Weak decays of light hadrons:

How present and future lattice calculations help constrain CKM elements.

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Outline

- Milestones for lattice calculations which impact phenomenology
  - F_π, F_K/F_π, $f^{K\pi}(0)$, B_K
  - [$f_B^2 B_B$, ξ, $B \to (\pi, \rho)\ell\mu$, $B \to (D, D^*)\ell\mu$]
- Simulation parameters needed to achieve milestones
  - Simple estimates
  - Case study: improved staggered simulations [MILC]
  - Possible Lattice Ensembles (PLE0–2)
  - Estimates of required CPU costs
- Guesstimated errors with possible lattice ensembles
- Thanks to Claude Bernard and Bob Sugar

Pion decay constant F_π

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 d|\pi^-(p)\rangle = iF_\pi p_\mu$$

- Why? Test of lattice methodology.
- Experiment: $\pi \to \ell\nu_\ell(\gamma)$ [PDG 2002]

$$F_\pi = 130.7 \pm 0.1 \text{ [}V_{ud}\text{]} \pm 0.36 \text{ [rad. corr.] MeV}$$

  - Total error small $\sim 0.3\%$
- Error milestones for lattice calculations
  - 3%: precision era begins
  - 1%: convincing precision
  - 0.3%: matches experimental error
- First milestone may have been reached with improved staggered fermions [MILC, Aubin 2003]

  $F_\pi = 129.3 \pm 1.1$ [stat] $\pm\, 3.5$ [sys]$\pm ??$[det$^{1/2}$] MeV (2.8% error)

Decay constant ratio F_K/F_π

- Why? Determine $|V_{us}|/|V_{ud}|$ and test CKM unitarity
- Experiment: taken from [W. Marciano, hep-ph/0402299]

$$\frac{F_K^2|V_{us}|^2}{F_\pi^2|V_{ud}|^2} = 0.07602 \pm 0.00023\text{[expt.]} \pm 0.00027\text{[rad.corr.]}$$

  - Total error is small: $2 \times 0.23\%$
- CKM unitarity? PDG 2002:
  - $|V_{ud}| = 0.9740(5)$; $|V_{us}| = 0.2196(26)$ (from $K_{\ell 3}$)
  - $|V_{ud}|^2 + |V_{us}|^2 = 0.9486(10) + 0.0482(11) = 0.9969(15)$
  - Errors in $|V_{ud}|$ and $|V_{us}|$ contribute equally to unitarity check

Decay constant ratio F_K/F_π (cont.)

$$\frac{F_K^2|V_{us}|^2}{F_\pi^2|V_{ud}|^2} = 0.07602 \pm 0.00023\text{[expt.]} \pm 0.00027\text{[rad.corr.]}$$

$$|V_{us}| = 0.2196(26)(\text{from} K_{\ell 3})$$

- Milestones for F_K/F_π:
  - 1%: determine $|V_{us}|$ from $K \to \ell\nu/\pi \to \ell\nu$ with same precision as from $K_{\ell 3}$
  - 0.25%: match experimental errors in $K \to \ell\nu/\pi \to \ell\nu$
- MILC result approaches first milestone: [MILC, Aubin 2004]

$$F_K/F_\pi = 1.201 \pm 0.008\,\text{[stat]} \pm 0.015\,\text{[sys]} \pm ??\text{[det}^{1/2}\text{]}$$ (1.4% error)

$K \to \pi\ell\nu$ ($K_{\ell 3}$) form factors

$$\langle\pi^-(p')|\bar{s}\gamma_\mu u|K^0(p)\rangle = f_+(t)(p+p')_\mu + f_-(t)(p-p')_\mu\,, \quad t=(p-p')^2$$

- Why? Determine $|V_{us}|$
- Experiment determines: $|V_{us}|f_+(0)$. Errors are [Cirigliano, 2001]

  $\pm 0.6\%$ [rate] $\pm\, 0.4\%$ [slope] $\pm\, 0.3\%$ [rad. corr.] $= 0.8\%$
- Standard theoretical estimate (ChPTh + models)

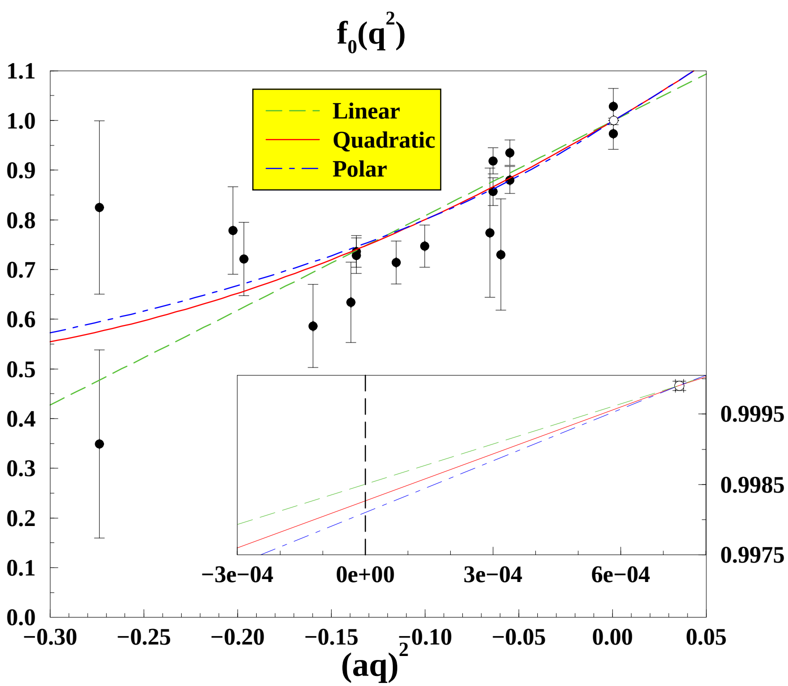
  $f_+(0) = 0.961 \pm 0.008$ [PDG2002, Leutywyler & Roos]
- Updated theoretical estimate (NNLO ChPTh + data)

