B_K, B_7 and B_8 with unquenched, improved staggered fermions

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Abstract

We propose to calculate kaon matrix elements of four-fermion operators using improved staggered quarks. Our major focus will be on a calculation of B_K, which is of particular interest to constraining elements of the CKM matrix, and on the related matrix elements $B_7^{3/2}$ and $B_8^{3/2}$, which enter into the prediction for ϵ'/ϵ. We propose to use the MILC coarse and fine Asqtad configurations, and calculate valence propagators using HYP-smeared staggered fermions. On each lattice we will calculate with 10 different valence quark masses, allowing 55 kaon masses, so as to provide sufficient data points for fitting to the rather elaborate form predicted by staggered chiral perturbation theory. We also propose a parallel spectrum calculation, determining in particular the masses and decay constants of flavor non-singlet pions of all tastes. This information is needed as input into the chiral perturbation theory fits to $B_{K,7,8}$ and also will provide cross-checks on the applicability of staggered chiral perturbation theory. The output of the proposed calculation will be a result for B_K and other kaon matrix elements with all errors controlled.

We are asking for CPU time on the QCDOC, since our codes are built on the Columbia Physics System, and use the level-3 QCD-API routines for the QCDOC. We are asking for a total of 1.2 Million node-hours ($\equiv 0.26$ Mnode-hrs on Jlab 4g), so that this is a "class B" proposal.

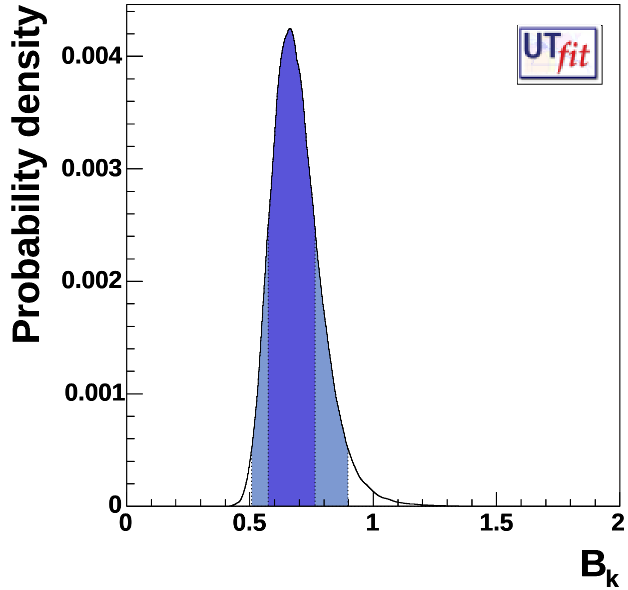
Status of calculations of B_K

$$\frac{8}{3} m_K^2 f_K^2 B_K = \langle \bar{K} | \bar{s}\gamma_\mu(1-\gamma_5) d \bar{s}\gamma_\mu(1-\gamma_5) d | K \rangle \,. \qquad (1)$$

- B_K is a key quantity needed for constraining CKM matrix
- Present status:
  - Quenched, degenerate s, d:
    $\Rightarrow \widehat{B}_K = 0.78(4)$ [Dawson]
  - Unquenched staggered, degenerate, coarse MILC lattice:
    $\Rightarrow \widehat{B}_K = 0.83(18)$ [Gamiz *et al.*]
    - Major error from use of 1-loop matching
  - Unquenched 2 and $2+1$ DWF:
    $\Rightarrow \widehat{B}_K = 0.78(9)$ [Dawson]
    - Major error is guess for continuum and non-degenerate extrapolations
- Need more precise results with fully controlled errors, using several fermion discretizations

Importance of precise calculation

- Predicted value from Unitarity Triangle fit: $\widehat{B}_K = 0.69 \pm 0.1$ [UTfit]
- $\Rightarrow$ error $\lesssim 10\%$ needed



Proposed calculation

- MILC coarse and fine ensembles to allow continuum and sea-quark mass extrapolations
  - No study of volume dependence possible, but expected to be small effect for B_K
- Use HYP-smeared valence staggered quarks
  - Smaller taste-violations than Asqtad [Hasenfratz & Knechtli]
  - Small one-loop matching contributions [Lee & Sharpe]
  - Different from [Gamiz *et al.*] who use thin links in operators
- 10 valence quark masses/lattice $(m_s/10 - m_s)$ giving 55 "kaon" masses
  - Needed for fits to Partially Quenched rooted Staggered Chiral Peturbation Theory result [Van de Water & Sharpe]
- Calculate B_K, $B_7^{3/2}$, $B_K^{3/2}$ using previously tested methods [Lee *et al.*]
- Based on present run, expect $\sim 1\%$ statistical errors
- Calculate spectrum of HYP-smeared valence quarks for all tastes
  - Needed in chiral fits
  - Code being tested on quenched configurations

