B_K in unquenched QCD using improved staggered fermions

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Collaboration

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Staggered ϵ'/ϵ Project

- Motivation:

  1. Calculate B_K and ϵ'/ϵ.
  2. Using improved staggered fermions (Cheap!).
  3. Measurement on MILC 2+1 flavor lattice.
  4. NLO in Staggered χPT.

- Effect:

  1. Compared with the experiments.
  2. Find a window for new physics.
  3. Or confirm the standard model.

Why staggered fermions?

- Advantage:
  1. Conserved $U(1)_V \otimes U(1)_A$.
  2. Quark mass is protected (No Residual Mass).
  3. Computationally cheap ($< 50 \times 4$).
  4. Discretization error = $\mathcal{O}(a^2)$.
  5. Easy to improve (no extra cost).
- Disadvantage
  1. Broken $SU(4)$ Flavor (Taste) Symmetry.
  2. Operator mixing matrix of 65536×65536.
  3. NPR is possible but difficult.

The History:

1. Perturbative matching (unimproved).
2. Numerical study (unimproved, quenched QCD)
3. Choosing the best improvement scheme (HYP or $\overline{\mathrm{Fat7}}$).
4. Perturbative matching for HYP/$\overline{\mathrm{Fat7}}$
5. Numerical study (HYP, quenched QCD)
6. Staggered χPT for B_K
7. Numerical study (HYP stag val & 2+1 AsqTad sea)

The 7th Stage:

Numerical Study with Mixed Actions

- 2+1 Flavor AsqTad Sea Quarks:

$$m_{ud}(\text{sea}) = 0.01$$
$$m_s(\text{sea}) = 0.05$$

- HYP Valence Quarks (9 comb $\longrightarrow$ 45 comb):

$$m_d(\text{valence}) = 0.01, 0.015, 0.02, 0.025, \cdots, 0.05$$
$$m_s(\text{valence}) = 0.01, 0.015, 0.02, 0.025, \cdots, 0.05$$

Simulation Parameters

parameter	value
β	6.76
lattice geometry	$20^3 \times 64$
# of configurations	593 $\rightarrow$ 640
$1/a$	1.588(19) GeV

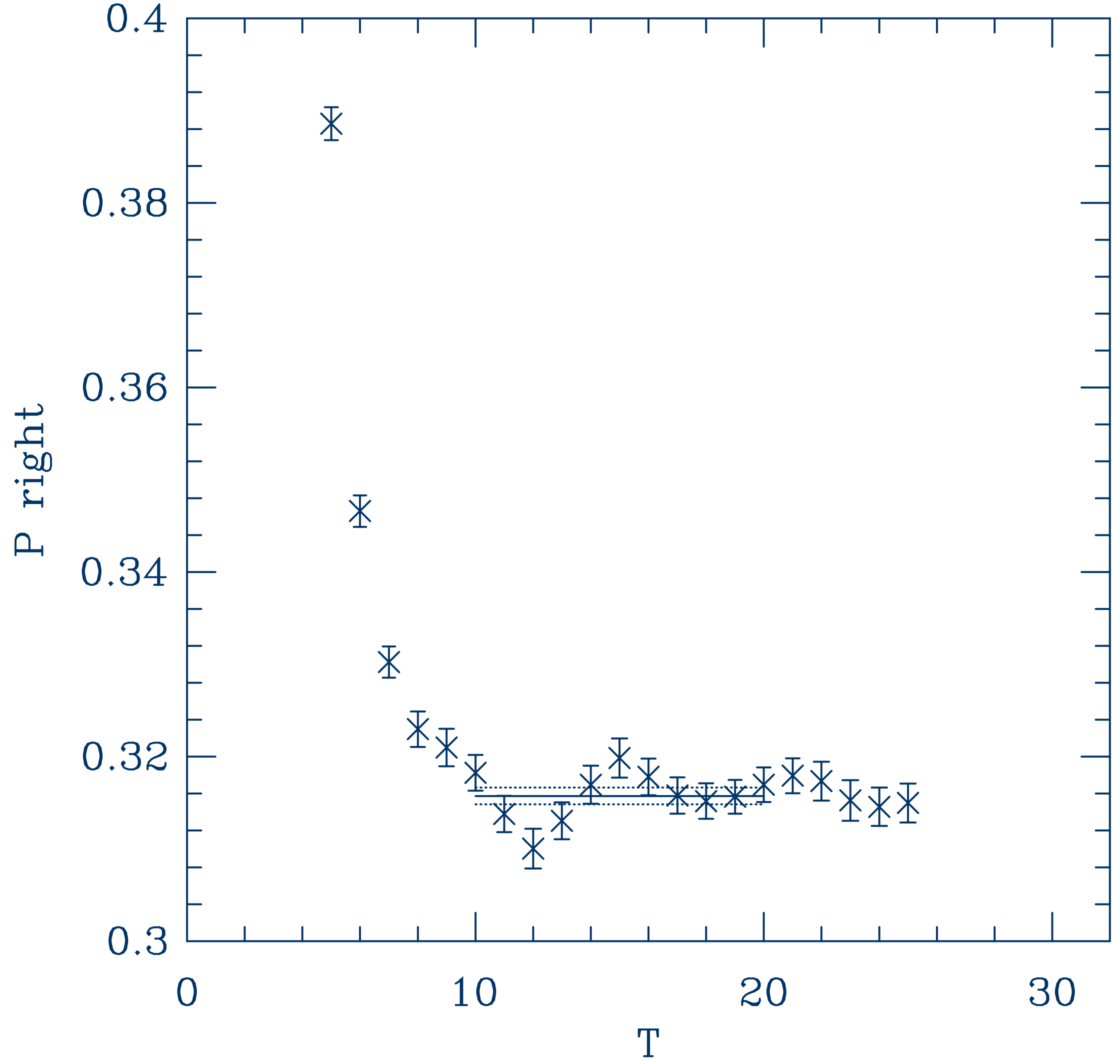
B_K (definition)