  $f_+(0) = 0.976 \pm 0.010$ [Bijnens & Talavera]
- Error milestones for lattice calculations
  - 1%: match present theory error
  - 0.5%: match experimental errors

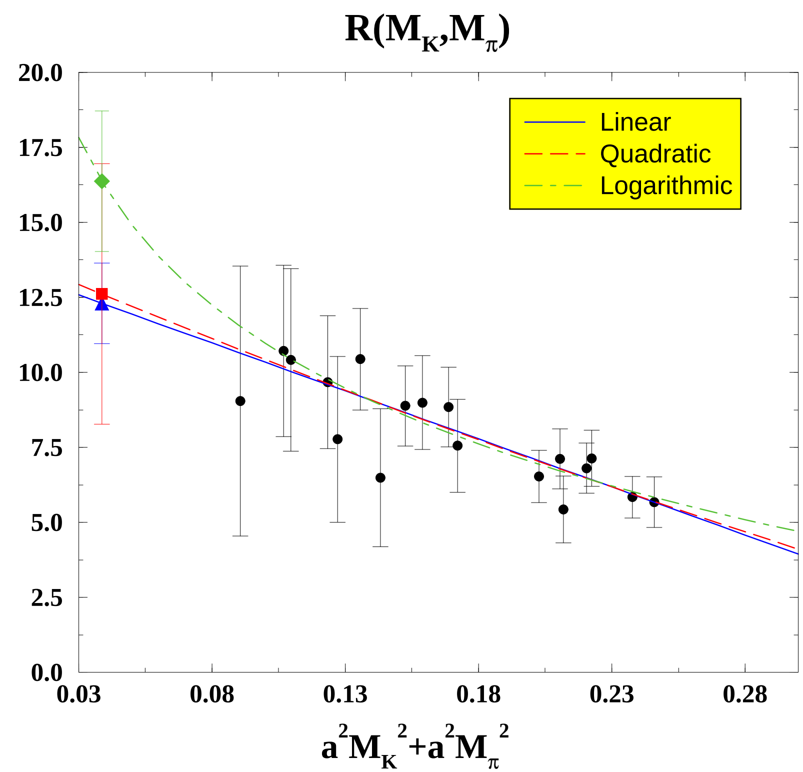
$K \to \pi \ell \nu$ ($K_{\ell 3}$) form factors (continued)

- Methodology now exists to reach 1% accuracy: [Becirevic 2004]

  $f_+(0) = 0.961 \pm 0.005$ [stat] ± 0.007 [sys] $\pm ???$ [quench]
- Need to extrapolate $t \to 0$ and $\widehat{m} = (m_u + m_d)/2 \to \widehat{m}^{\text{phys}}$



$q^2 = t$



$R = [1 + f_2 - f_+(0)]/(a^2(M_K^2 - M_\pi^2))^2$

CP violation in $K - \bar{K}$ mixing: B_K

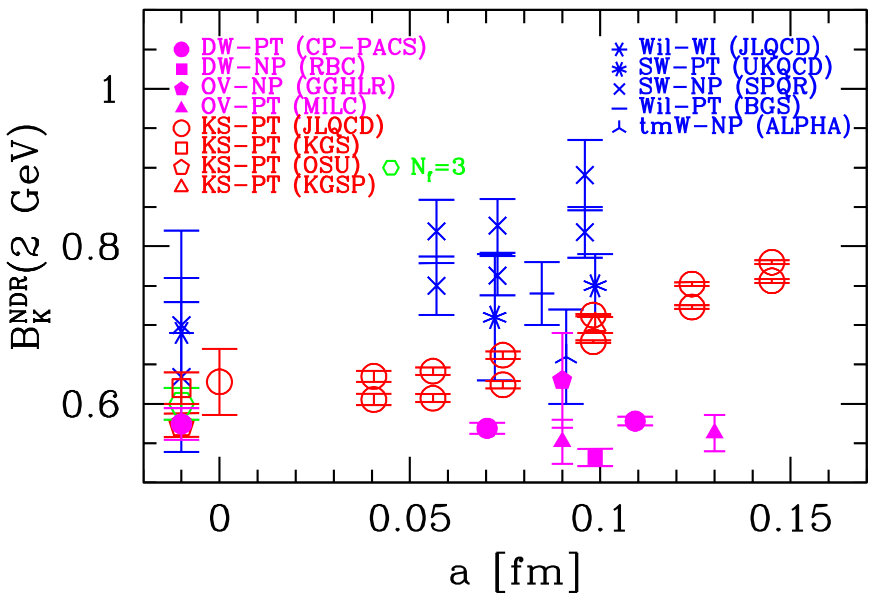
$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \bar{\eta} \left[\eta_2 S(x_t) A^2 \lambda^4 (1 - \bar{\rho}) + \mathrm{charm{-}contribs}\right] \hat{B}_K$$

- Why? Constrain $\bar{\rho}$ and $\bar{\eta}$
- Total non-lattice error is 9.5%, primarily from 2.2% error in $A\lambda^2 = |V_{cb}|$.
- Lattice $\hat{B}_K$ result used in CKM fits is: [Lellouch 00]

  $\hat{B}_K = 0.86 \pm 0.06$ [stat & match] $\pm\, 0.14$ [quench % chiral] $(18\%\ \mathrm{error})$

- current lattice error $\approx 2\times$ that from other sources
- Error milestones for lattice calculations of $\hat{B}_K$:
  - 10%: matches that from other sources
  - 5%: $\approx$ half that of other sources

