# $B_K$ and related matrix elements with unquenched, improved staggered fermions (Class A)

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http://www.phys.washington.edu/~sharpe/qcdoc/index.html

AND

# Non-perturbative renormalization with improved staggered fermions (Class B)

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http://www.phys.washington.edu/~sharpe/usqcd/index.html

# $B_K$ and related matrix elements with improved staggered fermions

# Overview

$$\frac{8}{3} m_k^2 f_K^2 B_K = \langle \bar{K} | \bar{s}\gamma_\mu(1-\gamma_5) d \bar{s}\gamma_\mu(1-\gamma_5) d | K \rangle$$

- Aim: Calculate $B_K$ and related matrix elements with percent-level precision
- We use HYP-smeared valence quarks on asqtad sea
- Present 2+1 flavor results
  - ➡ RBC-UKQCD: $B_K = 0.524(30)$ [2007] $B_K = 0.537(19)(FV)$[prelim]
  - ➡ ALV: $B_K = 0.527 \pm 0.006 \pm 0.020$
- Important to have results with several discretizations
  - ➡ Our prelim result (1-loop matching):

$$B_K = 0.512 \pm 0.014 \pm 0.034$$

# Running this year (09-10)

* Allocation: 25.06 Mnode-hrs on the QCDOC and 0.8 6n equivalent Mnode-hrs on the FNAL cluster
* Expect to use: 20 Mnode-hrs on the QCDOC
    * ➡ Mainly running on $64^3 \times 192$ ultrafine lattices using 4096 nodes
    * ➡ Some delays in wiring 4096 node partition

# Status and Proposed Running

| ID | $a$ (fm) | $am_l/am_s$ | Size | Configs. | spectrum | $B_K$ anal. |
|---|---|---|---|---|---|---|
| C1 | 0.12 | 0.03/0.05 | $20^3 \times 64$ | 564 | Y | Y |
| C2 | 0.12 | 0.02/0.05 | $20^3 \times 64$ | 486 | Y | Y |
| C3 | 0.12 | 0.01/0.05 | $20^3 \times 64$ | $9 \times 671$ | Y | Y |
| C3-2 | 0.12 | 0.01/0.05 | $28^3 \times 64$ | $8 \times 274$ | N | Y |
| C4 | 0.12 | 0.007/0.05 | $20^3 \times 64$ | $10 \times 651$ | Y | Y |
| C5 | 0.12 | 0.005/0.05 | $24^3 \times 64$ | 509 | Y | Y |
| C6 | 0.12 | 0.01/0.03 | $20^3 \times 64$ | 312 | Y | Y* |
| F1 | 0.09 | 0.0062/0.031 | $28^3 \times 96$ | 995 | Y | Y |
| F2 | 0.09 | 0.0031/0.031 | $40^3 \times 96$ | 853 | N | Y* |
| S1 | 0.06 | 0.0036/0.018 | $48^3 \times 144$ | $2 \times 743$ | N | Y# |
| S1 | 0.06 | 0.0036/0.018 | $48^3 \times 144$ | $7 \times 743$ | N | Proposed (GPU) |
| S2 | 0.06 | 0.0018/0.018 | $64^3 \times 144$ | $2 \times 826$ | N | Proposed (GPU) |
| U1 | 0.045 | 0.0030/0.015 | $64^3 \times 192$ | 497 | N | Y* |
| U1 | 0.045 | 0.0030/0.015 | $64^3 \times 192$ | 386 | N | Proposed (QCDOC) |

- Y*: have (or will have) data, but not yet analyzed
- Y#: partially analyzed

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Will be run this year on QCDOC

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# Proposal for 2010-11

* Complete running on ultrafine lattices, and calculate reweighting factors to use physical $m_s$ (sea)
  - ➡ 14.7 Mnode-hrs on QCDOC
* Use GPUs to increase statistics on superfine lattices and to include a smaller $m_l/m_s$
  - ➡ 120K GPU-hrs on Infiniband cluster
  - ➡ 540 K J/psi-equiv node-hrs for 20 Tbyte disk

