Overlap

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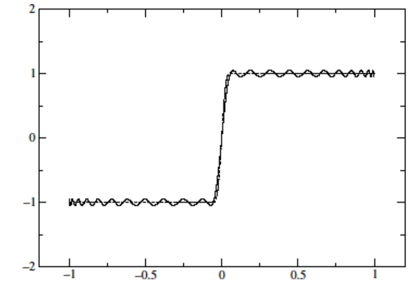
Overlap fermion

- Overlap operator
  - Satisfies the GW relation
  - Discontinuous at zero
  - Special treatment necessary (e.g. reflection/refraction)
  - Or, fix the global topology, for instance, by introducing

$$D = \frac{1}{a}\left[1 + \frac{X}{\sqrt{X^\dagger X}}\right],\ X = aD_W - 1$$
$$= \frac{1}{a}\left[1 + \gamma_5 \,\mathrm{sgn}(aH_W)\right],\ aH_W = \gamma_5(aD_W - 1)$$

$$\det\left[\frac{H_W(-m_0)^2}{H_W(-m_0)^2 + \mu^2}\right]$$

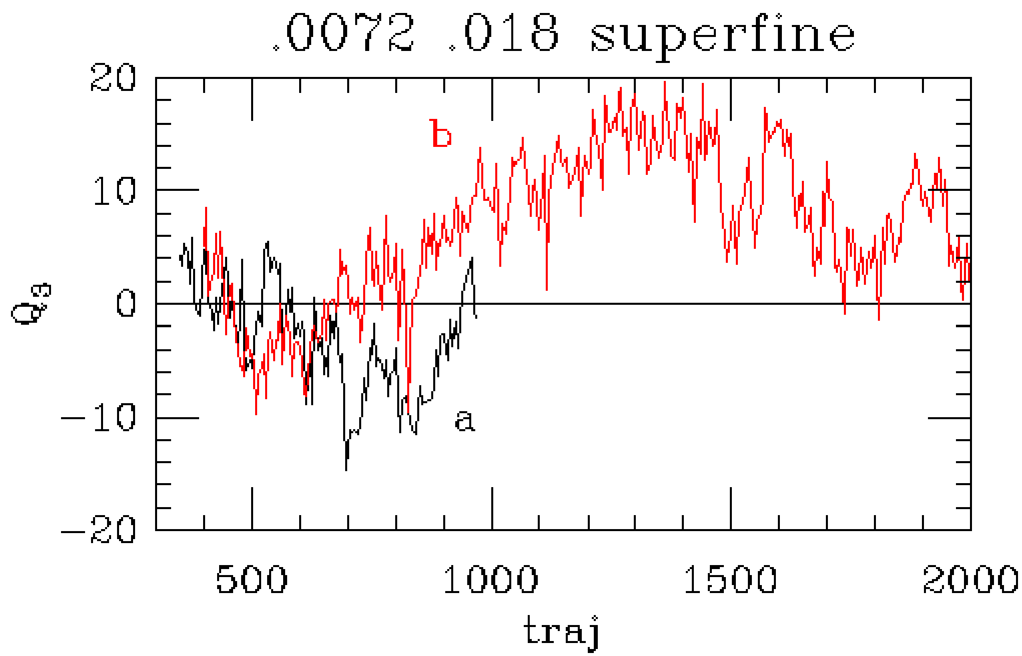
- Approximation
  - Rational approximation $\varepsilon[x] = x\left(p_0 + \sum_{l=1}^{N_{pole}} \frac{p_l}{x^2 + q_l}\right)$
  - Inversion requires inner-outer
    - Inner is multi-shift
    - May also use the 5D solver (see below)