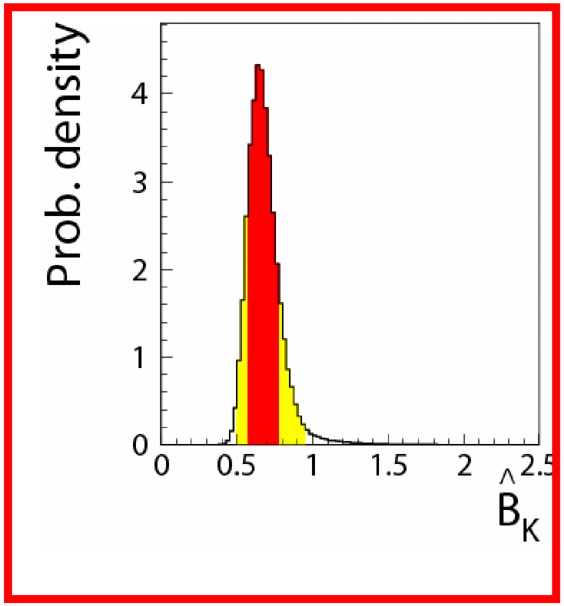
K^0-$\bar{K}^0$ ***mixing: summary (1)***

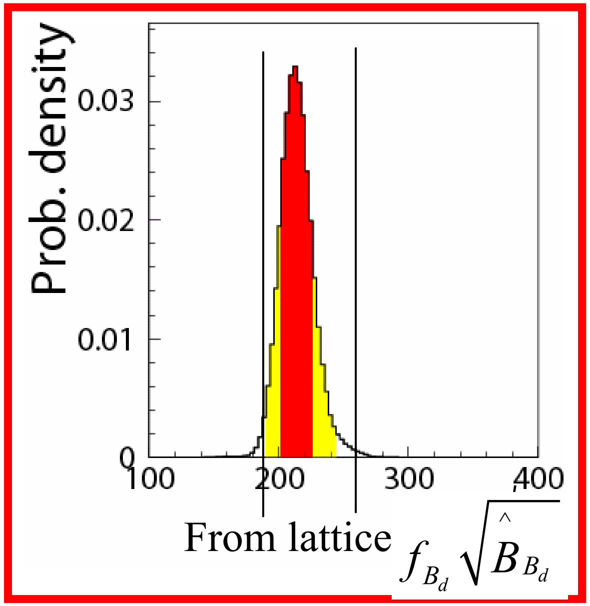


- Only one preliminary unquenched result (OSU '97)
- Results consistent in continuum limit
- Some GW results are slightly lower
- Reference result: quenched staggered JLQCD '98 calculation (weak point: perturbative renormalization)
- Quenching: $\delta B_K \sim 15\%$ (OSU $N_f = 3$ and QχPT)
- $m_d = m_s = m_s^{phys}/2 \to m_d \neq m_s$: $\delta B_K \sim 5\%$ (χPT)

(Sharpe '92, '96)

Indirect determination of the non-perturbative QCD parameters





Results of the fit

$$\hat{B}_K = 0.66 \begin{smallmatrix} +0.11 \\ -0.09 \end{smallmatrix}$$

$$[0.51 - 0.95] \quad @95\% \quad C.L.$$

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 212.5 \pm 12.0 \quad \text{MeV}$$

$$[190.5 - 243.5] \quad \text{MeV} \quad @95\% \quad C.L.$$

Inputs:

$$\hat{B}_K = 0.86 \pm 0.06 \pm 0.14$$

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 223 \pm 33 \pm 12 MeV$$

B_d–$\bar{B}_d$ mass difference: $f_{B_d}^2 \hat{B}_{B_d}$

$$\Delta M_d = \frac{G_F^2 M_W^2 M_{B_d}}{6\pi^2} \eta_c S(x_t) A^2 \lambda^6 \left[(1-\bar{\rho})^2 + \bar{\eta}^2\right] f_{B_d}^2 \hat{B}_{B_d}$$

- Total non-lattice error is 6% (from $A\lambda^2$, λ, η_c and ΔM_d in decreasing order)
- Lattice result used in recent fits is: [Ciuchini 03]

$$f_{B_d}\sqrt{B_{B_d}} = 223 \pm 33\,\text{[stat]} \pm 12\,\text{[syst]}\ \mathrm{MeV} \qquad [\approx 15\%\ \mathrm{error}]$$

- Current lattice error is ≈ 5 times error from other sources (since $f_{B_d}^2 B_{B_d}$ is relevant quantity)
- Error milestones for lattice calculations of $f_{B_d}\sqrt{B_{B_d}}$:
  - $\sim 8\%$: reduction by factor of 2
  - 3–4% comparable to that from other sources

B_s–$\bar{B}_s$ mass difference: ξ

$$\frac{\Delta M_d}{\Delta M_s} = \frac{M_{B_d}}{M_{B_s}} \lambda^2 \left[(1-\bar\rho)^2 + \bar\eta^2\right] \frac{1}{\xi^2}$$

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

- ΔM_s not measured, probably will be soon at Tevatron
- Present standard lattice result: [Ciuchini 03]

$$\xi = 1.24 \pm 0.04 \text{ [stat] } \pm 0.06 \text{ [syst]} \quad [\approx 6\% \text{ error}]$$

  - second error mainly from chiral extrapolation
- Error milestones in lattice calculation of ξ:
  - 3%: reduction by factor of 2
  - 1.5%: reduction by factor of 4

Form factors in $b \to u$ exclusive decays

- Why? In future, strongest constraint on V_{ub} likely to come from exclusive $B \to \pi\ell\nu$ and $B \to \rho\ell\nu$ decays
- Present status [Ciuchini 03]

$$|V_{ub}| \times 10^4 = 33.0 \pm 2.4 \,\text{[exp]} \pm 4.6 \,\text{[thy]} \quad (16\% \text{ error})$$