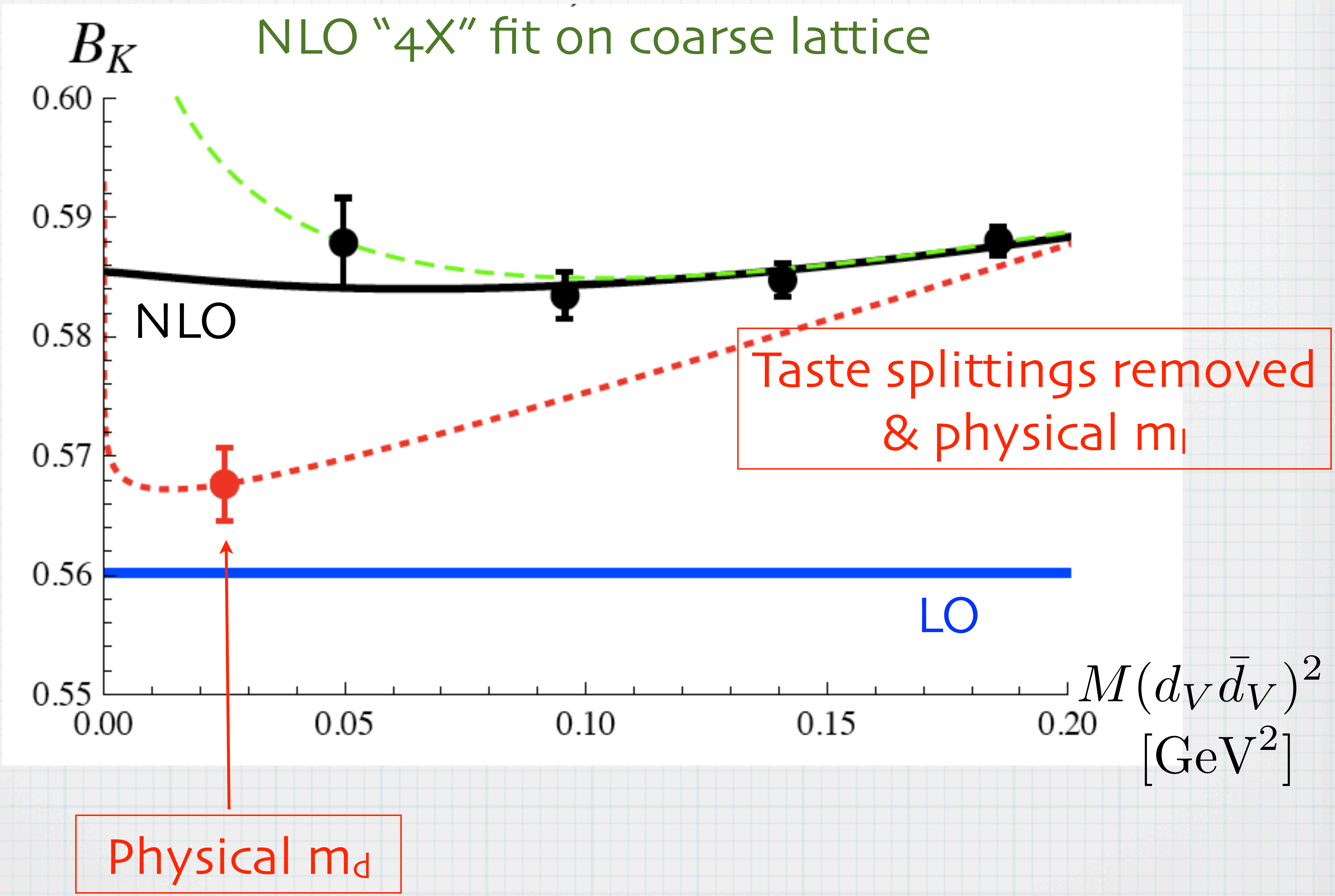
# Best results use SU(2) fitting

$$B_K = B_0(m_y, m_s, a^2, \alpha^2)(1 + \text{chiral logs} + d_1 m_x + d_2 m_\ell + d_3 m_x^2)$$

