Quark mass reweighting

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based on

Reweighting towards the chiral limit. ,
Low energy chiral constants from epsilon-regime simulations with improved Wilson fermions. ,
Epsilon regime calculations with reweighted clover fermions,
A. Hasenfratz, R. Hoffmann, S. Schäfer.

Fluctuations and reweighting of the quark determinant on large lattices
Martin Lüscher, Filippo Palombi

Take two flavours as an example

$$\det(D^\dagger D) \;=\; W \underbrace{\det(\tilde{D}^\dagger \tilde{D})}_{\Uparrow}$$

defines ensemble put in HMC

reweighting

$$\langle \mathcal{O} \rangle \;=\; \frac{\langle \mathcal{O}\, W \rangle_{\rm m}}{\langle W \rangle_{\rm m}}$$

$$\langle \mathcal{O} \rangle_{\rm m} \;=\; \frac{1}{Z} \int \mathrm{D}[U] \mathrm{e}^{-S_G[U]} \det(\tilde{D}^\dagger[U] \tilde{D}[U])$$

why?

Reweighting notation

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  $\Rightarrow$ smaller variance of correlators

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- oversampling of small $\lambda(D^\dagger D)$
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- **better sampling of topological sectors**

Fluctuations of reweighting factor

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O}\, W \rangle_\mathrm{m}}{\langle W \rangle_\mathrm{m}}$$

Expect trouble if W fluctuates too much.
Take $W = \mathrm{e}^{-X}$, cumulant expansion:

$$\frac{\langle W_l^2 \rangle_\mathrm{m}}{\langle W_l \rangle_\mathrm{m}^2} = \langle W_l \rangle \langle {W_l}^{-1} \rangle = \exp \left\{ \sum_{n=1}^{\infty} \tfrac{2}{(2n)!} \langle X_l^{2n} \rangle_\mathrm{con} \right\},$$

X extensive: expect reweighting to fail at large volume.

- Choose W to suppress contribution from large modes, say $\lambda > (100\,\mathrm{MeV})^2$

The ensembles (modified Dirac operators)

.

$$W = w_l(D^\dagger D)$$

l	$\tilde{D}_l$	$w_l(\nu^2)$	$w_l(\nu^2)\big\|_{\nu^2 \gg \mu^2}$
LP 1	$D + i\mu\gamma_5$	$\frac{\nu^2}{\nu^2+\mu^2}$	$1 - \frac{\mu^2}{\nu^2} + \mathrm{O}(\nu^{-4})$
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HHS	$D + \mu$		

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- Lüscher, Palombi:

$$W_{l,N} = \frac{1}{N}\sum_{k=1}^{N} \exp\left\{\left(\eta_k, \left[1 - w_l(D^\dagger D)^{-1}\right]\eta_k\right)\right\}$$

Some RMT and numerical results

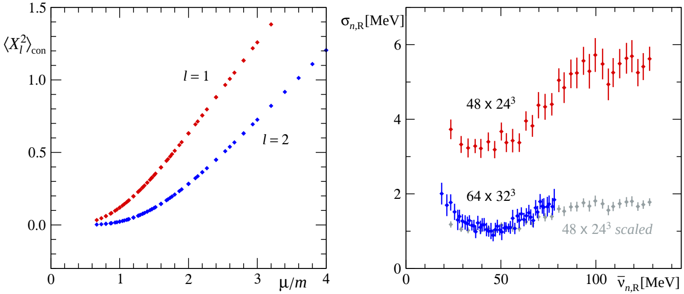

Figure: Values of $\langle X_l^2 \rangle_{\rm con}$ computed in random matrix theory at $m = 5, \ldots, 30$ MeV and $\mu = 20, \ldots, 44$ MeV, assuming $\Sigma = (250\ \mathrm{MeV})^3$ and $V = (4.5\ \mathrm{fm})^4$ (plot on the left). The plot on the right shows the widths of the distributions of the first 32 (48) eigenvalues of $(D^\dagger D)^{1/2}$ on a lattice of size 48×24^3 (64×32^3). In both cases the lattice spacing and the renormalized sea-quark mass are approximately equal to 0.08 fm and 25 MeV respectively. The grey points labelled "48×24^3 scaled" are the 48×24^3 data scaled by the ratio $(24/32)^4$ of the lattice volumes.

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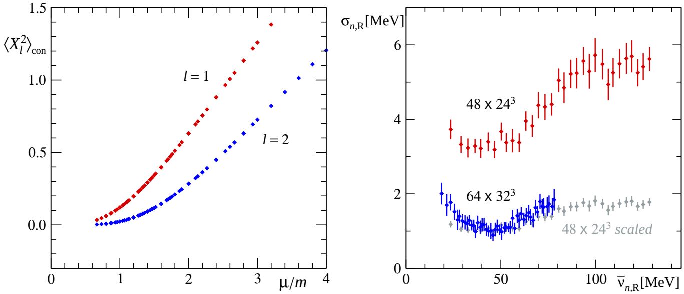

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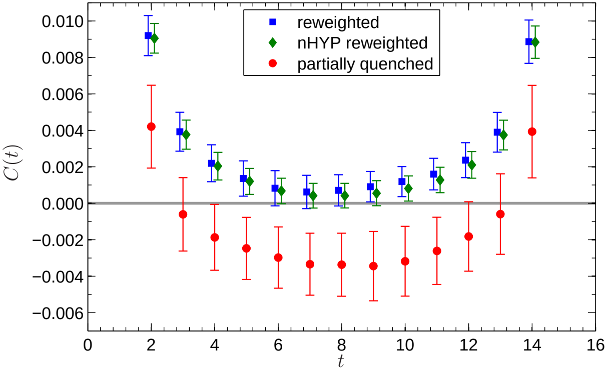
- This is promising

HHS results

performed complete simulation with reweighting from the p-regime to the ϵ-regime
nHYP action, $a \approx 0.12\,\mathrm{fm}$, $L = 1.9 \dots 2.8\,\mathrm{fm}$



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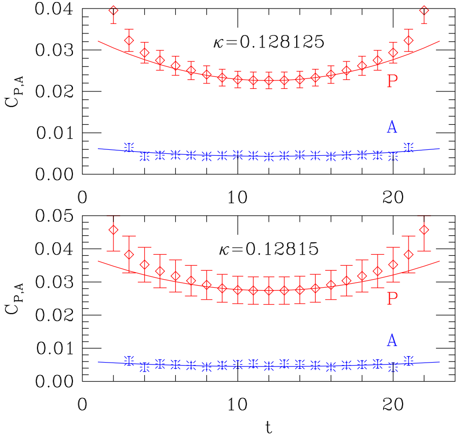

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