  - Theory error ($4.6 \times 10^{-4} = 14\%$) from comparing models
- Current lattice errors for $B \to \pi\ell\nu$ form factors are $\approx 15\%$ *in quenched approx* [Ryan 02]
- Error milestones in lattice calculation of form factors:
  - 14%: matches (present) theory error
  - 7%: matches (present) experimental error

Form factors in $b \to c$ exclusive decays

- Why? Determine V_{cb}
- Best current results use inclusive decays, rather than exclusive decays amenable to lattice treatment:

$$|V_{cb}| \times 10^3 = 41.4 \pm 0.7\,\text{[exp]} \pm 0.6\,\text{[thy]} \qquad (2.2\%\ \text{error})$$

- Exclusive $B \to D^* \ell \nu$ result does use lattice:

$$|V_{cb}| \times 10^3 = 42.1 \pm 1.1\,\text{[exp]} \pm 1.9\,\text{[thy]} \qquad (5\%\ \text{error})$$

- Current theory error (4.5%) dominated by lattice result
- Error milestones for lattice calculation of $B \to (D, D^*)\ell\nu$ form factors:
  - 2.5%: matches current exclusive experimental error
  - 1.5%: matches inclusive experiment or theory errors

Summary of milestones for lattice calculations

Quantity	Present error	Significant impact	Match “expt”
F_π	2.8% + det$^{1/2}$	1-3%	0.3%
F_K/F_π	1.4% + det$^{1/2}$	1%	0.25%
$f^{K\pi}(0)$	0.9% + quench	1%	0.5%
$\widehat{B}_K$	7% $\pm$ 16%	10%	10%
$f_B\sqrt{B_B}$	15% $\pm$ 6%	8%	3%
ξ	3% $\pm$ 5%	3%	$--$
$B \to (\pi, \rho)$	15% + quench	14%	7%
$B \to (D, D^*)$	4.5%	2.5%	1.5%

Error milestones for errors in lattice computations.

Note that results for the square of the last four quantities are sometimes presented.

What are attainable errors?

Sources of error in lattice calculations

- Statistical (how many independent configurations?)
- Finite volume (how large should size L_s be?)
- Continuum extrapolation (what should the minimum a be?)
- Chiral extrapolation, $\widehat{m} = (m_u + m_d)/2 \to \widehat{m}_{\rm phys}$ (what should the minimum m_π/m_ρ, or $\hat{m}/m_s^{\rm phys}$, be?)
- Matching factors

$$O_{\rm cont}(\mu) = Z(\mu a, g)\, O_{\rm lat}$$

  - Z can sometimes be calculated non-perturbatively, otherwise to one- or two-loop accuracy in pert. theory.
  - Significant source of error in $\widehat{B}_K$, $f_B\sqrt{B_B}$, and perhaps heavy-light form factors.

Minimum lattice spacing?

Rough (naive?) estimate

- Assume $O(a)$ improved action (very important)

$$\mathcal{Q}_{\rm lat} = \mathcal{Q}_{\rm cont}\left[1 + (a\Lambda_2)^2 + (a\Lambda_n)^n + \dots\right]$$

  - Improved Wilson, $n=3$; Staggered, G-W $n=4$
  - $\Lambda_2 \sim \Lambda_n \sim \Lambda_{\rm QCD}$, though typically larger for staggered than Wilson

- Assume simulations at $a_{\rm min}$ and $\sqrt{2}a_{\rm min}$, and linearly extrapolate in a^2. Resulting error is:

$$\delta\mathcal{Q}_{\rm cont}/\mathcal{Q}_{\rm cont} \approx (1 - 2^{n/2})\,(a_{\rm min}\Lambda_n)^n$$

- If require fractional error ϵ:

$$a_{\rm min} = \left(\frac{\epsilon}{0.01}\right)^{1/n}\left(\frac{0.4\text{GeV}}{\Lambda_n}\right)\begin{cases} 0.075\text{ fm} & n=3 \\ 0.133\text{ fm} & n=4 \end{cases}$$

Minimum quark mass?

- Chiral perturbation theory (schematic):

$$\mathcal{Q}_{\rm lat} \sim \mathcal{Q}_{\rm phys}\left[1 + c_1\left(\frac{m_\pi}{m_\rho}\right)^2 + c_2\left(\frac{m_\pi}{m_\rho}\right)^4 + \ldots\right]$$

  - $c_1 \sim c_2 \sim O(1)$

- Require fractional error ϵ given two values of m_π/m_ρ:

$$(m_\pi/m_\rho)_{\rm min} = 0.27 \times (\epsilon/0.01)^{1/4} \times c_2^{1/4}$$