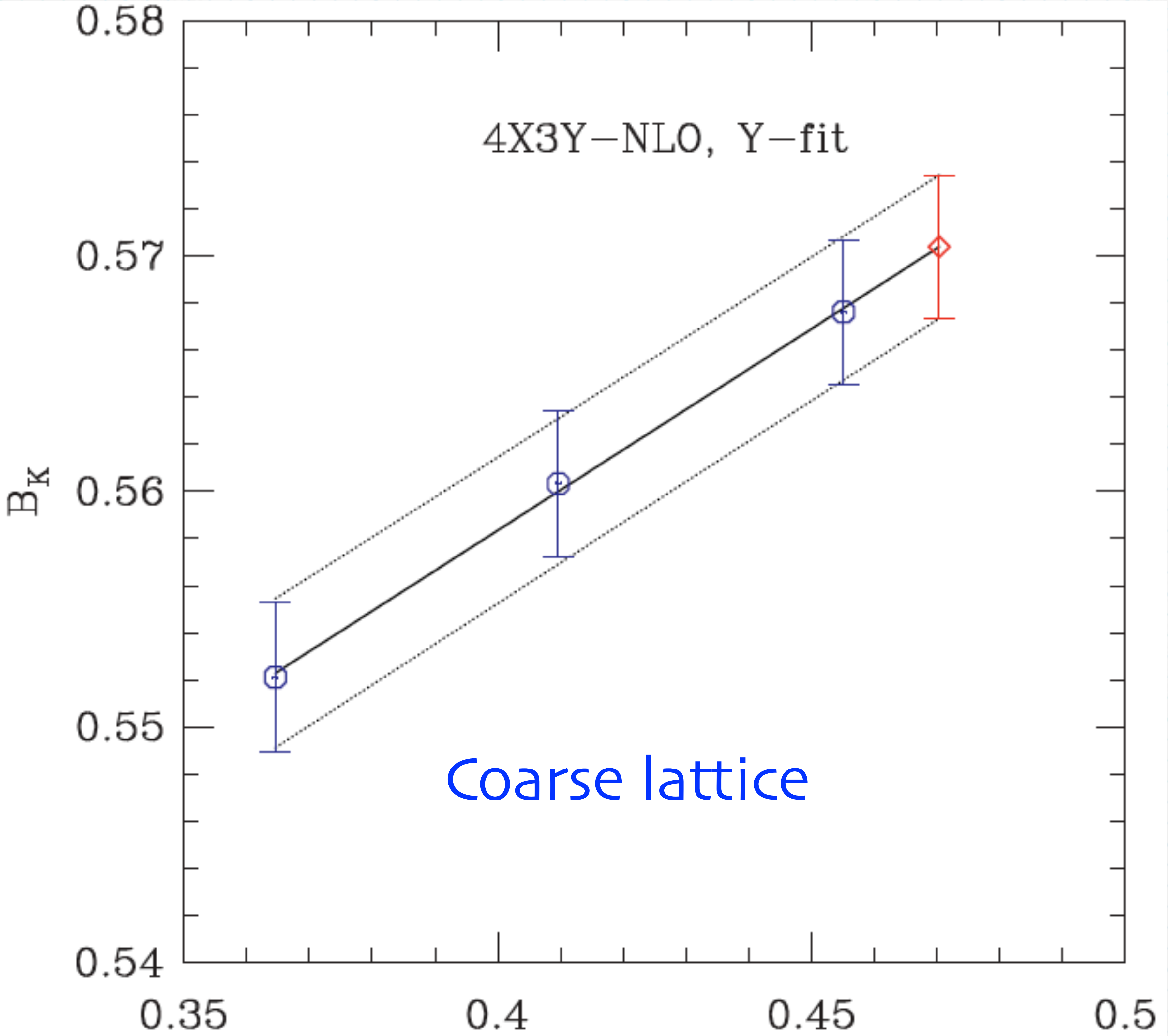
- $m_y$: Valence strange
- $m_s$: Sea strange
- chiral logs: Requires masses of all tastes of valence-valence light quark pions, and sea-sea taste I
- $m_x$: Valence u and d
- $m_\ell$: Sea u and d

$$\text{chiral logs} = \frac{1}{32\pi^2 f^2}\left\{\ell(X_I) + (L_I - X_I)\widetilde{\ell}(X_I) - 2\langle \ell(X_B)\rangle\right\}$$