Comments on topology

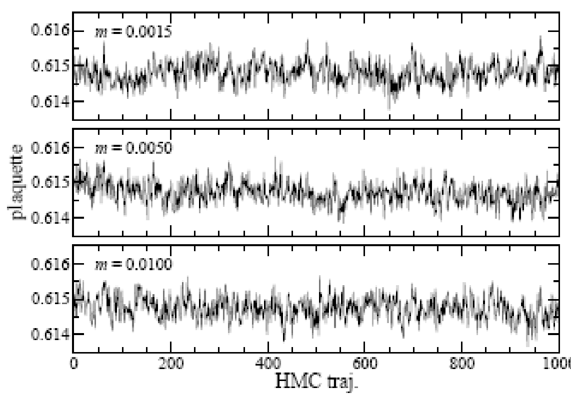
- Auto-correlation time for topological charge will be very large when *a* is small, say 0.05 fm.
  - As far as the evolution is continuous; property of the continuum (pure) gauge theory
  - Already some indication by RBC-UKQCD, MILC; will become more significant for smaller *a* and *m*.
  - Not feasible to accumulate, say 1,000 independent configs.
- Finite volume effect is expected
  - Typically scale as $1/(m\Sigma V)$.
  - At m=5MeV, $\Sigma=(250\text{ MeV})^3$, $V=(5\text{ fm})^4$, it is ~ 3%.
  - Even with some topology changes, this effect will remain.
  - Finite volume correction is necessary anyway; fixing the topology is a theoretically cleaner way (even for Wilson fermions).

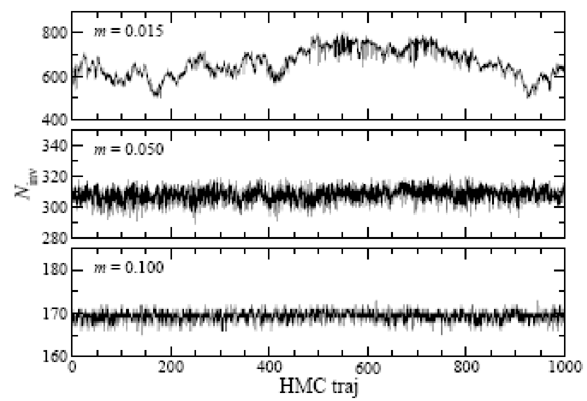
MILC superfine (a=0.06 fm), from Hetrick at Lattice 2007

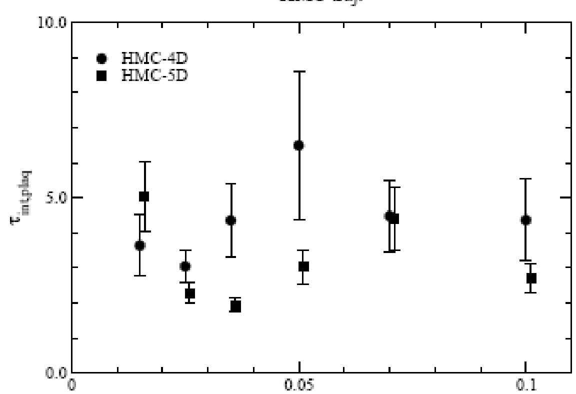


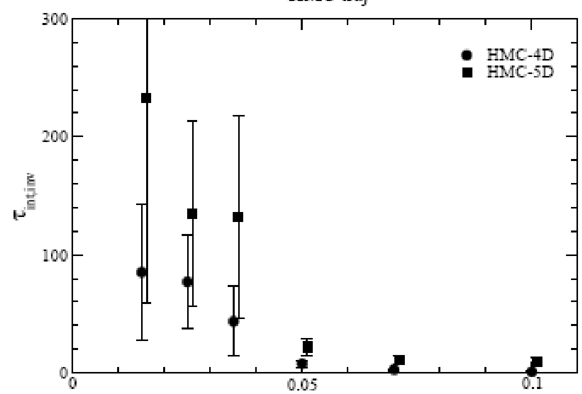
Comments on auto-correlation

- Not clear, but
  - Some indication of increase toward chiral limit, if N_{inv} is looked at. This should be sensitive to the lowest eigenvalue.
  - Plaquette should not be used for this measurement.









Should count another factor of $1/m$?

Time for overlap

- 5D solver
  - Overlap solver may be rewritten in a form of a 5D matrix inversion. x3-4 faster than the inner-outer solver.
  - (probably) no significant difference in the condition number and in the time for inversion, compared to DWF.
  - Thus, overlap is indeed quite similar to DWF, once the topology is fixed.
    - Still, some factor of x2-3 remains due to a technical problem of the 5D solver (two inversions necessary for $(D^\dagger D)^{-1}$).

(Borici, 2004, Edwards et al., 2006)

Schur decomposition

- One can solve $S\psi_4 = \chi_4$ by solving (example: N=2 case)

$$M_5 \begin{pmatrix} \phi \\ \psi_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \chi_4 \end{pmatrix}, \quad M_5 = \left(\begin{array}{cccc|c} H_W & -\sqrt{q_2} & & & 0 \\ -\sqrt{q_2} & -H_W & & & \sqrt{p_2} \\ & & H_W & -\sqrt{q_1} & 0 \\ & & -\sqrt{q_1} & -H_W & \sqrt{p_1} \\ \hline 0 & \sqrt{p_2} & 0 & \sqrt{p_1} & R\gamma_5 + p_0 H \end{array}\right) = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$$

$S = D - CA^{-1}B$: overlap operator (rational approx.)

