4} (\mathrm{CP-PACS}, 2001) \\ &= (-4.0 \pm 2.3) \times 10^{-4} (\mathrm{RBC}, 2001) \end{aligned}$$

- New Testament: in quenched QCD,
  1. We can NOT calculate reliably the leading contribution to $Re(\epsilon'/\epsilon)$ from Q_6 due to the ambiguity in Q_6.
  2. We can NOT calculate reliably even the sub-leading contribution to $Re(\epsilon'/\epsilon)$ from Q_4 due to the ambiguity in Q_4.

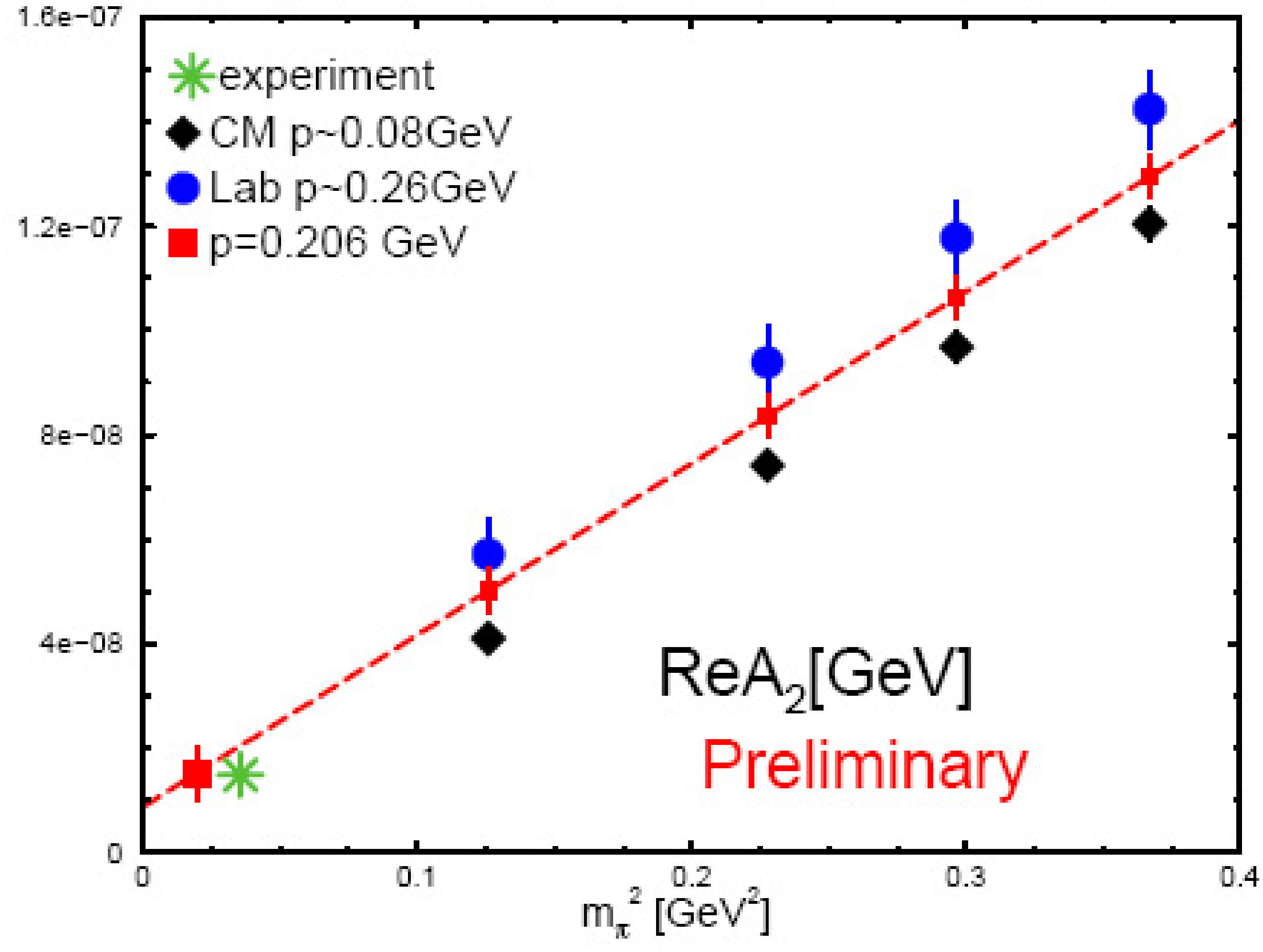
Solution to the problem

- We need to work on partially quenched QCD ($N_F = 2{+}1$ or $N_F = 3$).

Direct calculation of $K \to \pi\pi$

- Lellouch & Luscher (2001): finite volume effect on the CM frame $K(0) \to \pi(\vec{p})\pi(-\vec{p})$
- Ishizuka (2003): projection method
- C. Kim (2005): H parity BC / G parity BC