- Standard model:

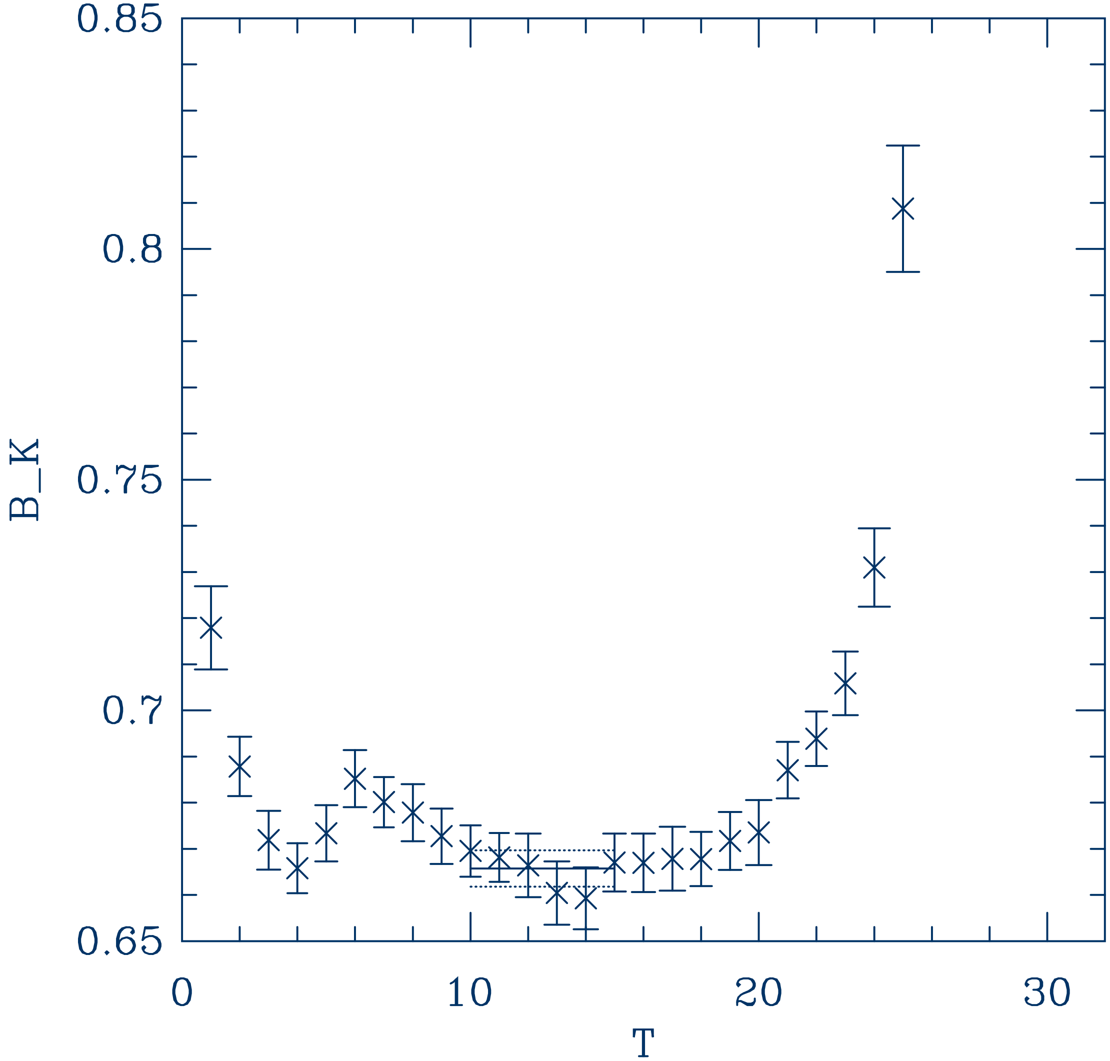
$$\epsilon \quad \propto \quad B_K$$

$$B_K \quad = \quad \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle}$$

Kaon Mass $(m_s = m_d = 0.03)$



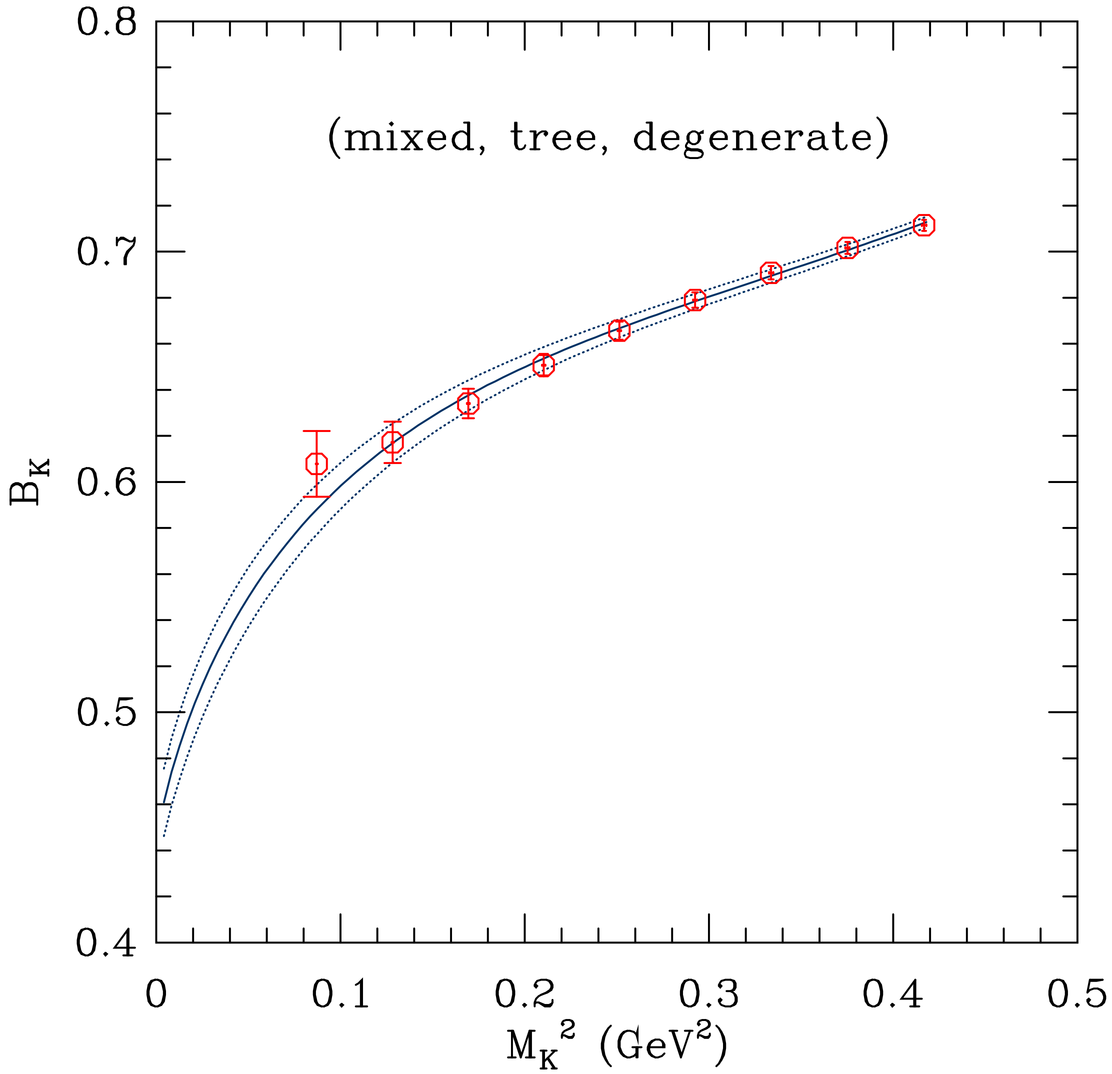
B_K with $m_s = m_d = 0.03$



PQ Chiral Behavior of B_K ($m_s = m_d$)

$$
\begin{aligned}
B_K &= c_1 \left(1 + \frac{1}{48\pi^2 f^2} \Big[I_{\rm conn} + I_{\rm disc} \Big] \right) + c_2 m_{xy}^2 + c_4 m_{xy}^4 \\
I_{\rm conn} &= 6 m_{xy}^2 \tilde{l}(m_{xy}^2) - 12 l(m_{xy}^2) \\
I_{\rm disc} &= 0 \\
l(X) &= X \log(X/\Lambda^2) + \text{F.V.} \\
\tilde{l}(X) &= -[\log(X/\Lambda^2) + 1] + \text{F.V.}
\end{aligned}
$$