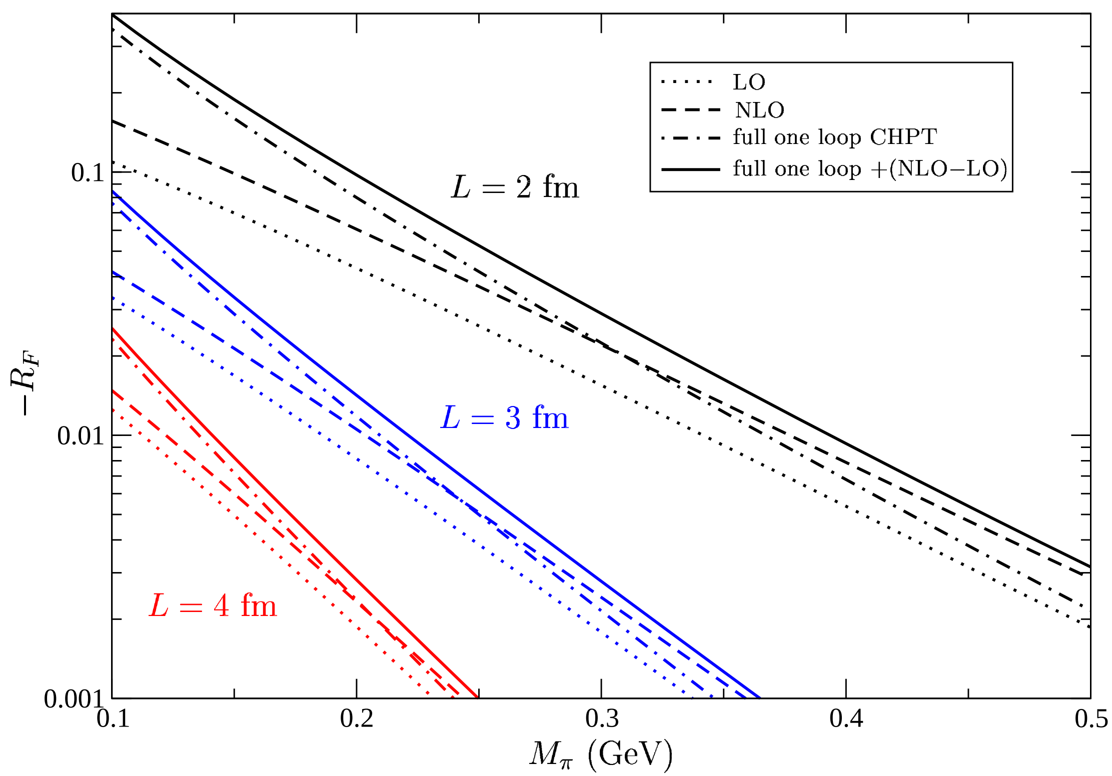
  - $(m_\pi/m_\rho)_{\rm min} = 0.27$ corresponds to $(\hat{m}/m_s^{\rm phys})_{\rm min} = 0.08$

- Can improve extrapolation using more quark masses, but at the price of needing higher order terms in the chiral exp.

- Useful to use partially quenched simulations ($m_{\rm Valence} \neq m_{\rm Sea}$)—no unitarity problems for quantities considered here

Minimum box size?

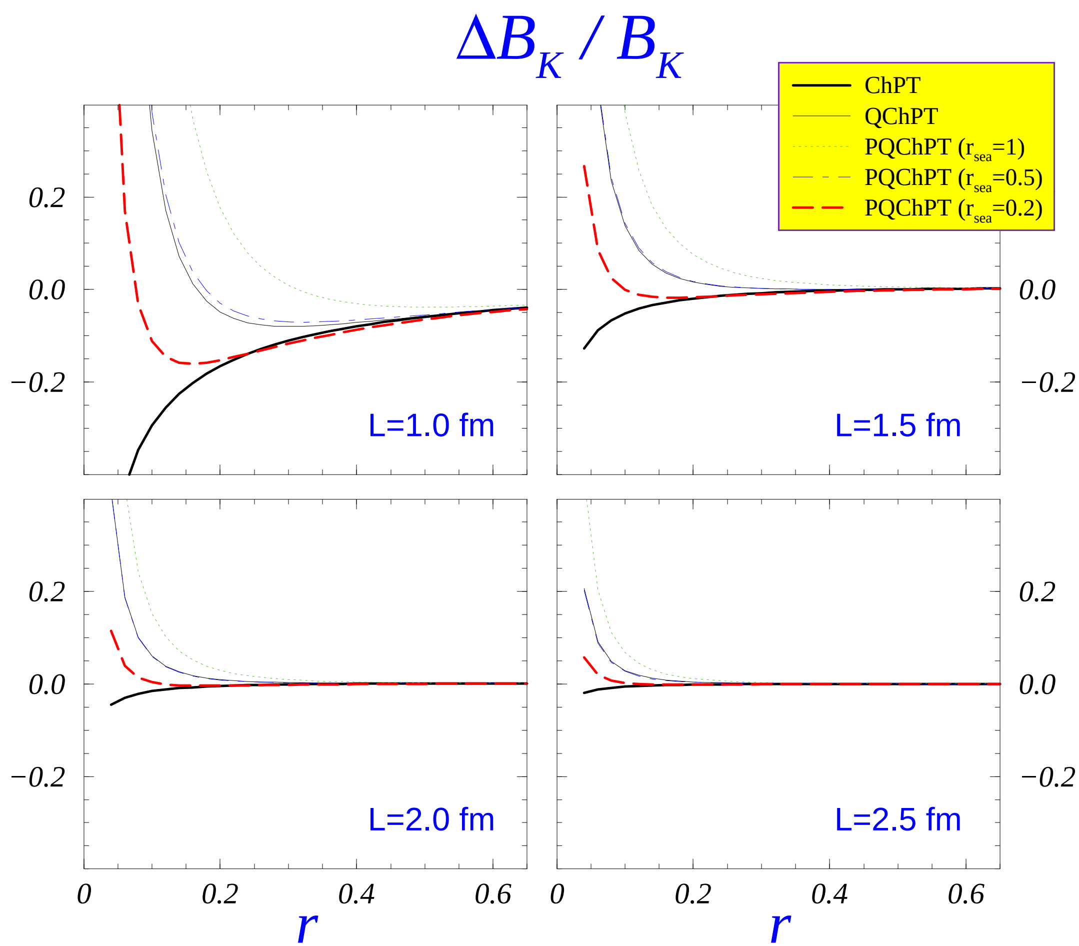
Finite volume shifts important when aim for 1% precision



Relative error for F_π [Colangelo, hep-lat/0403025]

