$K \rightarrow \pi\pi$ decays and Exaflops

HEP Exascale Workshop

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RBC and UKQCD Collaborations

Outline

- Physics background
- Operator renormalization
- ~~Chiral perturbation theory~~
- Techniques for $\pi - \pi$ final states
- Error estimates

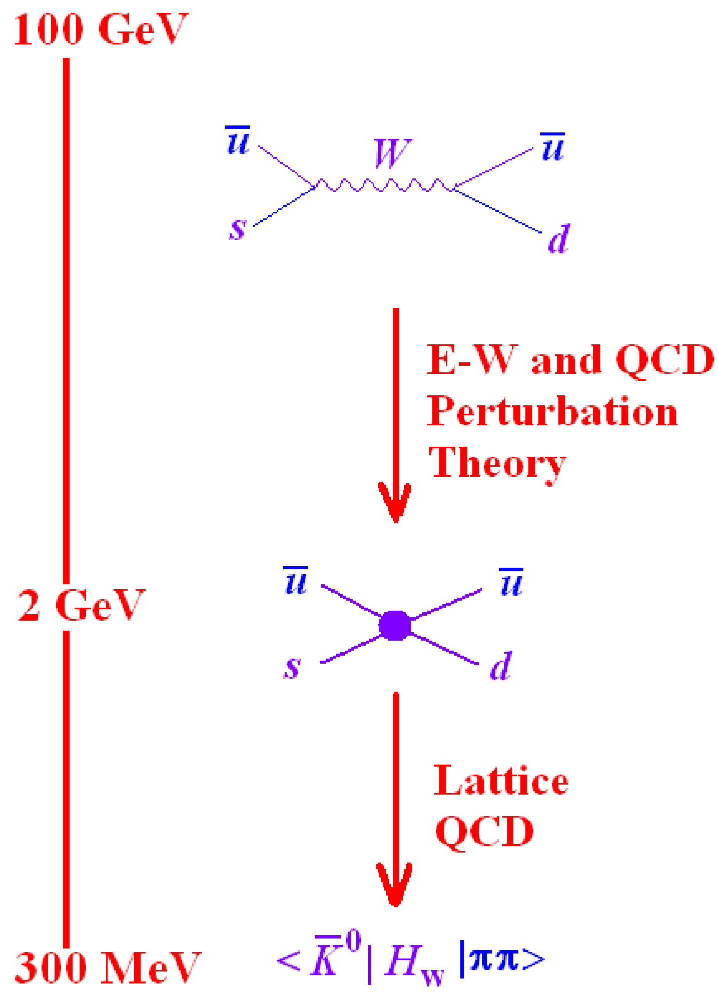
Physics Background

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{ts}^*} \frac{V_{us}^*}{V_{ud}} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Four quark operators

- **Current-current operators**

$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$

$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**

$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$

$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**

$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$

$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Status

- Long-standing problem in particle physics.
- Especially important given the connection with CP violation:

  $$\varepsilon'/\varepsilon = 16.5\ (2.6) \times 10^{-4}$$

- Natural target for lattice QCD.
- Even 10-20% errors would be of great value.

Challenges

- Match lattice and continuum operator normalizations.
- Difficult $\pi - \pi$ final state
  - Physical decay: $p \sim 205$ MeV
  - Euclidean, large time limit: $p \sim 0$ MeV
- Eye diagrams contain quadratic divergences
- $\Delta I = 1/2$ amplitudes require disconnected graphs

Chiral Perturbation Theory

- Use soft-pion techniques to relate difficult $K \rightarrow \pi\,\pi$ to much easier $K \rightarrow \pi$ and $K \rightarrow |0>$.
- 2000 RBC quenched calculation suggested this might be possible with eventual 20-30% errors?
- Recent RBC/UKQCD, 2+1 flavor calculation with lighter quarks shows that this approach does not work:
  - $\Delta I = 3/2$ amplitudes are poorly described by NLO ChPT.
  - $\Delta I = 1/2$ amplitudes involve too many LEC's to even test NLO ChPT.
- Must use two pion final states.

Computational Strategy

Operator Renormalization

- RI/MOM scheme, gauge-fixed off-shell Green's functions.
- Earlier quenched and recent 2+1 flavor calculation demonstrate errors ~few % errors are feasible.
- Sub-percent statistical errors possible from 5-10 configurations (Dirk Broemmel, Southampton)
- Non-exceptional kinematics gives sub-percent infrared effects at $\mu = 1.7$ GeV.
- Largest uncertainty comes from $\mu = 2$ GeV QCD perturbation theory. Remove by step-scaling
  - Compare RI/MOM Green's functions on a sequence of ensembles with small volumes, $L \sim 1/2^N$
  - Match with continuum perturbation theory at $\mu = 1.7 \cdot 2^N$ GeV
  - Error ~1/N

Operator Renormalization (con't)

- Seven $\Delta S = 1$ operators divide into three groups which mix:
  - $O_{(27,1)}$
  - O_7 and O_8
  - O_2, O_3, O_5, O_6
- Accurately handled by RI/MOM (Chris Dawson, Shu Li)
- Mixing with lower dimension operators are a small effect and easily treated.
- Effects of a single gluonic operator not yet considered.