# Fit to SU(2) rSPQMAChPT

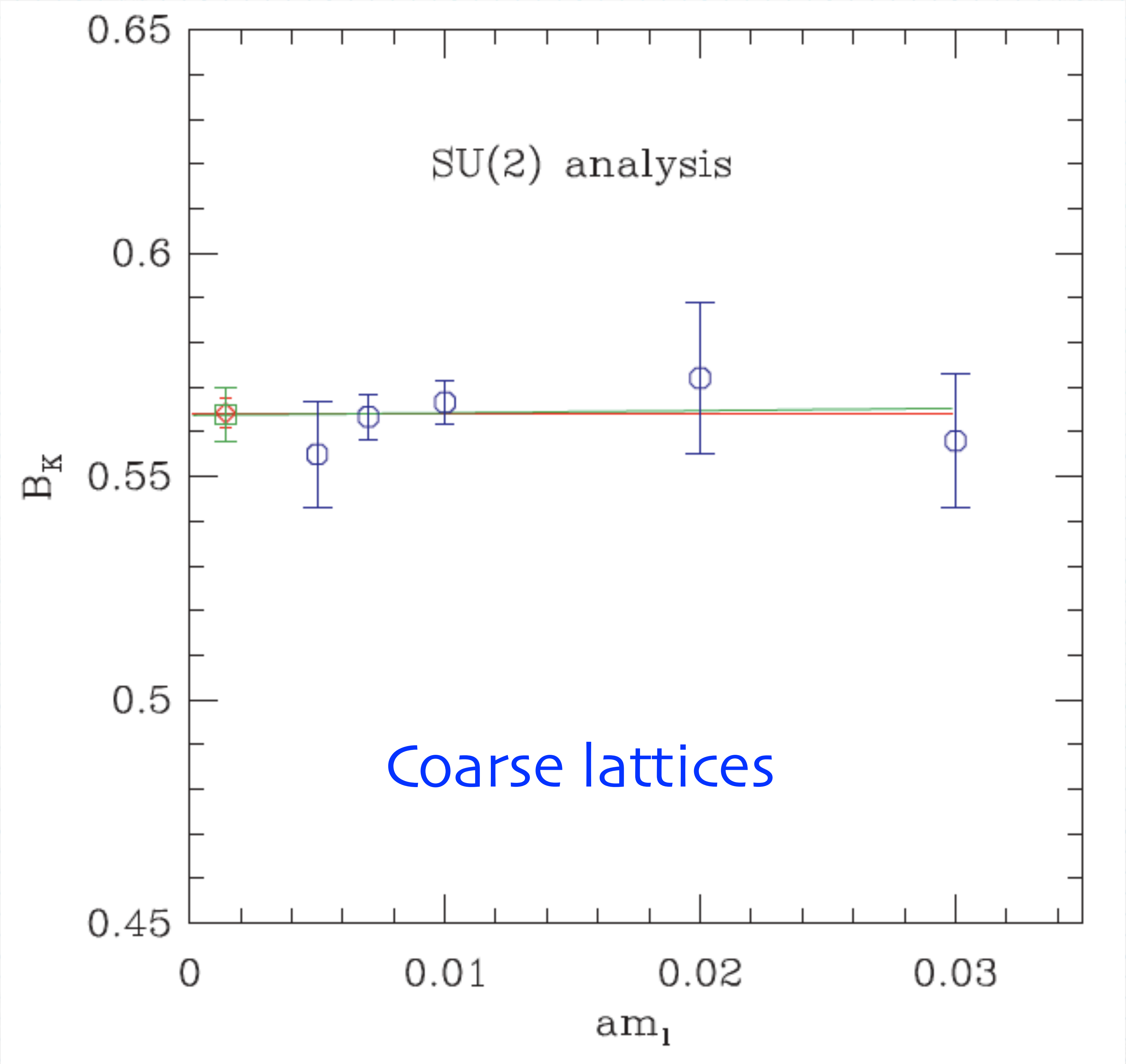


# Analytic fit to valence $m_s$

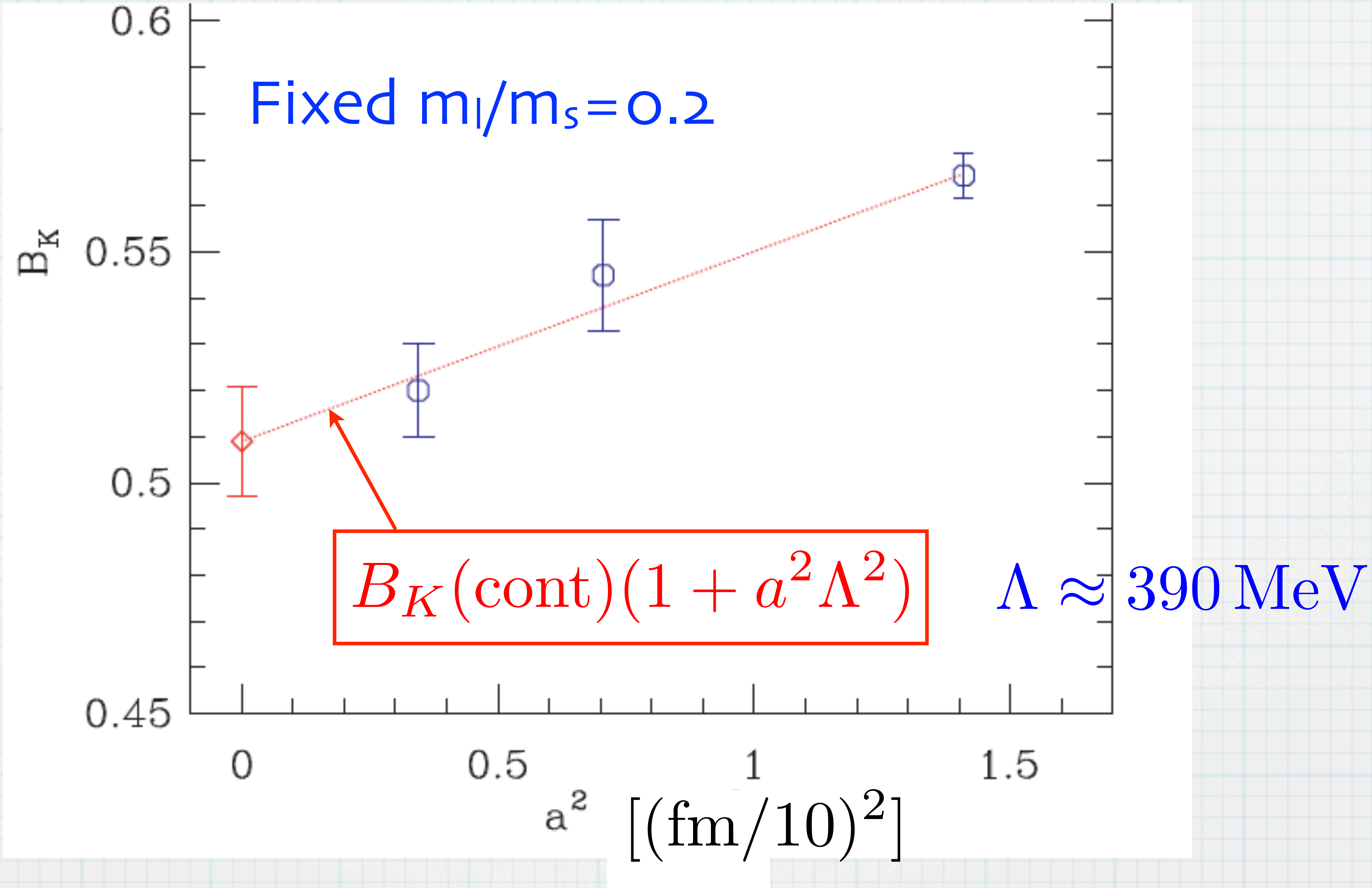



$M(s_V \bar{s}_V)^2$ [GeV$^2$]

# Dependence on sea $m_l$



# Continuum extrapolation




Aims in 2010-11: Ultrafine result & reduced errors

# SPC questions (1)

- A specific concern is the smallness of the chiral extrapolation systematic error. This appears to be about 10 times smaller than the one quoted by the RBC-UKQCD collaboration at Lat09 for DWF.

| cause | error (%) | description |
|---|---|---|
| statistics | 2.8 | 4X3Y-NNLO fit (ensemble C3) |
| discretization | 1.4 | diff. of (S1) and $a = 0$ |
| fitting (1) | 0.15 | X-fit: NLO vs. NNLO |
| fitting (2) | 0.5 | Y-fit: linear vs. quadratic |
| fitting (3) | 0.25 | constant vs linear $am_l$ dependence |
| finite volume | 0.89 | $20^3$ (C3) versus $28^3$ (C3-2) |
| matching factor | 6.4 | $\alpha_s^2$ on (S1) |

Table 2: Preliminary error budget for $B_K$ obtained using SU(2) SChPT fitting.

★Answer: comparing apples & oranges
★The 0.25 is error of only part of chiral extrap.
★RBC-UKQCD use analytic chiral fit to estimate error (no chiral log); we assume ChPT

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# SPC questions (2)

- We are also interested in your experience running on, and coding for, the GPU cluster

★Code mainly in development on nVidia GTX 295 @ Seoul National U (SNU)

★Speeds on GTX 285 @ JLab very similar