- Dominant effect from pion loops
- For quantities considered here, can calculate leading order volume shift using ChPTh (no new parameters)
- Uncertainty due to NLO known for m_π and F_π (shown here)
- Relative shift (R_F) $< 1\%$ for lattices discussed below
- Uncertainty in R_F still smaller

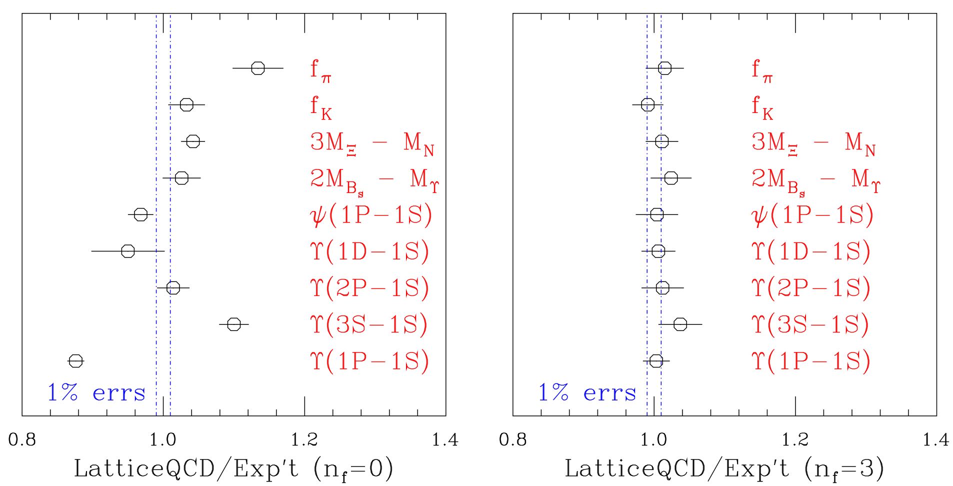
Minimum box size? (continued)



Leading order ChPT results for relative volume shift in B_K, with $r = \hat{m}/m_s$ [Becirevic, hep-lat/0311028]

The MILC case study

- 2+1 simulations with improved staggered dynamical quarks
- Detailed study of continuum and chiral extrapolations for $F_{\pi,K}$ and $M_{\pi,K}$ [Aubin *et al*, hep-lat/0309088,0402030]
  - Uses "staggered ChPT", including $O(a^2)$ errors
  - Somewhat more complicated than needed for Wilson fermions
- Uses $\det(\mathrm{N_f}=2) \approx \sqrt{\det(\mathrm{stagg})}$ trick



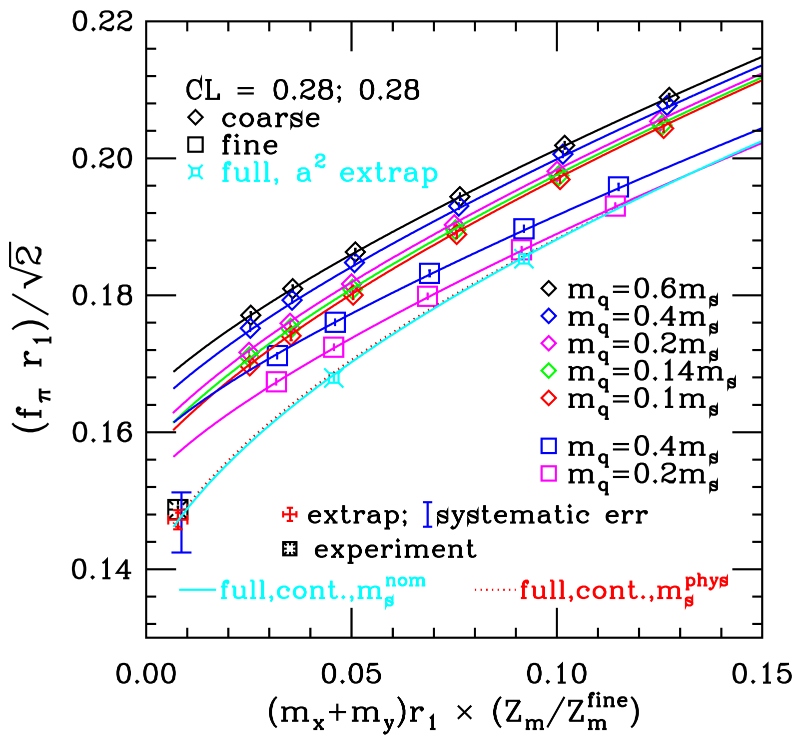
- Lattice action non-local at $O(a^2)$ $\Rightarrow$ unknown systematic errors
- Test by comparing to experiment [Davies *et al*, PRL92, 022001]
- Scale set by $\Upsilon(2S) - \Upsilon(1S)$