- Christ, Kim, Sharpe, Sachrajda, Yamazaki (2005): finite volume effect on the Lab frame
- T. Yamazaki (2006): on the Lab frame
  $K(\vec{p}) \to \pi(\vec{p})\pi(0)$

$Re(A_2)$ (quenched, Yamazaki)



K_{l3} decays

Precise determination of V_{us}

- Vector form factor:

$$\langle \pi(p') | V_\mu | K(p) \rangle = (p_\mu + p'_\mu) \cdot f_+(q^2) + q_\mu \cdot f_-(q^2)$$

where $q_\mu = p_\mu - p'_\mu$.

- $K \to \pi l^+ \nu_l$ decay rate:

$$\Gamma = \frac{G_F^2}{192\pi^3} M_K^5 |V_{us}|^2 |f_+(0)|^2 I (1+\delta)$$

- We know $|V_{us} \cdot f_+(0)| = 0.2173$ from experiment.

Scalar Form Factor

- $f_0(q^2)$

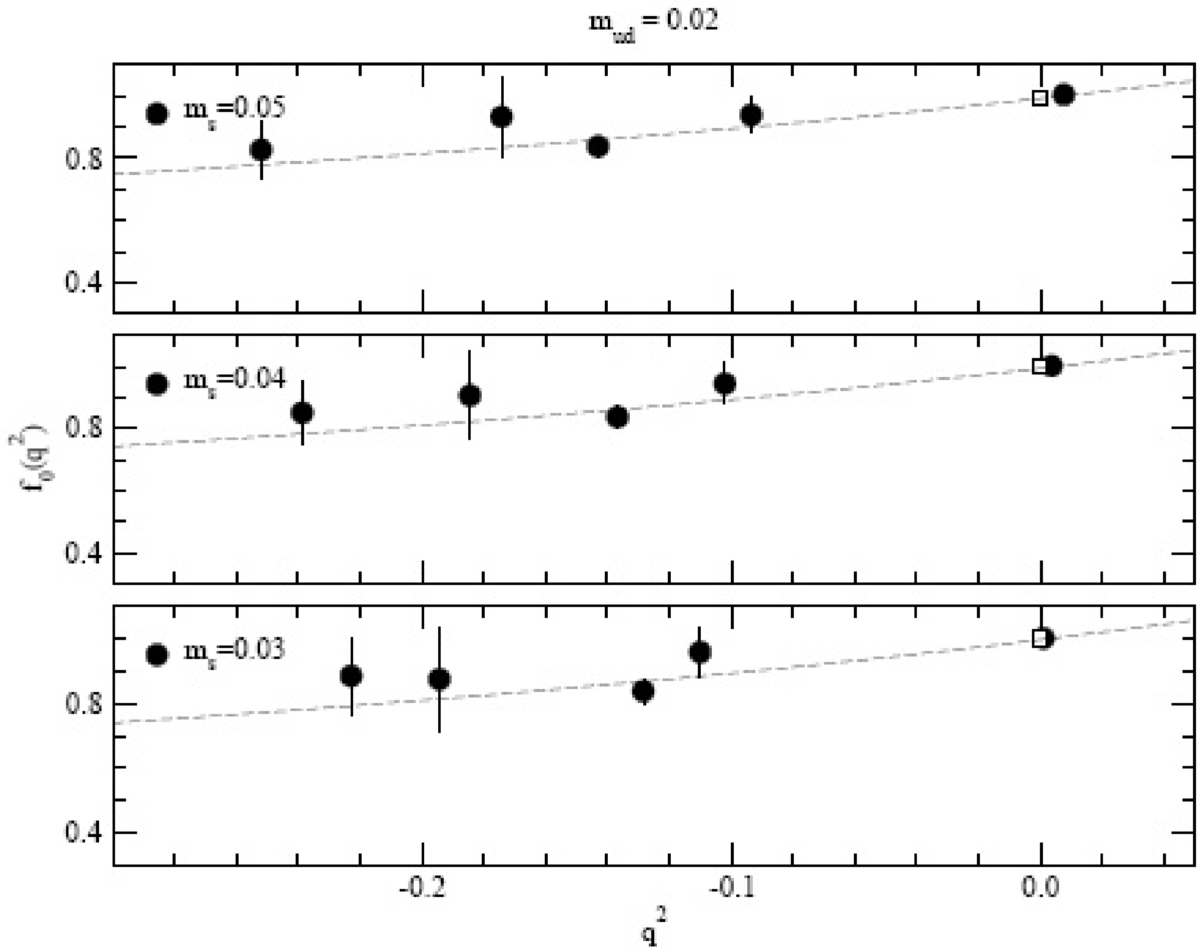
$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$$f_0(0) = f_+(0)$$