★Tested on $28^3$x96 (fine) lattices on 4 GPUs/2 CPUs with Infiniband

★CUDA v2.3 for staggered CG (double precision)

★Partially coalesced memory access: 5.3 GFlop/s/GPU

★"Pure GPU speed" = 10.3 GFlop/s/GPU (13% peak)

★Threads should be a multiple of 32

★Expect doubling of speed on FERMI GTX 480

# Non-perturbative renormalization for improved staggered fermions

# Aims of NPR project

* Calculate $Z_m$ (and other Z-factors) for asqtad quarks
    - ➡ Reduce truncation errors in MILC's determination of light-quark masses
    - ➡ Check recent HPQCD determination using $m_s/m_c$
* Calculate Z-factors for bilinears and four-fermion operators with HYP valence fermions on asqtad sea
    - ➡ Reduce dominant error in $B_K$
    - ➡ Compare to perturbation theory

# Status of 2009-10 running

| lattices | size | $am_\ell : am_s$ | valence masses | status |
|---|---|---|---|---|
| coarse | $20^3 \times 64$ | $0.01 : 0.05$ | $0.01, 0.02, 0.03$ | 16 completed |
| coarse | $20^3 \times 64$ | $0.02 : 0.05$ | $0.01, 0.02, 0.03$ | 16 completed |
| coarse | $20^3 \times 64$ | $0.03 : 0.05$ | $0.01, 0.02, 0.03$ | 16 completed |
| fine | $28^3 \times 96$ | $0.0062 : 0.0310$ | $0.0062, 0.0093, 0.0124$ | 16 to be run this year |
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- Asqtad running will use 350K 6n-node hrs
- 9 different momenta; unquenched and PQ data for bilinears
- HYP code tested, ready to calculate propagators once asqtad running completed