PQ Chiral Behavior of $B_K(m_s = m_d)$



χ^2 Fitting for $B_K(m_s = m_d)$

parameters	unit	average	error
c_1	1	0.4422	0.0145
c_2	GeV^{-2}	$-$1.4616	0.1611
c_3	GeV^{-2}	$-$	$-$
c_4	GeV^{-4}	1.4674	0.1913
χ^2/d.o.f.	1	0.5396	0.3

PQ Chiral Behavior of B_K $(m_s \neq m_d)$

$$
\begin{aligned}
B_K &= c_1\left(1 + \frac{1}{48\pi^2 f^2}\left[I_{\rm conn} + I_{\rm disc}\right]\right) + c_2 m_{xy}^2 \\
&\quad + c_3 \frac{(m_X^2 - m_Y^2)^2}{m_{xy}^2} + c_4 m_{xy}^4 \\
I_{\rm conn} &= 6 m_{xy}^2 \tilde{l}(m_{xy}^2) - 3 l(m_X^2)\left(1 + \frac{m_X^2}{m_{xy}^2}\right) - 3 l(m_Y^2)\left(1 + \frac{m_Y^2}{m_{xy}^2}\right) \\
I_{\rm disc} &= (I_X + I_Y + I_\eta)/m_{xy}^2 \\
I_X &= \tilde{l}(m_X^2)\frac{(m_{xy}^2 + m_X^2)(m_U^2 - m_X^2)(m_S^2 - m_X^2)}{m_\eta^2 - m_X^2}
\end{aligned}
$$

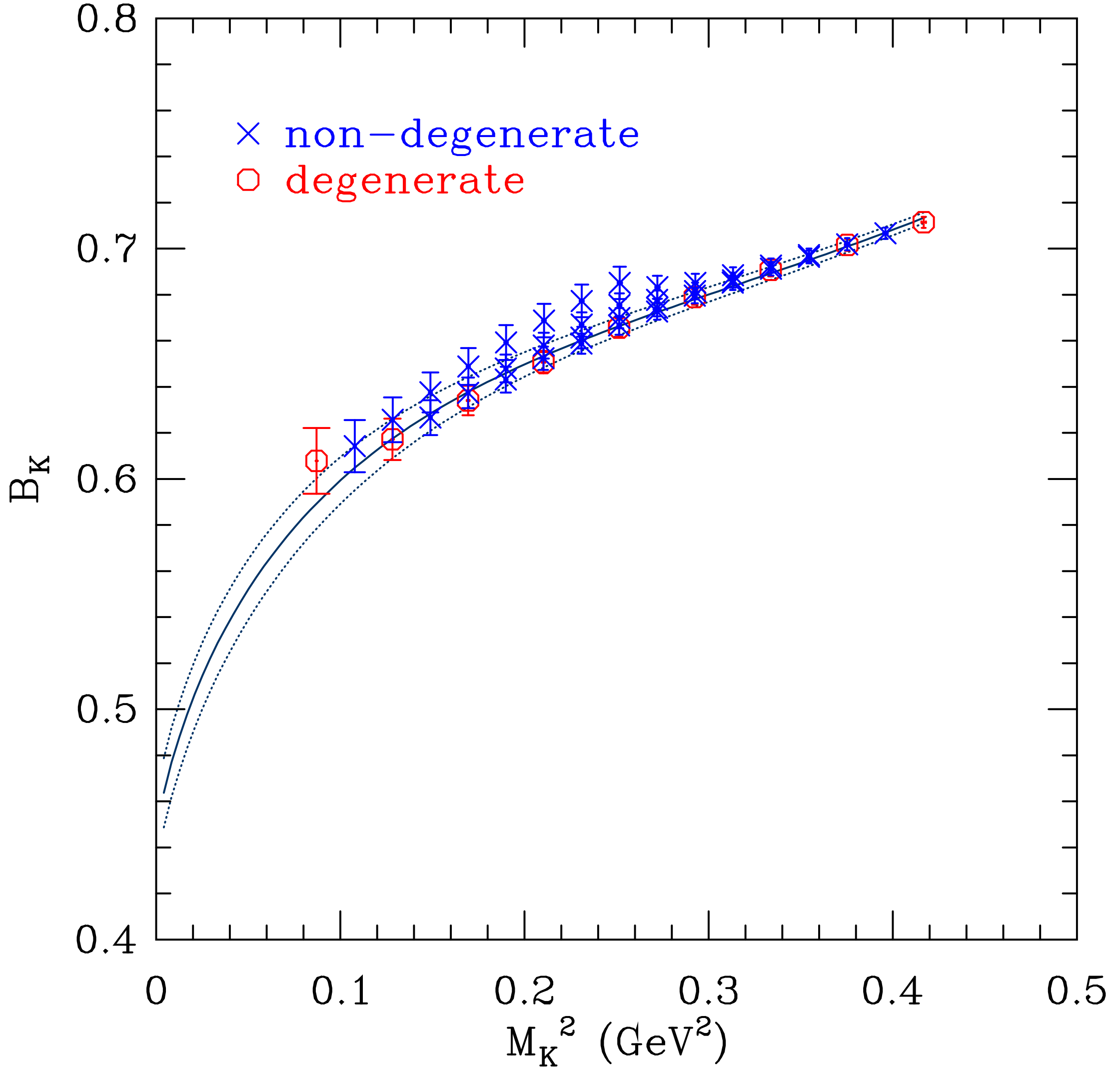
$$-l(m_X^2)\frac{(m_{xy}^2+m_X^2)(m_U^2-m_X^2)(m_S^2-m_X^2)}{(m_\eta^2-m_X^2)^2}$$