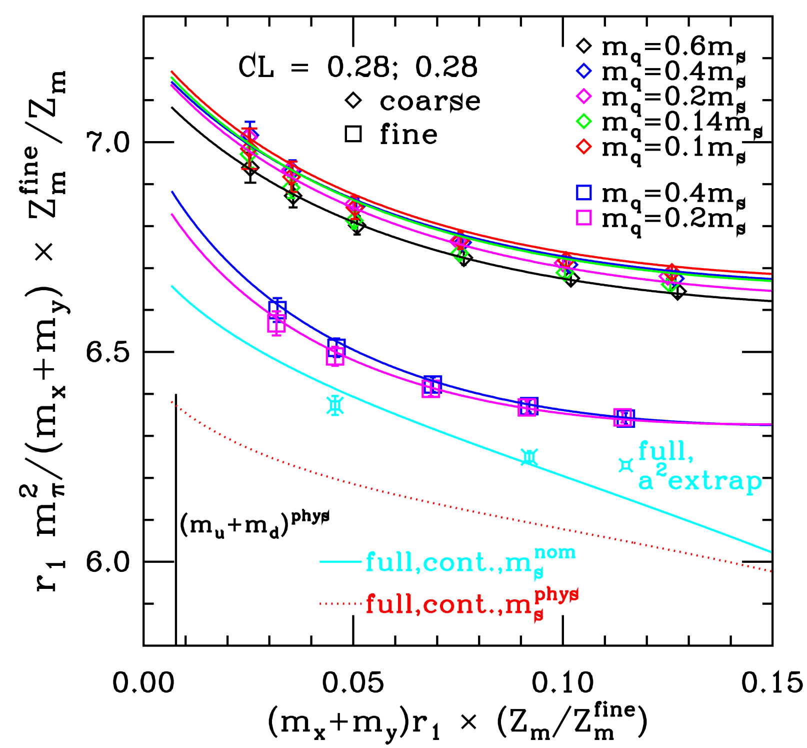
MILC study—lattice parameters

a(fm)	$\widehat{m}/m_s^{\rm phys}$	Size	L_s (fm)	$N_{\rm conf}$
0.12	0.125	$24^3 \times 64$	2.9	≈ 137
0.12	0.175	$20^3 \times 64$	2.4	≈ 447
0.12	0.25	$20^3 \times 64$	2.4	≈ 608
0.12	0.5	$20^3 \times 64$	2.4	≈ 485
0.12	0.75	$20^3 \times 64$	2.4	≈ 262
0.09	0.22	$28^3 \times 96$	2.5	$\approx 583/4$
0.09	0.44	$28^3 \times 96$	2.5	$\approx 531/4$

- Used 8 valence quarks with $m_V/m_s = 0.14 - 1$ on each set
- Parameters on fine lattice
  - $(a\widehat{m})_{\rm min} = 0.0062$, $(am_\pi)_{\rm min} = 0.15$, $(m_\pi)_{\rm min} = 0.33{\rm GeV}$
  - $m^2_{\rm NGB}(\hat{m} = 0) \approx (0.16{\rm GeV})^2 - (0.28{\rm GeV})^2 \sim \alpha_s^2 a^2$

MILC study: fits





- Fits use NLO *staggered* ChPT, NNLO analytic and some NNNLO analytic terms
- 240-416 points, 40-48 params (20 tightly constrained)
- Statistical errors 0.1-0.4%
- [Thanks to Claude Bernard]

MILC case study—results and errors

- $F_\pi = 129.3 \pm 1.1$ [stat] ± 3.5 [syst] MeV
  - Statistical error: 0.9% after extrapolations
  - Scale error: 2.2% (from $\Upsilon(2S) - \Upsilon(1S)$)
  - Combined continuum and chiral extrapolation error: 1.5%
  - Finite volume shifts (included) $\approx 0.5\%$
- $F_K/F_\pi = 1.201 \pm 0.008$ [stat] ± 0.015 [syst]
  - Statistical error: 0.7%
  - Essentially no scale error since ratio
  - Combined continuum and chiral extrapolation error: 1.2%
- CPU cost: 480/4 fine lattices at $\hat{m}/m_s = 0.22$ required 0.09 TFlops-years
- Total project required 0.6 TFlops-years

Possible lattice ensembles

- Need to control continuum and chiral extrapolations
  - PLE0: $a = 0.09$ fm, $\widehat{m}/m_s = 0.2$, $L_s = 2.5$ fm,
    - Already exists for staggered fermions with $N_{\rm conf} \approx 150$
  - PLE1a: halve a^2 ($a = 0.06$ fm)
  - PLE1b: halve $\widehat{m}$ ($\widehat{m}/m_s = 0.1$) at fixed $m_\pi L_s = 4.2$
  - PLE2: halve a^2 *and* $\hat{m}$, at fixed $m_\pi L_s = 4.2$
- Costs with improved staggered fermions [Gottlieb 02, Sugar]

$$\mathrm{TFlops-years} = 0.09 \left(\frac{N_{\rm conf}}{120}\right)\left(\frac{L_s}{2.5fm}\right)^4\left(\frac{L_t}{3.43L_s}\right)\left(\frac{0.2}{\widehat{m}/m_s}\right)^{2.5}\left(\frac{0.09fm}{a}\right)^7$$

- Costs with improved Wilson [Ukawa 02]

$$\mathrm{TFlops-years} = 0.25\times 2.8 \left(\frac{N_{\rm conf}}{1000}\right)\left(\frac{L_s}{3fm}\right)^5\left(\frac{L_t}{2L_s}\right)\left(\frac{0.6}{m_\pi/m_\rho}\right)^{6}\left(\frac{0.1fm}{a}\right)^7$$

  - [Farchioni *et al*, hep-lat/0403014] claim much lower cost for Wilson fermions

Costs of future lattice ensembles

	a(fm)	size	L_s (fm)	$\frac{\widehat{m}}{m_s}$	$\frac{m_\pi}{m_\rho}$	am_π	CPU (stag)	CPU (Wils)
PLE0	0.09	$28^3 \times 96$	2.5	0.2	0.39	0.15	0.09	1.6
PLE1a	0.06	$42^3 \times 138$	2.5	0.2	0.39	0.10	1.5	26
PLE1b	0.09	$40^3 \times 96$	3.6	0.1	0.3	0.105	1.5	34
PLE2	0.06	$60^3 \times 138$	3.6	0.1	0.3	0.07	25	550

Costs in TFlops-years; $N_{\text{conf}} = 120$, $m_\pi L_s = 4.2$

- Notation below: PLE1 $=$ PLE1a + PLE1b
- Estimates are conservative
- Overhead for lattices at larger a and $\hat{m}$ is $\approx 2 - 3$
- Cost of twisted-mass Wilson similar to Wilson