# Status of 2009-10 running

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Completed

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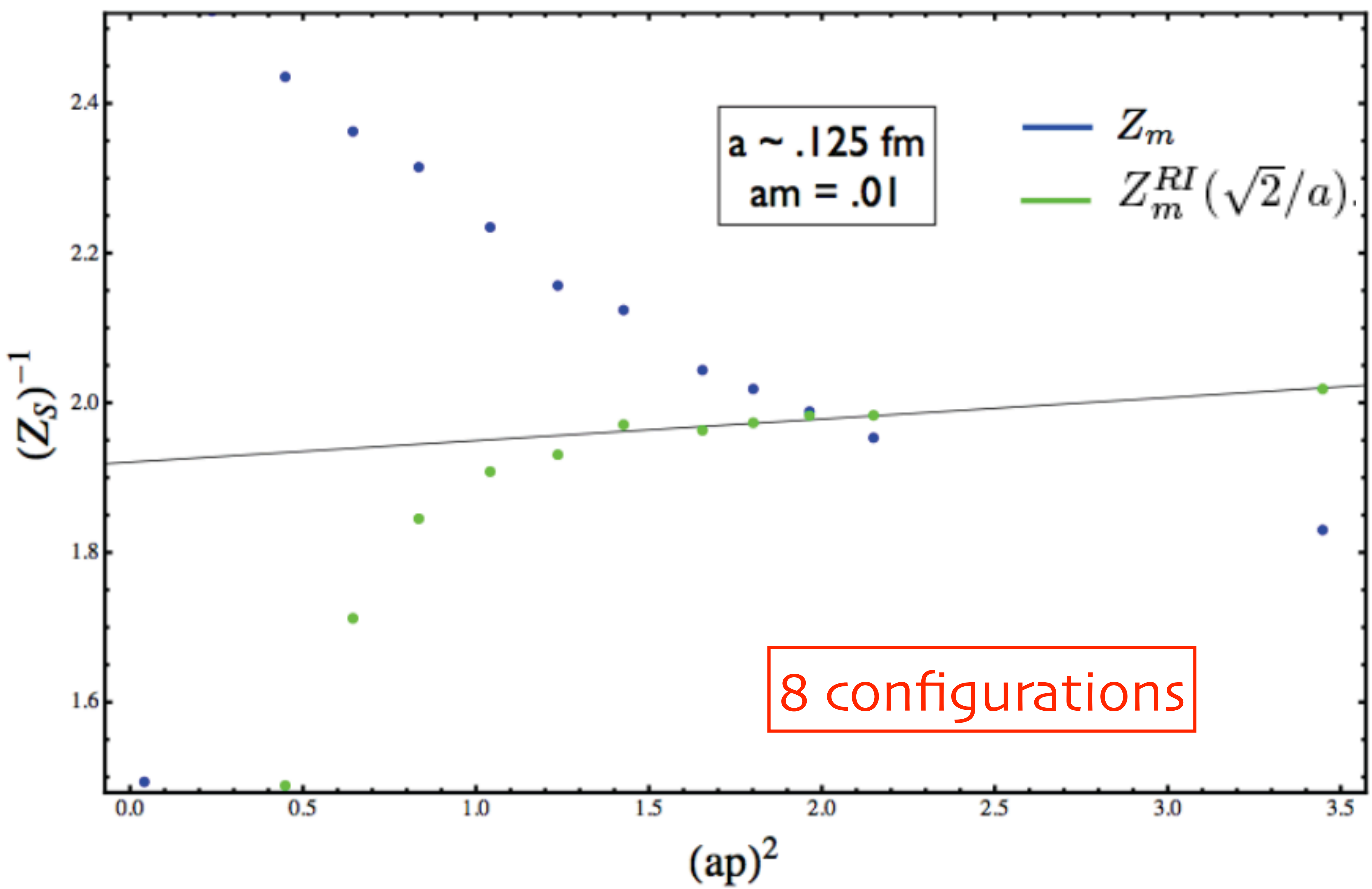
# Results for $Z_m$