$$-l(m_X^2)\frac{2(m_{xy}^2+m_X^2)(m_U^2-m_X^2)(m_S^2-m_X^2)}{(m_Y^2-m_X^2)(m_\eta^2-m_X^2)}$$

$$-l(m_X^2)\frac{(m_U^2-m_X^2)(m_S^2-m_X^2)-(m_{xy}^2+m_X^2)(m_S^2-m \ldots}{m_\eta^2-m_X^2 \ldots}$$

$$\begin{aligned} I_Y &= I_X(m_X^2 \leftrightarrow m_Y^2) \\ I_\eta &= l(m_\eta^2)\frac{(m_X^2-m_Y^2)^2(m_{xy}^2+m_\eta^2)(m_U^2-m_\eta^2)(m_S^2-m_\eta^2)}{(m_X^2-m_\eta^2)^2(m_Y^2-m_\eta^2)^2} \end{aligned}$$

PQ Chiral Behavior of $B_K(m_s \neq m_d)$



χ^2 fitting for $B_K(m_s \neq m_d)$

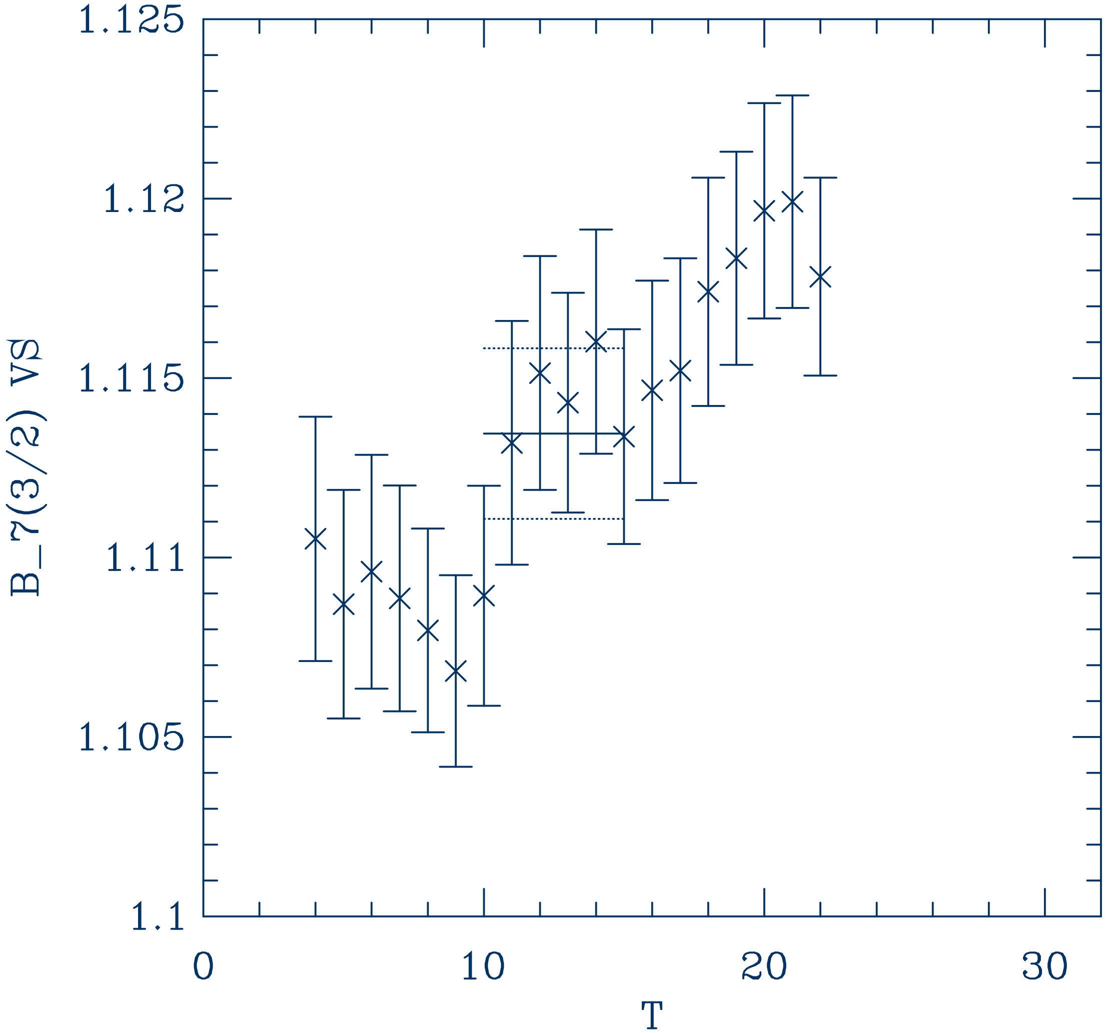
parameters	unit	average	error
c_1	1	0.4451	0.0149
c_2	GeV^{-2}	-1.5049	0.1672
c_3	GeV^{-2}	0.0305	0.0083
c_4	GeV^{-4}	1.5366	0.2021
χ^2/d.o.f.	1	0.1888	0.0935

