All-to-all

Shoji Hashimoto (KEK)

all-to-all propagator

- Quark propagator from any y to any x

$$D^{-1}(x,y) = \sum_{k=1}^{N_{ev}} \frac{1}{\lambda^{(k)}} u^{(k)}(x) u^{(k)\dagger}(y) + \sum_{d=1}^{N_d} \left[D_{high}^{-1} \eta^{(d)} \right](x) \eta^{(d)}(y)$$

Low mode contribution

High mode propagation
From the random noice

Random noise

- One may specially treat the low-mode contributions that can be calculated exactly.

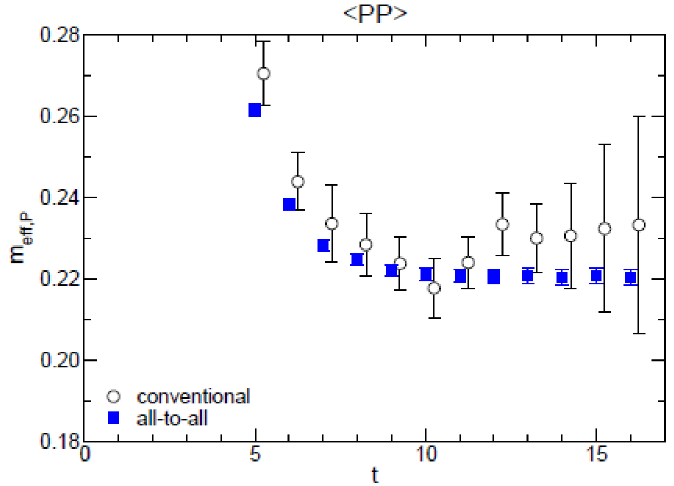
Use of all-to-all propagators

- Flavor singlet
  - η' mass, decays, nucleon structure, strange quark content, pion scalar form factor, …
  - Absolutely necessary
- n-point functions
  - three-point functions calculable with the (sequential) source method, but it requires more inversions depending on momentum configs; can be avoided by all-to-all.
- Statistics improvement
  - By averaging over source points; useful even for two-point functions.

Better statistics

- By averaging over the source points
  - Much better signal
  - Low-mode averaging (only the low-lying modes averaged) works equally well for pions.
  - Another benefit from stored low-modes: can be used to accelerate inversions (*exact* deflation)
  - Needs enormous storage:
    - 8x2x3x3x4xL^3T bytes for a gauge config
    - 8x2x3x4xL^3TxN_{ev} bytes for low-modes

pion effective mass

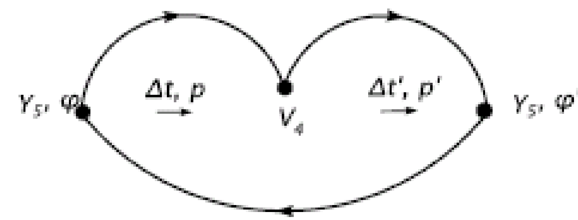


$$D_m^{-1}(x,y) = \sum_{k=1}^{N} \frac{u_k(x)u_k^\dagger(y)}{\lambda_k + m} + D_m^{(h)-1}(x,y)$$

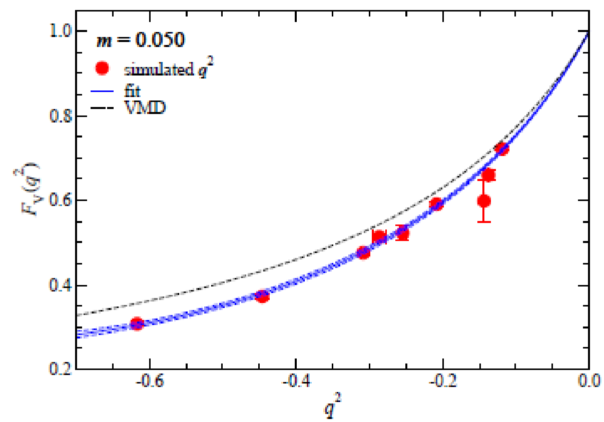
$$C(x,y) = C^{ll}(x,y) + C^{hh}(x,y) + C^{hl}(x,y) + C^{lh}(x,y)$$

n-point functions

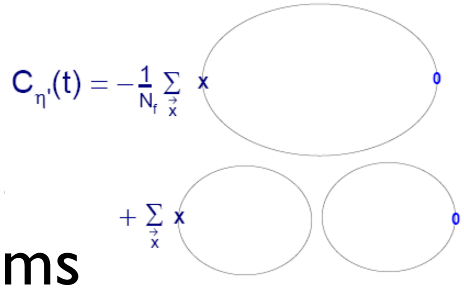
- Many momentum configurations can be calculated at once with all-to-all
  - Example: pion form factor on a 16^3x32 lattice
  - Will be more beneficial on large lattices
  - Needs further storage:
    - $8 \times 2 \times 3 \times 4 \times L^3 T \times N_d$ bytes for all-to-all
    - Typically $N_d = 3 \times 4 \times T$
    - On a 32^3x64 lattice, 400MBxN_d = 300GB per config, 300TB per 1,000 indep configs.



pion form factor



Flavor singlet



- Disconnected diagrams
  - η' is a typical example.
  - All-to-all essential.
  - Some success; works best when dominated by lowmodes.

$$C^{(\mathrm{disc})}_{\gamma_5\gamma_5,\phi,\phi'}(t) = (D^{-1})_{\mathrm{low}}(D^{-1})_{\mathrm{low}} + (D^{-1})_{\mathrm{low}}(D^{-1})_{\mathrm{high}} + (D^{-1})_{\mathrm{high}}(D^{-1})_{\mathrm{low}} + (D^{-1})_{\mathrm{high}}(D^{-1})_{\mathrm{high}}$$

  - Statistics remains the problem.
    - No further improvement expected by simply adding more noises. Larger volume + more independent configs will help.

Const: due to fixed topology
Fall-off: η' mass