CPU estimates

$a(fm)$	am_ℓ/am_s	size	number	CPU (Mnode-hrs)
0.12	0.03/0.05	$20^3 \times 64$	564	0.040
0.12	0.02/0.05	$20^3 \times 64$	485	0.034
0.12	0.01/0.05	$20^3 \times 64$	593*	$---$
0.12	0.007/0.05	$20^3 \times 64$	493	0.035
0.12	0.005/0.05	$24^3 \times 64$	400	0.047
0.12	0.03/0.03	$20^3 \times 64$	350	0.025
0.12	0.01/0.03	$20^3 \times 64$	349	0.025
0.086	0.0124/0.031	$28^3 \times 96$	531	0.12
0.086	0.0062/0.031	$28^3 \times 96$	583	0.13
0.086	0.0031/0.031	$40^3 \times 96$	500	0.57
TOTAL				1.2

$*$ Underway on cluster at Seoul National University

- Time for B_K, $B_{7,8}$ versus spectrum is in ratio 1:5
- Total includes 10% overhead for gauge fixing, HYP smearing and I/O

Ancilliary theoretical work

- 1-loop perturbative matching factors for the improved gauge action
- Extending staggered chiral perturbation theory calculation to mixed staggered set-up (HYP-valence, Asqtad-sea)
  - Simplifies for B_K because no "valence-sea" mesons at one-loop
  - Tentative conclusion: introduces new parameters $\delta_{V,A}$
- Extend staggered chiral perturbation theory analysis to $B_{7,8}^{3/2}$
- Non-perturbative renormalization of staggered operators (just beginning)

