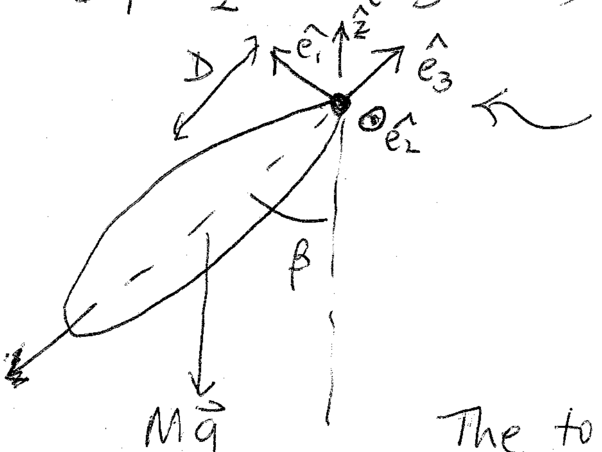
$\boxed{s_\beta = \sin\beta \quad c_\beta = \cos\beta}$

Problem 1b, alternative solution.

The simplest approach is to use $\dot{\vec{L}} = \vec{\Gamma}$ as in the original solutions.

Many students attempted to use Euler's equations, in particular

$$I_1 \dot{\omega}_2 - (I_3 - I_1)\,\omega_3 \omega_1 = \tau_2$$



Need to be careful to specify axes. These $\hat{e}_i$ are body-fixed (as needed for Eulers eqs.) at the instant shown.

The torque is $\tau_2 = MgD\, s_\beta$ at this instant.

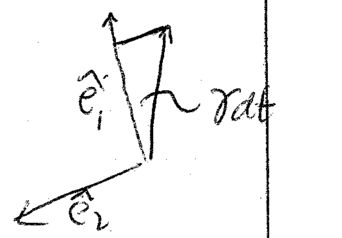
At this instant $\vec{\omega} = \dot{\gamma}\,\hat{e}_3 + \dot{\alpha}\,\hat{z}$

$$= (\dot{\gamma} + c_\beta \dot{\alpha})\,\hat{e}_3 + s_\beta \dot{\alpha}\,\hat{e}_1$$

so $\omega_3 = \dot{\gamma} + c_\beta \dot{\alpha}$ $\quad \omega_1 = s_\beta \dot{\alpha}$ $\quad \omega_2 = 0$

Those following this path then all said $\dot{\omega}_2 = 0$, arguing that $\vec{\omega}$ remains in the $\hat{e}_1, \hat{e}_3$ plane. This is wrong - the body-fixed axes rotate.

Thus a short time later, dt,



$$\vec{\omega} = \dot\gamma\, \hat{e}_3 + \dot\alpha\, \hat{z}$$

$$= (\dot\gamma + c_\beta \dot\alpha)\, \hat{e}_3 + s_\beta \dot\alpha\, \hat{e}_1 \cos(\dot\gamma dt) \underbrace{- s_\beta \dot\alpha\, \hat{e}_2 \sin(\dot\gamma dt)}$$

$$\Rightarrow \dot\omega_2 = - s_\beta \dot\alpha \dot\gamma$$

Thus Euler gives

$$- I_1 \cancel{s_\beta} \dot\alpha \dot\gamma - (I_3 - I_1)(\dot\gamma + c_\beta \dot\alpha)(\cancel{s_\beta} \dot\alpha) = MgD \cancel{s_\beta}$$

or

$$\boxed{- I_3 \dot\gamma \dot\alpha + (I_1 - I_3)\, c_\beta\, \dot\alpha^2 = MgD}$$

which is the desired equation.

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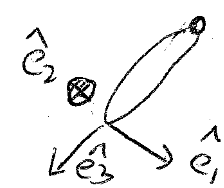
In part (a), several students used the Euler angle formulae given in the sheet, but did not get the angles correctly. To match to the standard definition, one must substitute $\beta \to \pi - \beta$ & $\gamma \to -\gamma$. Also, at the instant shown $\gamma = 0$. Finally, this is for axes



In this bases $\omega_1 = - \dot\alpha\, s_\beta$ $\quad \omega_2 = 0$

$$\omega_3 = - \dot\alpha\, c_\beta - \dot\gamma$$

This agrees with the original solutions (since the bases vectors here are opposite)