Domain Wall Fermions
Computational Requirements

HEP Exascale Workshop

December 09, 2008

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RBC and UKQCD Collaborations

Scaling DWF to Exaflops

- Present status:
  - 2+1 Flavor
  - $m_\pi = 280$ MeV
  - $L = 2.7$ fm, $a = 0.09$ fm
  - 32^3 x 64, 2-5 Tflops,
  - BG/P, 1-2K racks (Argonne)
- Exaflops target:
  - 2+1+1 Flavor
  - $m_\pi = 130$ MeV
  - $L = 6.0$ fm, $a = 0.05$ fm
  - 128^3 x 256 ,10-100 Pflops,

Hasenbusch Preconditioning

- Define $\mathcal{D}(m) \;=\; D_{\mathrm{DWF}}(m)^{\dagger} D_{\mathrm{DWF}}(m)$

- Hasenbusch method:

$$\det\Big(\mathcal{D}(m_l)\Big) \;=\; \det\left(\frac{\mathcal{D}(m_l)}{\mathcal{D}(m_H)}\right)\det\Big(\mathcal{D}(m_H)\Big)$$

- Our application (set $m_H = m_s$):

$$\frac{\det\Big(\mathcal{D}(m_l)\Big)\left[\det\Big(\mathcal{D}(m_s)\Big)\right]^{1/2}}{\left[\det\Big(\mathcal{D}(1)\Big)\right]^{3}} \;=\; \det\left(\frac{\mathcal{D}(m_l)}{\mathcal{D}(m_s)}\right)\det\left[\frac{(\mathcal{D}(m_s))}{(\mathcal{D}(1))}\right]^{1/3}\det\left[\frac{(\mathcal{D}(m_s))}{(\mathcal{D}(1))}\right]^{1/3}\det\left[\frac{(\mathcal{D}(m_s))}{(\mathcal{D}(1))}\right]^{1/3}$$

Expected Scaling Behavior

Space-time volume

Finite volume accept-reject

Critical slowing down

For $m_h \sim m_s$ scales as $<\bar{q}q> a^3$

$$\mathbf{Cost} = C_2 \cdot L^4 \cdot L \cdot \frac{1}{m_\pi} \cdot$$

$$\left\{ \frac{1}{m_h} \int_{m_h}^{\lambda_{\max}} \frac{\lambda}{\lambda^2 + {m_h}^2} \rho(\lambda) d\lambda + \frac{1}{m_l} \int_0^{m_h} \frac{\lambda}{\lambda^2 + m_l^2} \rho(\lambda) d\lambda \right\}$$

Hasenbusch CG

Light quark CG

HMC step size

HMC step size

Summary

$$\mathbf{Cost} = \left(\frac{L}{\mathbf{fm}}\right)^5 L_s \left(\frac{\mathrm{MeV}}{m_\pi}\right)\left(\frac{\mathbf{fm}}{a}\right)^6 \left\{C_0 + C_1\left(\frac{\mathrm{MeV}}{m_K}\right)^2\left(\frac{\mathbf{fm}}{a}\right) + C_2\left(\frac{\mathrm{MeV}}{m_\pi}\right)^2\left(\frac{a}{\mathbf{fm}}\right)^2\right\}\frac{\mathrm{Tflops}\cdot\mathrm{day}}{\mathrm{ind.\ conf.}}$$

$C_0 = 2.8\ 10^{-10}$ $C_1 = 3.3\ 10^{-5}$ $C_2 = 3.8\ 10^{-3}$

- Assume auto-correlation time of 40 time units for $m_\pi = 320$ MeV and $a = 0.114$.
- Describes RBC/UKQCD $a = 0.114$ and 0.086 fm, $L = 2.7$ fm, $m_\pi \geq 280$ MeV behavior.
- Caveat: Under estimates $L = 4.1$ fm, $m_\pi = 210$ MeV by 2x.

DWF Ensembles

a **(fm)**	m_l/m_s	m_π **(MeV)**	**Size**	L_s	***L*** **(fm)**	**MD time units**	**TF-Yr**	
0.114	0.186	310	24^3 x 64	16	2.7	7,000	0.6	QCDOC
0.086	0.15	275	32^3 x 64	16	2.7	13,000	3.0	BG/P
0.114	0.039	136	64^3 x 128	16	7.4	20,000	610	BG/Q
0.061	0.039	136	100^3 x 200	16	6.0	38,000	4,200	Exascale
0.02	0.15	260	100^3 x 200	16	2.0	58,000	13,500	Exascale
0.050	0.039	136	128^3 x 256	16	6.4	45,000	18,000	Exascale

Conclusion

- Exascale resources will allow simultaneous study:
  - Chiral fermions
  - Small lattice spacing: $a = 0.05$ fm
  - Large volume: $L = 6$ fm
  - Physical light quarks: $m_\pi = 135$ MeV
- May be adequate for dynamical charm quarks?
- Bottom quarks will still require heavy quark methods.