Operator Renormalization (con't)

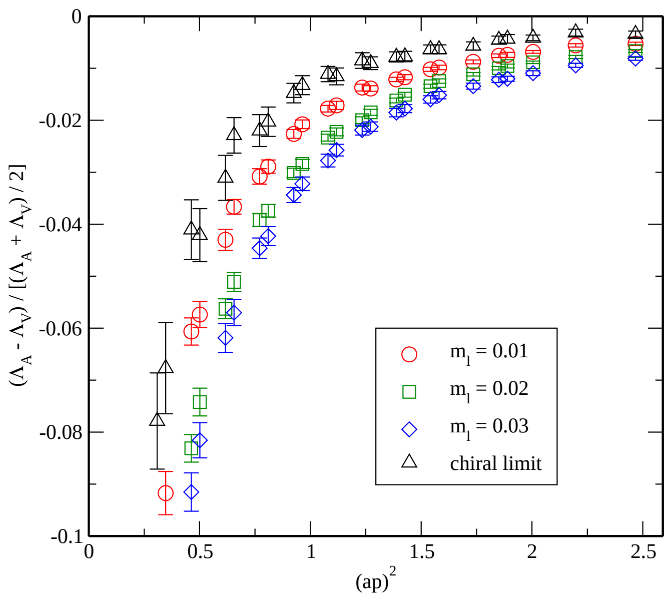
	1	2	3	5	6	7	8
1	1.218(33)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0.0033(53)	-0.0063(33)
2	0.0(0.0)	1.062(84)	0.076(77)	0.001(33)	0.016(29)	0.0026(80)	0.0026(68)
3	0.0(0.0)	0.13(20)	1.30(27)	-0.180(89)	0.120(99)	0.044(22)	-0.037(26)
5	0.0(0.0)	-0.08(24)	-0.03(21)	1.00(12)	0.269(93)	-0.016(23)	-0.034(24)
6	0.0(0.0)	-0.64(72)	-0.31(92)	-0.67(37)	1.97(38)	0.130(93)	-0.14(10)
7	-0.00030(89)	0.006(20)	0.024(25)	-0.0012(75)	-0.0074(92)	1.084(26)	0.294(29)
8	0.0002(14)	0.052(55)	0.138(76)	0.007(29)	-0.010(21)	0.060(22)	1.711(97)

[Shu Li]

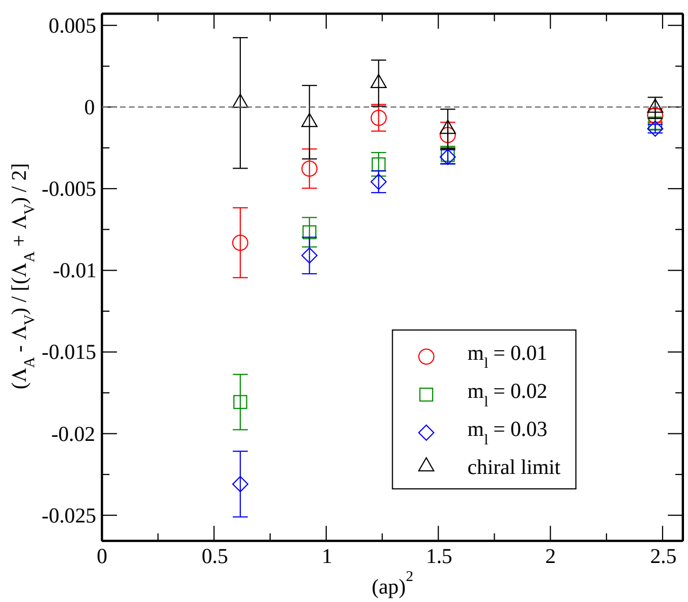
Inverse of renormalization matrix in 7 operator basis for unitary mass 0.01 and μ = 1.92 GeV. Done with 75 configurations, 16^3 x 32 1/a=1.73 GeV

Operator Renormalization (con't)

$(\Lambda_V - \Lambda_A)/(\Lambda_V + \Lambda_A/2)$



Exceptional momenta



Nonexceptional momenta

Y. Aoki, et al

Phys.Rev.D78:054510,2008,

arXiv:0712.1061 [hep-lat]