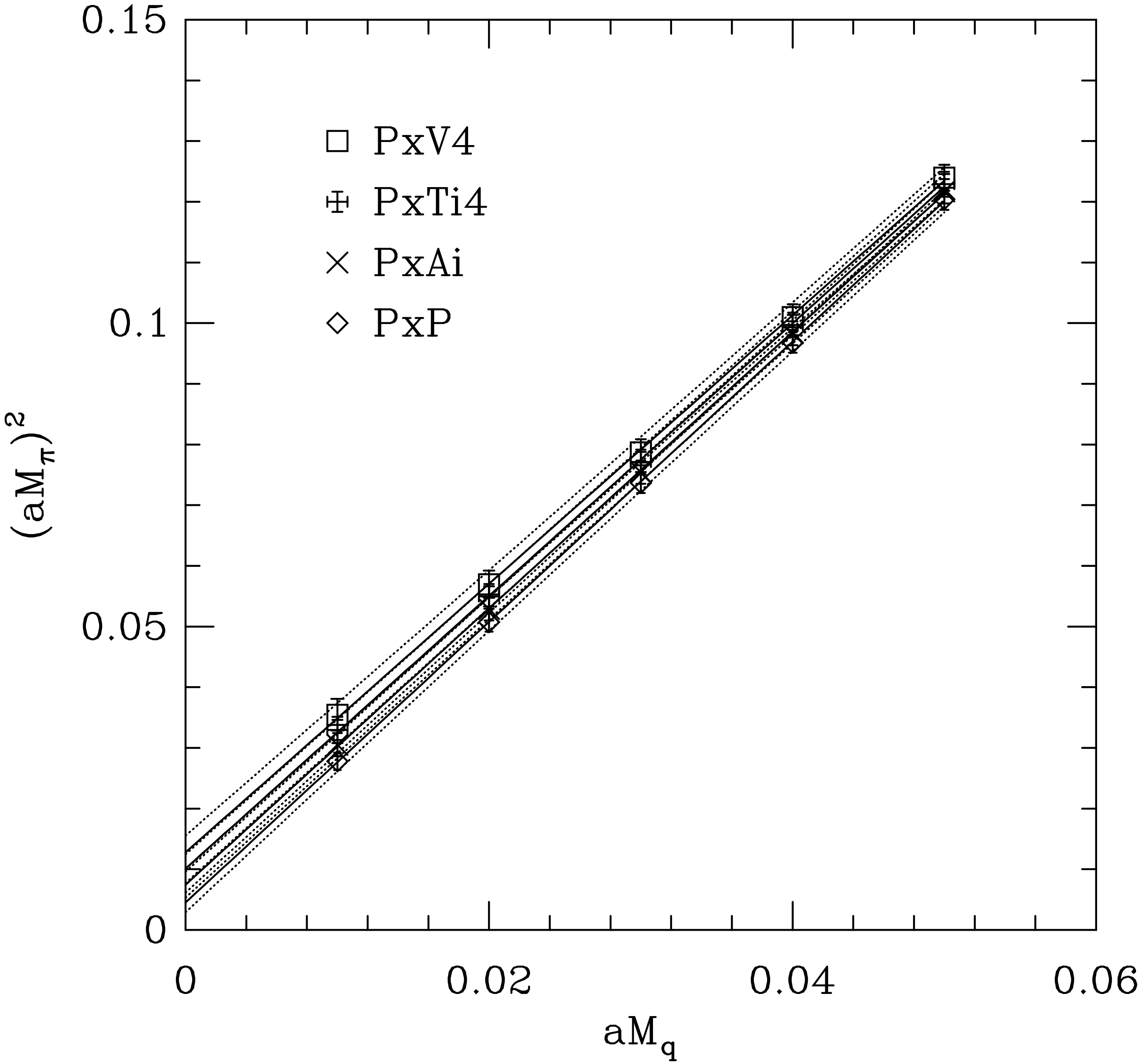
Code details

- Written in C++, built on top of CPS library, using level-3 CG
- Expect 20% efficiency on QCDOC, or 160 MFlop/s/node
- Present codes calculate all propagators for each s, d mass combination
  - Changing to single calculation of each propagator and reading in using parallel I/O
  - Saves factor of 11 in CPU
- 2500 CG iterations for $m = m_s/10$ (coarse), 8000 total for 10 masses
  - Scale up by $a_{\text{coarse}}/a_{\text{fine}}$ for fine lattice
- CG amounts to about 2/3rd of total CPU
- Local volume is $10^3 \times 4$ for $20^3 \times 64$ lattices (128 nodes) . . . and $10^3 \times 6$ on $40^3 \times 96$ lattices (1024 nodes)
- Complete calculation estimated to take 1.1 hours on $40^3 \times 96$ lattices, so will need to have ~ 100 stored on disk (120 GByte)

Storing data?

- Could store lightest mass propagators for future calculations (HYP valence propagators have not been generated previously)
  - Two $U(1)$ wall sources/config for B_K
  - Eight "cubic" wall sources/config for spectrum
- Storing 3 lightest masses for all the fine lattices requires 0.8 TBytes
- Need input from community/SPC

Quenched spectrum test ($\beta = 6,\ 16^3 \times 64$)



Taste breaking in spectrum?

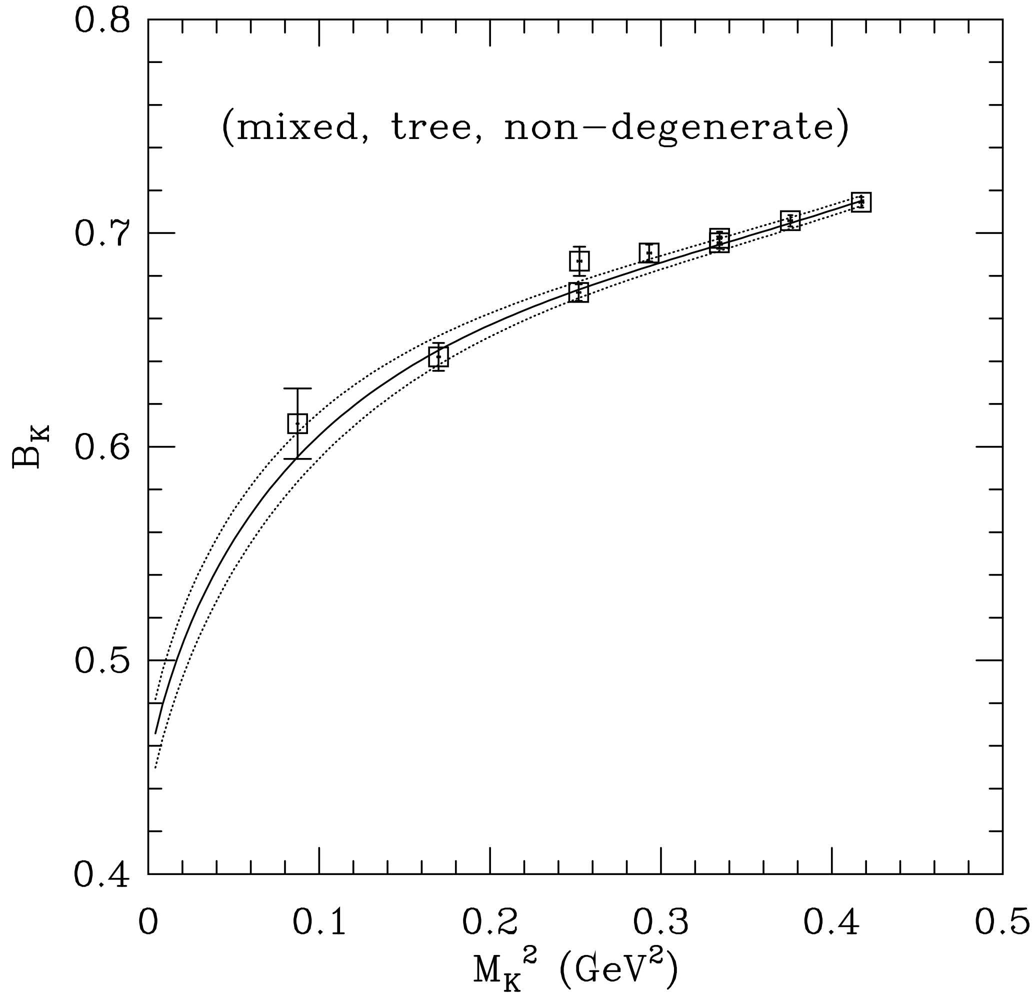
- Measure taste breaking at $m_G/m_\rho \approx 0.55$ with