B_7 (definition)

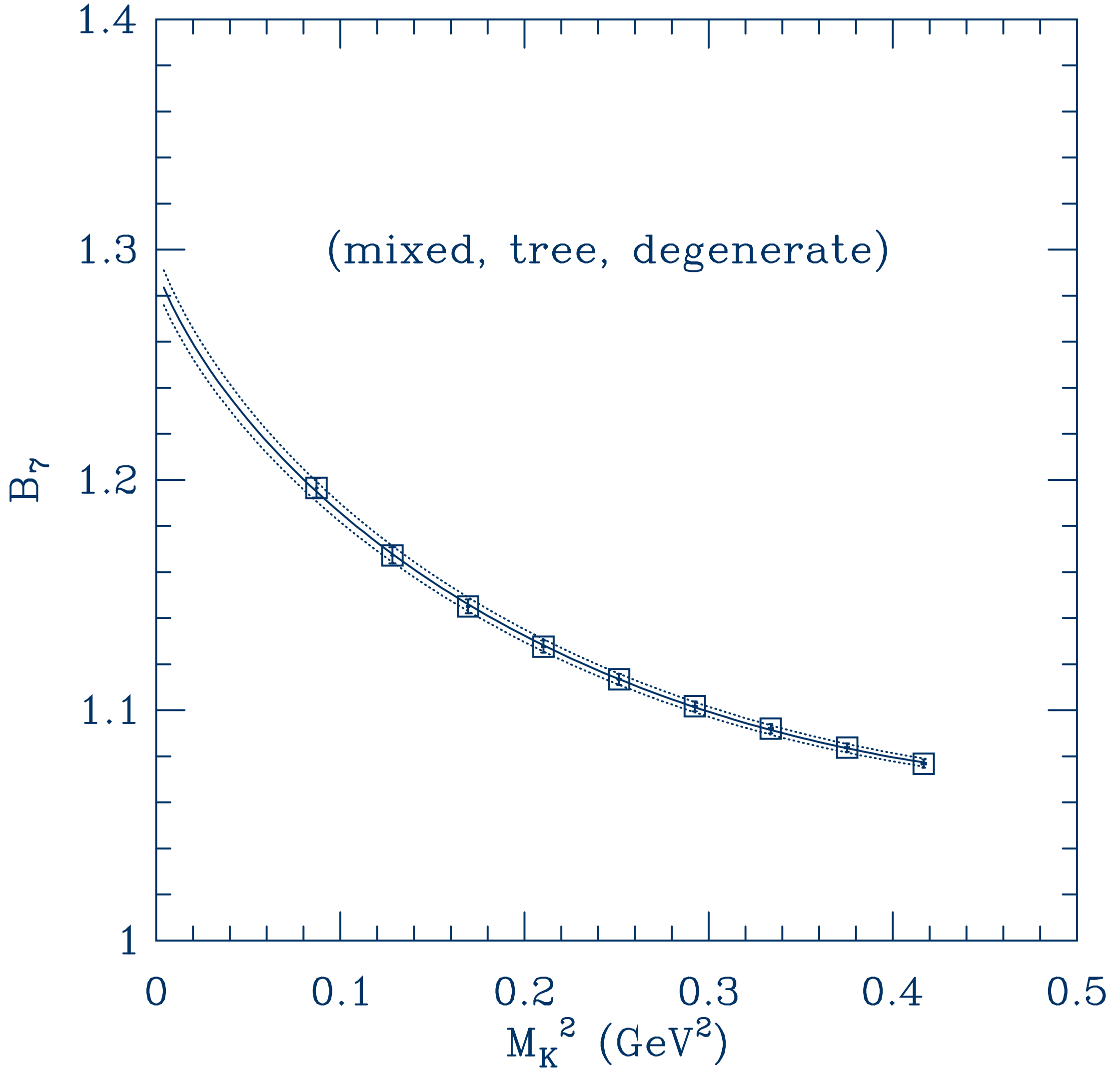
- Standard model:

$$B_7 = \frac{\langle K^+ | Q_7^{\Delta I = 3/2} | \pi^+ \rangle}{\text{Vacuum Saturation}}$$

$B_7(m_s = m_d = 0.03)$



Simple Fit of $B_7(m_s = m_d)$

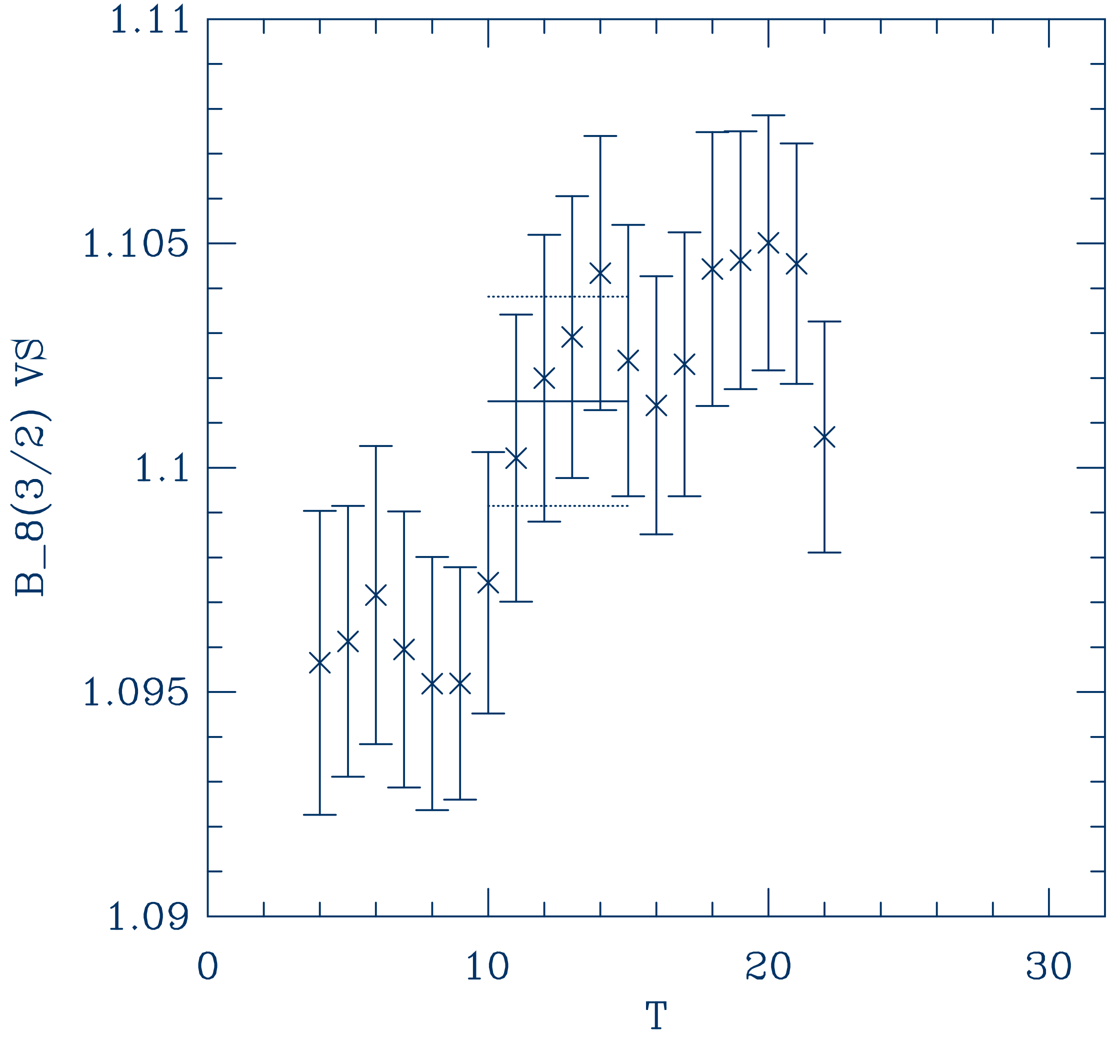


B_8 (definition)