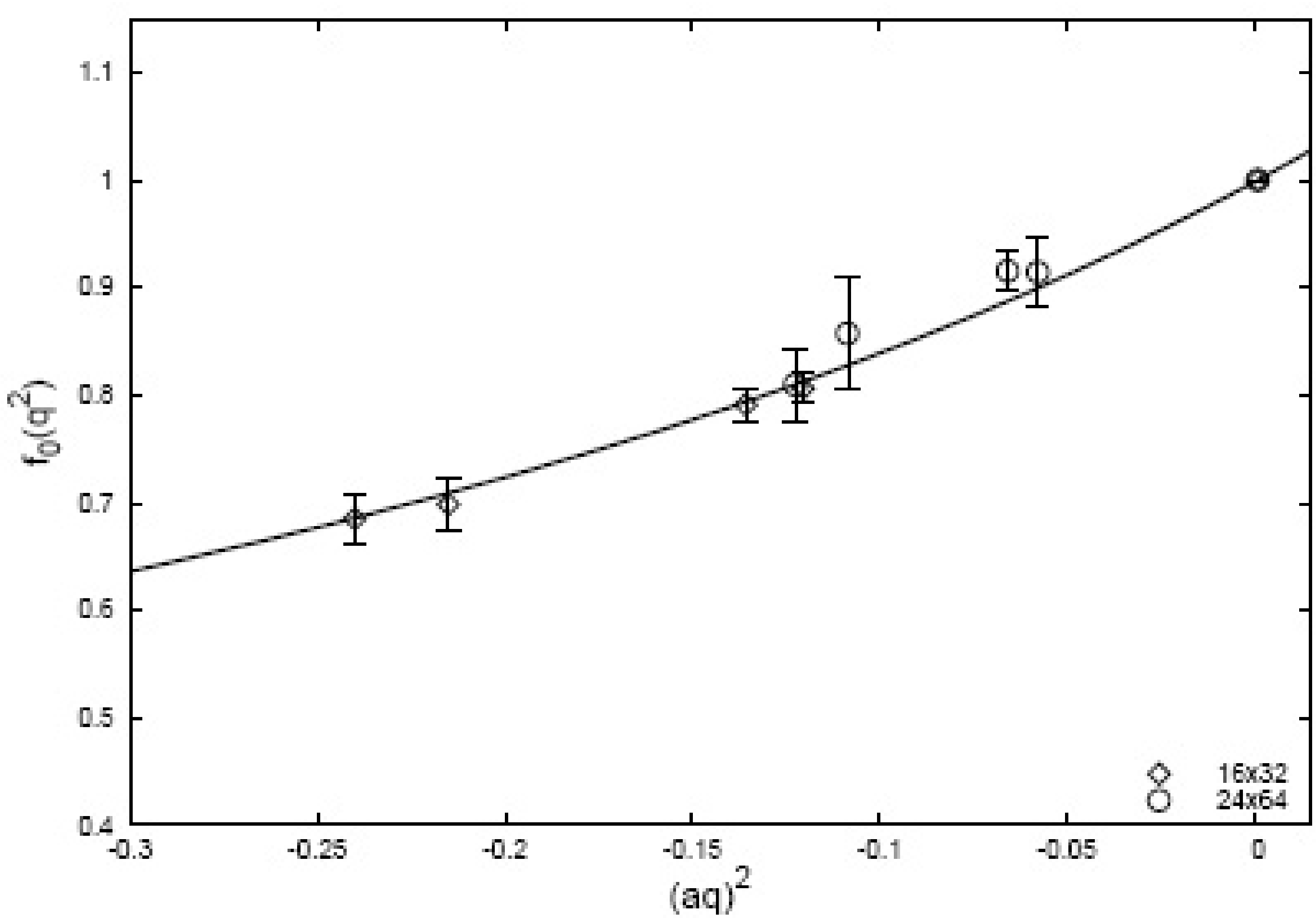
- $\xi(q^2)$

$$\xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}$$

$f_0(q^2)$ ($N_F = 2$, Soni and RBC)



$f_0(q^2)$ ($N_F = 2+1$, Zanotti and RBC)



Kaon Distibution Amplitude

Leading twist-2 Kaon DA

- Operator Representation:

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 P(z,-z)s(-z)|K^+(p)\rangle_{z^2=0} =$$
$$i f_K p_\mu \int_{-1}^{1} d\xi \ \exp(i\xi p\cdot z)\ \phi_K(\xi,\mu^2)$$
$$\int_{-1}^{1} d\xi\ \phi_K(\xi,\mu^2) = 1$$

- Partonic Picture:
  The s quark carries xp momentum with $x = (1+\xi)/2$.
  The $\bar{u}$ anti-quark carries $(1-x)p$ momentum.

n-th moment of Kaon DA

- Conformal Expansion (Gegenbauer polynomial):

$$\phi_K(\xi,\mu^2) \;=\; \frac{3}{4}(1-\xi^2)\left(1+\sum_{n=1}^{\infty} a_n(\mu^2) C_n^{3/2}(\xi)\right)$$

- Light-cone OPE and n-th moment:

$$\langle \xi^n \rangle(\mu^2) = \int_{-1}^{1} d\xi\; \xi^n\; \phi_K(\xi,\mu^2)$$

$$\mathcal{O}_{\mu_0,\mu_1,\cdots,\mu_n} = i^n \bar{u}(0)\gamma_{\mu_0}\gamma_5 \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n}\, s(0)$$

$$\langle 0|\mathcal{O}_{\{\mu_0,\mu_1,\ldots,\mu_n\}}(0)|K(p)\rangle = i f_K\, p_{\{\mu_0} \ldots p_{\mu_n\}}\, \langle \xi^n \rangle$$

Relationship between a_n and $\langle \xi^n \rangle$

- The first moment:

$$a_1 = \frac{5}{3}\langle \xi \rangle$$

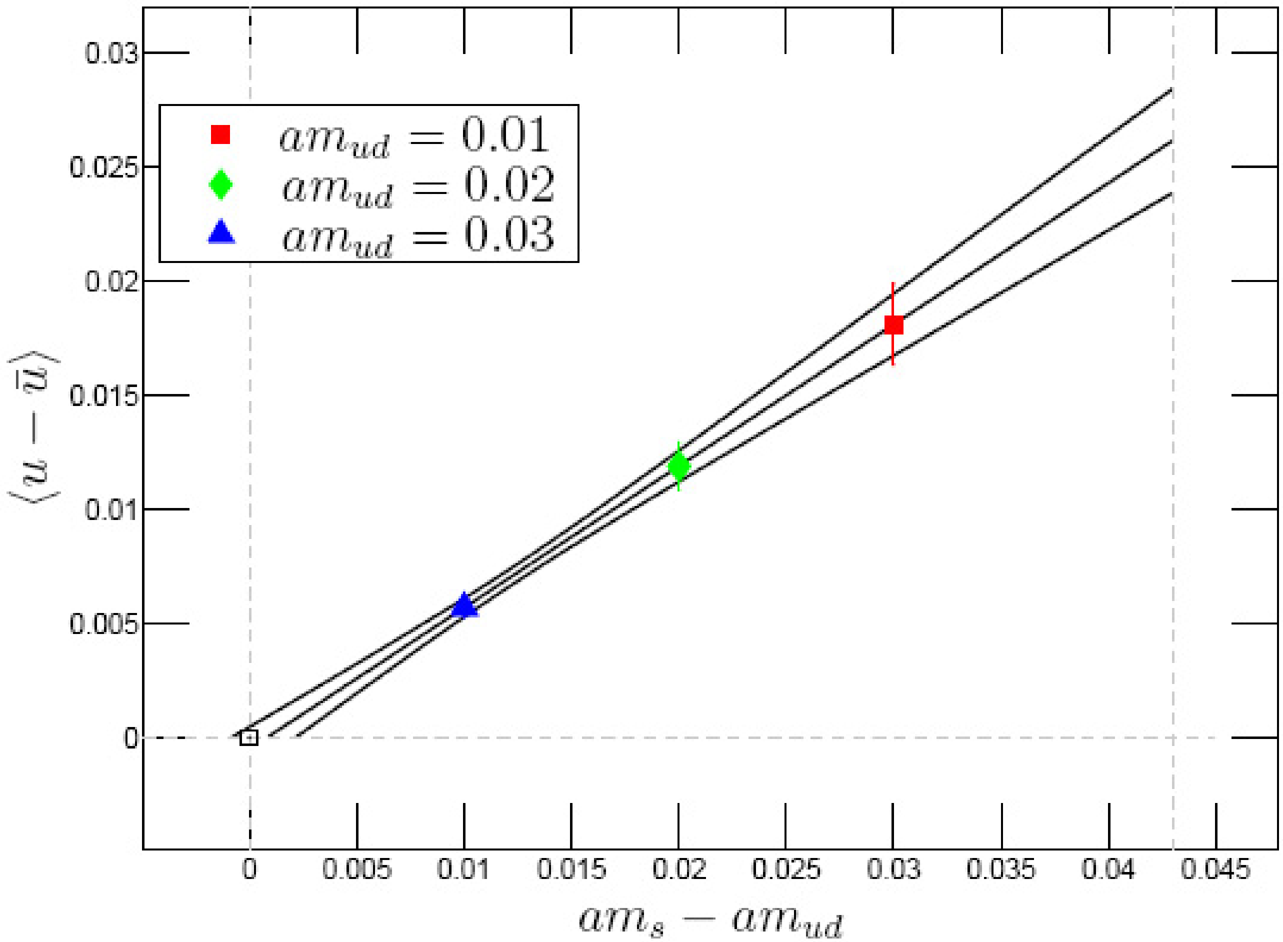
- The second moment:

$$a_2 = \frac{7}{12}(5\langle \xi^2 \rangle - 1)$$

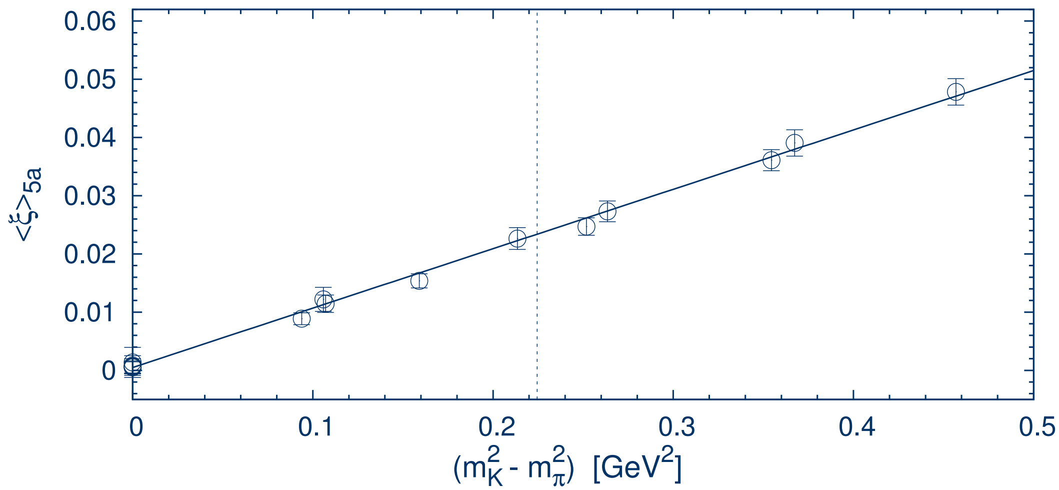
- The third moment:

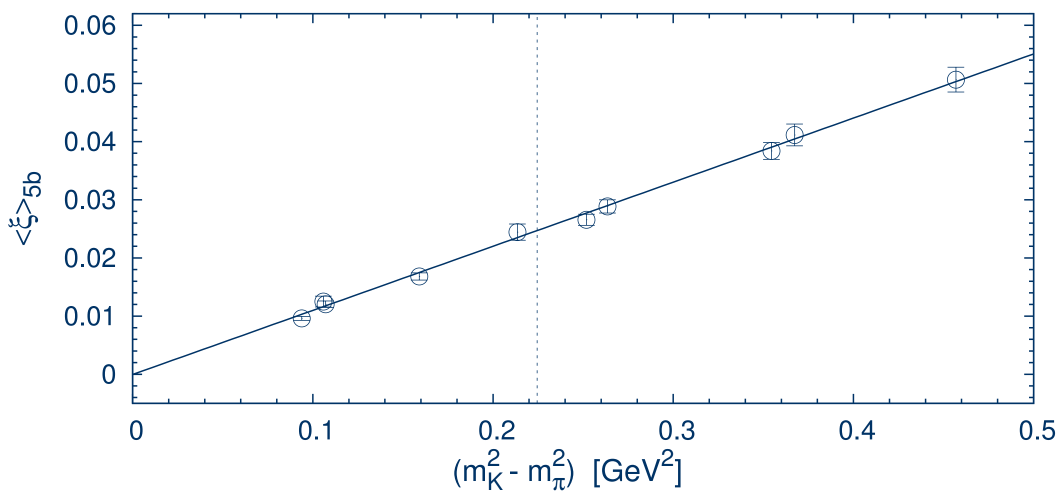
$$\cdots$$

$\langle \xi \rangle_K$ ($N_F = 3$, DWF, UKQCD, Flynn)



$\langle \xi \rangle_K$ ($N_F = 2$, Clover, QCDSF)





Suggestions

- In this talk, I proved that HYP is significantly more efficient than AsqTad in the improvement for staggered fermions.
  unimproved stag $<$ AsqTad $<$ HYP/$\overline{\text{Fat7}}$
- In 2002, I proposed $\overline{\text{Fat7}}$ which is equivalent to HYP but much more efficient for dynamical simulation with $N_F = 2+1$.
- I propose that we should produce $\overline{\text{Fat7}}$ ensembles as well as AsqTad ensembles (MILC gauge conf).

Comments

- Thank the Lattice 2006 organizers for the invitation.
- Feel sorry that I could not cover some of the topics and issues in Kaon physics.
- Welcome to Lattice 2006.