$$\delta_2 = \frac{(M_\pi a)^2 - (M_G a)^2}{(M_\rho a)^2 - (M_G a)^2}$$

- Taste $\xi_5\xi_i$: $\delta_2 = 0.013,\ 0.05,\ 0.24$ for HYP/Asqtad/Unimproved
- Taste ξ_4: $\delta_2 = 0.039,\ 0.06,\ 0.35$ for HYP/Asqtad/Unimproved
- Confirms hopes/expectations

Preliminary unquenched results [Lee *et al.*]

- MILC coarse lattices: $20^3 \times 64$, $am_\ell/am_s = 0.01/0.05$
- Tree-level matching, $am_{\rm HYP} = 0.01 - 0.05$, degen. and non-degen

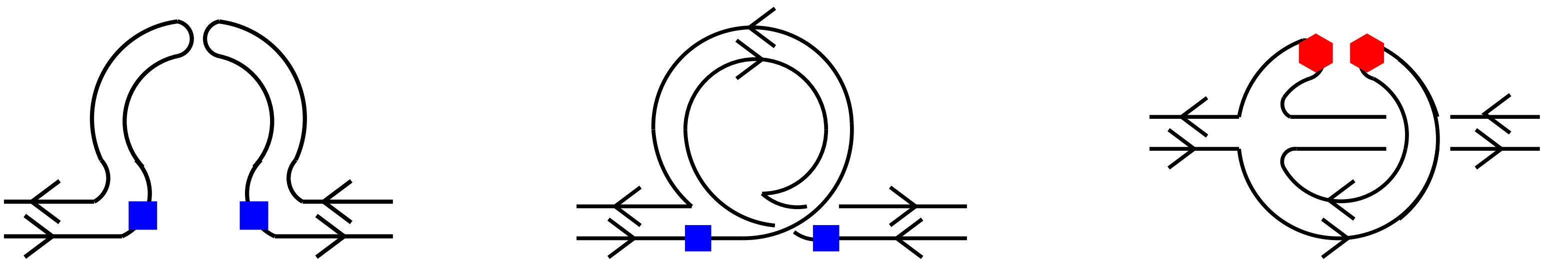


Summary

- Important to calculate B_K, $B_{7,8}^{3/2}$ using many methods with different systematics
- We propose to do so using valence HYP-smeared staggered fermions on MILC coarse and fine ensembles
- More challenging than bilinear matrix elements due to Z-factors, operator mixing and taste-breaking
- Use many degenerate and non-degenerate quark combinations to do chiral fits and try to remove contributions from unphysical operators
- 2 lattice spacings allow first attempt at a continuum extrapolation
- Dominant error due to two- and higher-loop matching
- Requesting 1.2 M node-hrs on QCDOC = 0.26 M node-hrs on 4g

B_K at one-loop in Chiral Perturbation Theory

Quark line diagrams contributing to B_K at 1-loop:



(a) (b) (c)

- No valence-sea meson contribution (cancel in ratio defining B_K)
- Sea-sea mesons enter only through (a)—vanish if $m_s = m_d$