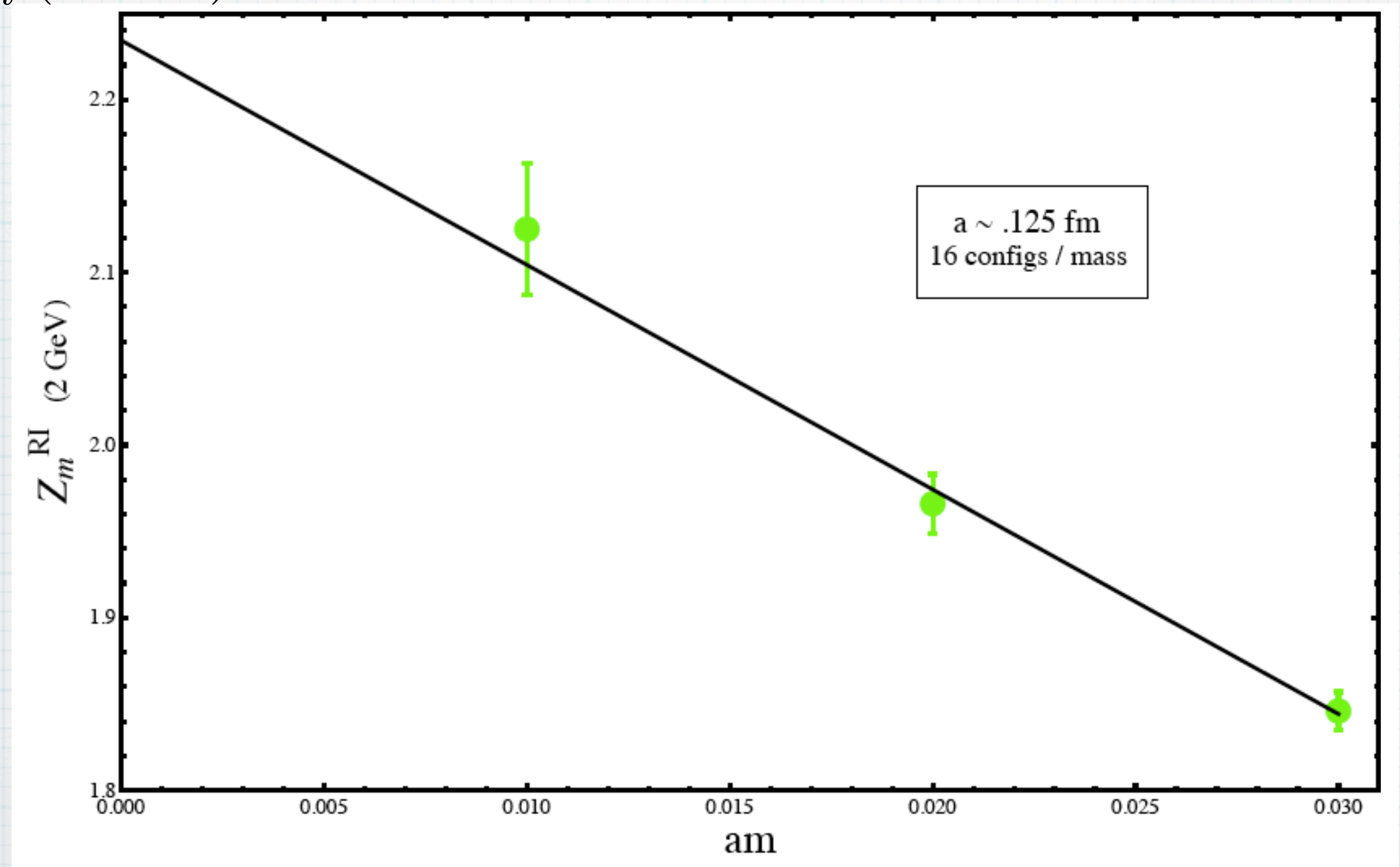
Lytle, Lat 2009



# Chiral extrap. (unquenched)

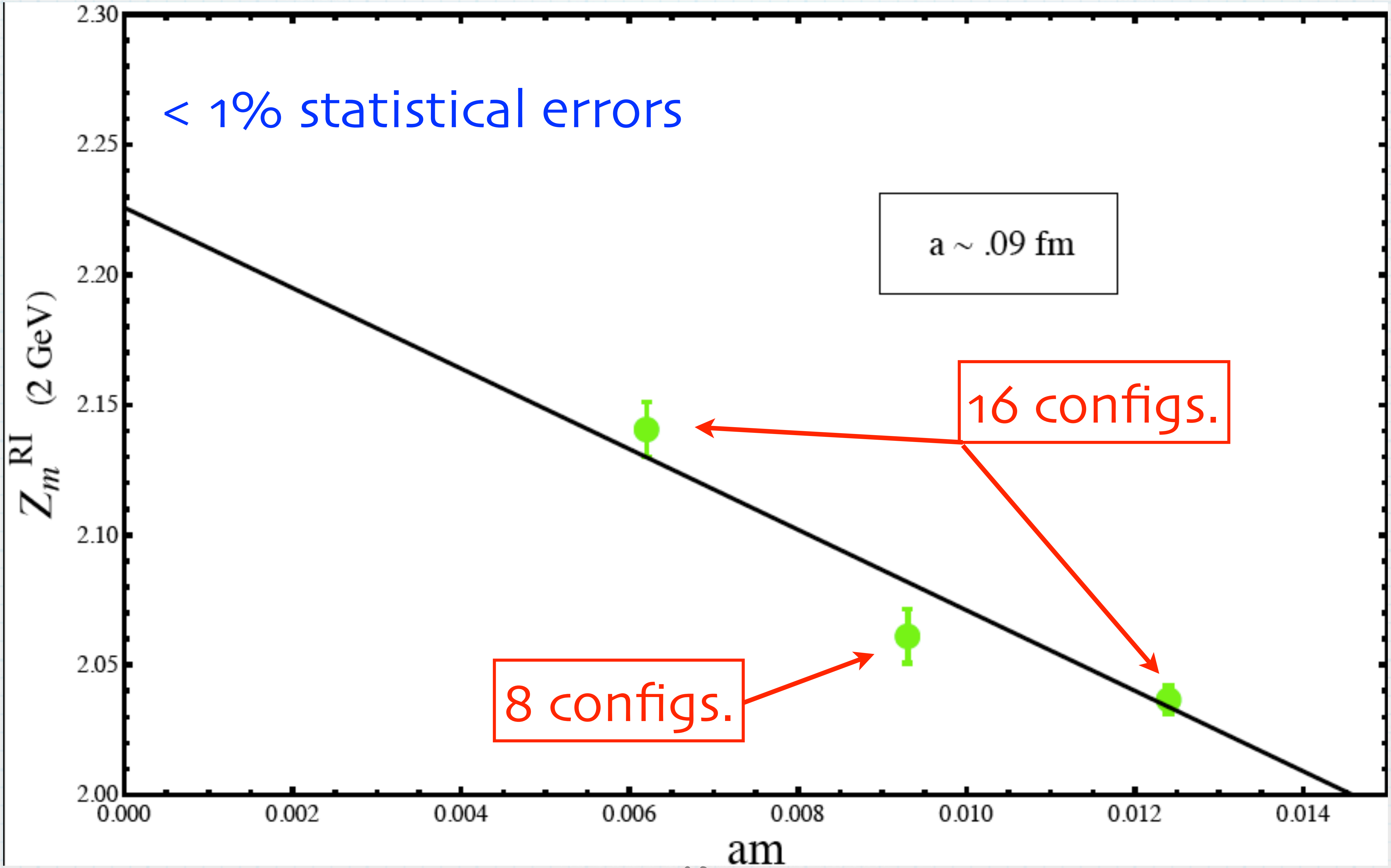
$Z_m^{\mathrm{RI}}(2\,\mathrm{GeV})$

Lytle, preliminary



# Chiral extrap. (unquenched)

Preliminary, fine lattices



# Result for $m_q$

On coarse lattices:

$$m_s(\overline{\mathrm{MS}},\,2\,\mathrm{GeV}) = \underbrace{Z^{\overline{\mathrm{MS}},\,\mathrm{RI}}(2\,\mathrm{GeV})}_{\approx 0.77}\,\underbrace{Z_m^{\mathrm{RI}}(2\,\mathrm{GeV})}_{\approx 2.2}\,\underbrace{m_{s,0}}_{65\,\mathrm{MeV}}$$

$$= 106 \pm 6\,\mathrm{MeV}$$ Lytle, Lat 2009

$$\longrightarrow 106 \pm 2\,\mathrm{MeV}$$ prelim. (stat. error only)

Compare to 2-loop matching:

$$m_s(\overline{\mathrm{MS}},\,2\,\mathrm{GeV}) = 84 \pm 5\,\mathrm{MeV}$$

Statistical error will be about 1% on fine lattices

# Proposal for 2010-11

* Extend NPR to HYP fermions
* Calculate Z-factors for bilinears and four-fermion operators (focusing on $B_K$)
  * 32 configurations, 9 momenta
  * Coarse and fine lattices with 3 light sea and 3 light valence masses

| calculation | cost (Jpsi-equivalenet core-hrs) |
|---|---|
| Asqtad bilinear vertex calculations | 50K |
| HYP-smeared inversions | 650K |