Ensembles with Chirally Symmetric fermions

- As resources increase, domain-wall or overlap dynamical fermions become increasingly attractive
- Exact or near exact chiral symmetry $\Rightarrow$ continuum & chiral extrapolations decoupled.
- Remove uncertainty from $\sqrt[2]{\mathrm{Det}}$ and $\sqrt[4]{\mathrm{Det}}$
- Unclear what size of fifth dimension (N_5) or number of conjugate gradient iterations will be needed
- Current RBC benchmark: cost is $(1-2) \times N_5$ cf. *imp. stagg.*
  - Perhaps 12–24$\times$ cost of comparable staggered ensembles
- Leads one to consider ensemble "DWF1"
  - $\widehat{m}$ and a^2 comparable to PLE1
  - Computer time comparable to PLE2 (staggered), i.e. 25 Tflops-years!
  - Expect errors comparable to PLE2
- Warning: other estimates *much* less optimistic [Jansen, 02]

Estimating future errors

WARNING

- Estimates are approximate
- Less reliable as go further into the future
- I have tried to be conservative
- Very hard to estimate how extrapolations will improve with better ensembles
- Thanks (but no blame) to Claude Bernard for help

F_π: estimated future errors

- Use MILC study as guide for staggered *and* Wilson fermions
  - staggered errors are nominally smaller [$O(\alpha_s a^2)$] than Wilson [$O(a^2)$], but extrapolations are complicated by taste violations [$O(\alpha_s^2 a^2)$]
- Estimated fractional errors in F_π:

Source	PLE0	PLE1	PLE2
Statistical	0.9%	0.6%	0.3%
Scale	2.2%	1.5%	1.0%
Extrapolation	1.5%	0.75%	0.5%
Total	2.8%	1.8%	1.2%

- Error in matching factors subdominant
  - staggered: $Z = 1$
  - Wilson: calculated non-perturbatively, with stat. error $\approx 0.25\%$
- Conclusion: 1% error eventually attainable

F_K/F_π: estimated future errors

- Again use MILC study as guide
- Estimated fractional errors in F_K/F_π:

Source	PLE0	PLE1	PLE2
Statistical	0.7%	0.5%	0.25%
Extrapolation	1.2%	0.6%	0.4%
Total	1.4%	0.8%	0.5%

  - Scale and matching factor errors largely cancel

- Conclusion: 1% milestone attainable
  - $|V_{us}|$ from $K \to \ell\nu$ competitive with that from $K \to \pi\ell\nu$
- Error $< 1\%$ possible
  - ultimately this may give best determination of $|V_{us}|$

$K \to \pi\ell\nu$ form factor: estimated future errors

- Use [Becirevic *et al*, hep-ph/0403217] as guide
  - PLE0 has smaller ensemble (120 vs. 460 lattices)
  - PLE0 is more chiral ($\hat{m}/m_s = 0.2$ vs. 0.5)
  - PLE0 is further from continuum ($a^{-1} = 2.2$ vs. $2.7; \mathrm{GeV}$)
- Estimated fractional errors in $f_+^{K\pi}(0)$:

Source	quench	PLE0	PLE1	PLE2
Statistical	0.5%	1.0%	0.6%	0.3%
Extrapolation	0.7%	0.5%	0.3%	0.2%
Total	0.9%+??	1.1%	0.7%	0.4%

  - Matching factor errors tiny: $\sim 0.03\%$
  - Can reduce statistical error with larger ensemble
- Conclusion: 1% milestone attainable and can be exceeded
  - $|V_{us}|$ from $K \to \pi\ell\nu$ can first be made more robust and then improved

B_K: estimated future errors

- Use [JLQCD, 98] unimproved stagg. simulation as guide
  - PLE0 is more chiral ($\hat{m}/m_s = 0.2$ vs. ≈ 0.5)
  - PLE0 further from continuum ($a = 0.09$ vs. 0.05fm), but uses improved action
  - JLQCD use 5 values of a for continuum extrapolation
  - PLE0 will have larger statistical errors since improved stagg.
- Estimated fractional errors in B_K:

Source	quench	PLE0	PLE1	PLE2
Statistical	1.0%	2.0%	1.2%	
Extrapolation	$1\% \pm 5\%??$	$5\% \pm 5\%$	2.5%	
Matching	5%	9%	3%	
Total	$7\% \pm 16\%??$	12%	5%	3%

  - PLE0 assumes 1-loop matching
  - PLE1/2 assume 2-loop or non-perturbative matching
- Conclusion: 10% milestone attainable and can be exceeded

Heavy-light quantities: est. future errors

- Estimated fractional errors:

Source	PLE0	PLE1	PLE2
$f_B\sqrt{B_B}$	8-13%	4-5%	3-4%
ξ	4%	3%	1.5-2%
$B \to \pi\ell\nu$	10-13%	5.5-6.5%	4-5%
$B \to D^*\ell\nu$	3-4%	2%	1.2%

  - PLE0 assumes 1-loop matching and staggered (or Wilson) chiral perturbation theory
  - PLE1/2 assume 2-loop or non-perturbative matching and chiral pert. theory

Future errors vs. Milestones

Source	PLE0	PLE1	Milestone 1	Milestone2
F_π	2.8%	1.8%	1-3%	0.3%
F_K/F_π	1.4%	0.8%	1%	0.25%
$f_+^{K\pi}(0)$	1.1%	0.7%	1%	0.5%
B_K	12%	5%	10%	10%
$f_B\sqrt{B_B}$	8-13%	4-5%	8%	3%
ξ	4%	3%	3%	
$B \to \pi\ell\nu$	10-13%	5.5-6.5%	14%	7%
$B \to D^*\ell\nu$	3-4%	2%	2.5%	1.5%

- Milestones are attainable with dedicated machine sustaining 10-100 TFlops