$\Delta I = 3/2$ Amplitudes

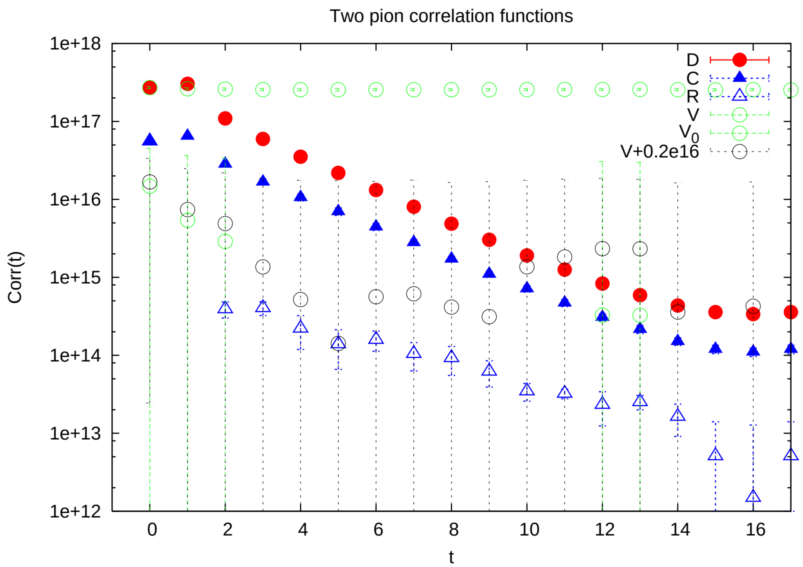
- Use K^0 at rest.
- Impose twisted boundary conditions on valence u quark
- Apply Lellouch-Luscher finite volume correction.
- Rely on Sachrajda and Villadoro to show all other finite volume effects ~ e^{-mL}. (More general proof needed.)
- Otherwise standard, 3-point measurement.
- Changhoan Kim's PhD thesis.
- Should be known on the 5 - 10% level at the petascale.
- 1% accuracy with exascale computation?

$\Delta I = 1/2$ Amplitudes

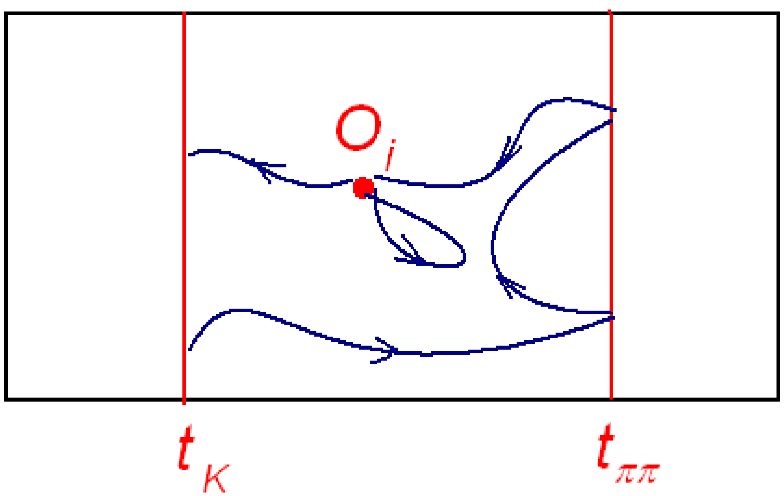
- Much harder than $\Delta I = 3/2$ amplitudes.
- How do we accurately evaluate disconnected graphs?
- How do we extract $p = 206$ MeV $\pi - \pi$ state?

Disconnected graphs

- Current testing:
  - 2+1 flavors
  - 16^3 x 32
  - $m_\pi = 310$ MeV



[Qi Liu]



- π - π correlator
- 32 wall sources, one for each t.
- 6 configurations
- 12 hrs/config. at 1/8 Tflops.

π - π States with p = 206 MeV

- Tune wave functions and perform three-state fits.
- Luscher-Wolf:
  - Use hermitian N x N correlation matrix
  - $3 \leq t \leq 5$?
  - Extract lowest three states: $|0>$, $|\pi\pi(p=0)>$, and $|\pi\pi(p=206\ \mathrm{MeV})>$.
- Use G-parity boundary conditions
  - New dynamical configurations
  - 2 x volume
  - Completely removes $|\pi\pi(p=0)>$ from signal and noise

ΔI = 1/2 Experiments at the Terascale

- First RBC/UKQCD experiments
- 32^4, $1/a$=1.35 GeV lattice
- $L_s = 32$
- Auxilary determinant
- m_{res} = 0.0018/a ~ 3 MeV
- L = 4.5 fm
- m_π = 180 MeV
- Target: 20% accuracy

$\Delta I = 1/2$ at the Exascale

- 128^4, a=0.05 fm lattice
- 128 sources per configuration.
- Luscher-Wolf and G parity
- Cost scaled from 1st experiments:

  5 Pflops days per configuration.
- Accuracy
  - 1% renormalization.
  - 1% scaling (64^4, 1/a=0.1 done earlier).
  - ? % statistics

Conclusion

- Accurate (1%-10%) $K \rightarrow \pi\pi$ calculation possible with exascale resources (100 Pflops sustained).