| HYP-smeared bilinear vertex calculations | 160K |
| HYP-smeared four-fermion vertex calculations | 640K |
| Storage | 140K |
| total | 1640K |

# SPC questions (1)

- What are the expected final errors for $Z_m$ and hence, e.g., the strange quark mass? How does this compare with existing numbers?

★ Existing 2+1 flavor results:

- MILC: $m_s(\overline{\mathrm{MS}},\, 2\,\mathrm{GeV}) = 89 \pm 5\,\mathrm{MeV}$
- RBC-UKQCD: $m_s(\overline{\mathrm{MS}},\, 2\,\mathrm{GeV}) = 94 \pm 3 \pm 5_{\mathrm{NPR}}\,\mathrm{MeV}$
- HPQCD: $m_s(\overline{\mathrm{MS}},\, 2\,\mathrm{GeV}) = 92.4 \pm 1.5\,\mathrm{MeV}$
- PACS-CS $m_s(\overline{\mathrm{MS}},\, 2\,\mathrm{GeV}) = 73 \pm 1\mathrm{MeV}(\pm\ \mathrm{trunc.\ error})$

★ We expect our errors to be comparable to those of MILC/RBC-UKQCD, and dominated by systematics

# SPC questions (2)

- What are the expected final errors for the four-quark operator matching?

★It is too early to tell.

# SPC questions (3)

- The $B_K$ project is working on MILC superfine lattices and plans to go onto ultrafine lattices. Are there plans for non-perturbative matching on these ensembles as well?

★Yes! But we have to show first that the method works on the coarse and fine lattices.