- Even-odd preconditioning
- Low-mode projection of H_W in lower-right corner

Acceleration by solving $\left(1 - M_{ee}^{-1} M_{eo} M_{oo}^{-1} M_{oe}\right) x_e = b_e'$

- Need fast inversion of the "ee" and "oo" block; easy if there is no projection operator
- $M_{ee(oo)}^{-1}$ mixes in the 5th direction, while $M_{eo(oe)}$ is confined in the 4D blocks

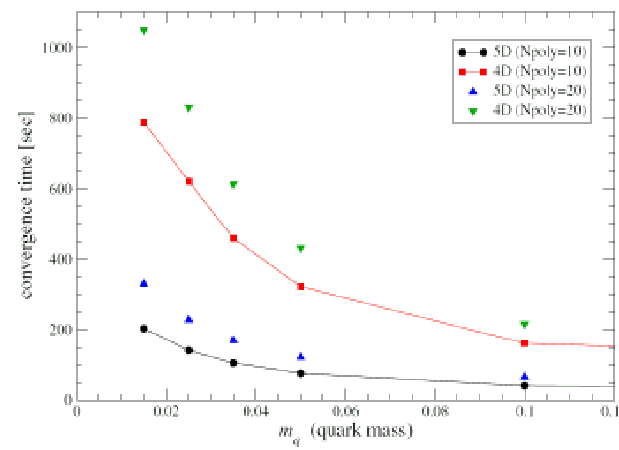
Low-mode projection

- Lower-right corner must be replaced by

$$R(1-P_H)\gamma_5(1-P_H) + p_0 H_W + \left(m_0 + \frac{m}{2}\right)\sum_{j=1}^{N_{ev}} \mathrm{sgn}(\lambda_j) v_j \otimes v_j^\dagger, \quad P_H = 1 - \sum_{j=1}^{N_{ev}} v_j \otimes v_j^\dagger$$

- Inversion of $M_{ee(oo)}$ becomes non-trivial, but can be calculated cheaply because the rank of the operator is only $2(N_{ev}+1)$.

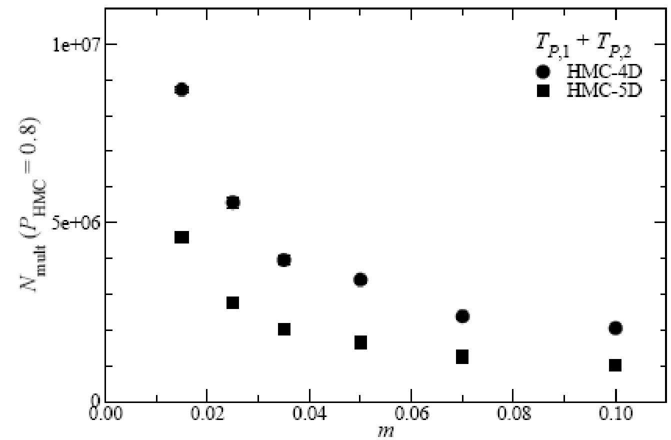
$$\{x_e, \gamma_5 x_e, \{v_{je}, \gamma_5 v_{je}\}\}$$



Scaling formula

$$C_{op} = k\left(\frac{20\,\mathrm{MeV}}{\overline{m}}\right)^{c_m}\left(\frac{L}{3\mathrm{fm}}\right)^{c_L}\left(\frac{0.1\mathrm{fm}}{a}\right)^{c_a}\ \mathrm{TFlops.yr}$$

- From a most recent data on a 24^3x48 run (N_f=2+1)
  - c_m=1, c_L=5, c_a=6 are assumed. Actual measurement on a 16^3x32 gave c_m=0.7 (up to auto-correlation).
  - 1,000 independent configs (separated by 10 traj assumed, maybe wrong)
  - k=23
    - Scale to (A) = 69 PFlops·year
    - Scale to (B) = 53 PFlops·year



- k=19, if c_m=0.7
  - Scale to (A) = 37 PFlops·year
  - Scale to (B) = 42 PFlops·year