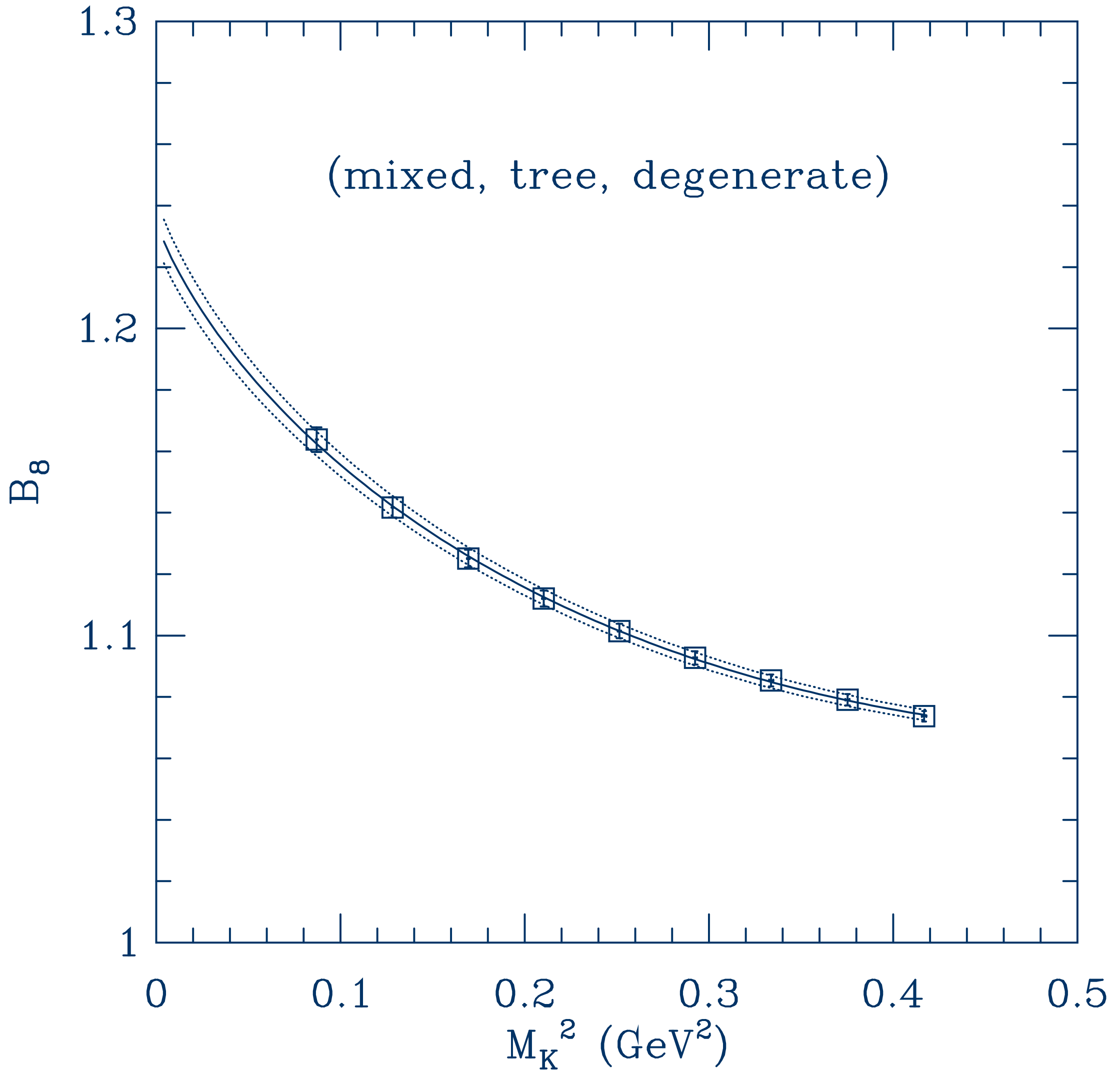
- Standard model:

$$B_8 = \frac{\langle K^+ | Q_8^{\Delta I = 3/2} | \pi^+ \rangle}{\text{Vacuum Saturation}}$$

$B_8(m_s = m_d = 0.03)$



Simple Fit of $B_8(m_s = m_d)$



Summary and Conclusion

- This is an attempt to calculate B_K using mixed actions (2+1 AsqTad Sea + HYP valence).
- The results are preliminary.
- The partially quenched chiral perturbation formula does not fit well the data in the light quark mass region.
- We expect that the chiral logs become less steep by the contribution from the non-Goldstone pions (Sharpe and Van de Water).
- We will incorporate the contribution from non-Goldtone pions into the data analysis soon.

- We will calculate the one-loop matching factors for the mixed action soon.
- For B_7 and B_8, we need to understand the chiral behavior from the staggered chiral perturbation.
- Extension to other MILC ensembles is in progress (SciDAC) in collaboration with Chulwoo Jung (BNL).