Basics of Magnetic Reconnection

Cory D. Schillaci

University of Washington

(Dated: March 18, 2009)

I. INTRODUCTION

Magnetic reconnection is an exciting phenomenon in plasma physics. See Figure 1 for a depiction of a reconnection event. Breaking and reconnecting the field lines in a plasma allows the release of huge bursts of energy. On a solar scale, reconnection is thought to be responsible for solar flares and coronal mass ejections. The aurorae are also generally considered to be related to reconnection events in the Earth's magnetosphere. Tokamaks and other laboratory setups with plasmas display irrefutable evidence of reconnection. While its effects are seen both near and far from home, reconnection is still poorly understood, making it one of the most important problems in plasma physics today. In the last 20 years, computational advances have allowed simulation of three dimensional reconnection events. This was a great advance over two dimensional theories, which fail to capture many of the subtleties present in real reconnection processes. Plasma physics is an often controversial field, possibly due to its complexity, and debates over mechanisms for reconnection are far from settled. Classical reconnection theories require the poorly understood assumption of anomalous resistivity in order to achieve physically reasonable time scales, but a new theory based on decoupling of the electron and ion flows gives very promising results.

This paper will develop the basic concept of reconnection. Starting from the concept of flux tubes, field lines will be given a mathematical treatment. A discussion of the problem of defining reconnection in three dimensions, which makes use of the previous concepts,

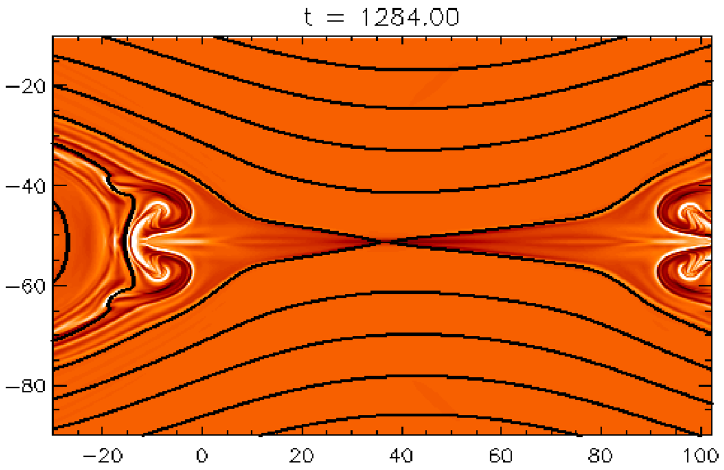

FIG. 1: Field line reconnection in two dimensions. This event is of the collisionless type described in Section IV. Taken from `http://terpconnect.umd.edu/~drake/`.

follows. Finally, a brief discussion of some classical models and the relatively new whistler wave regime is given.

II. FLUX TUBES AND FIELD LINES

The idea of magnetic reconnection seems intuitive. As the name implies, during dynamical evolution of a plasma the magnetic landscape of a plasma sometimes changes, causing field lines to “break” and “reconnect”. Such qualitative definitions are certainly useful, but plasmas are highly complex systems. A more specific definition is therefore an essential tool in order to understand this very specific process.

First, the idea of a magnetic field line should be made more rigorous. When field lines are introduced in typical introductory physics classes, they are given no formal definition. Usually, students are taught that these lines “follow the field” or, in the case of electric fields, “follow the path a charged particle would take.” Field line density is said to be proportional to field strength. This model is troublesome, as the choice of where to begin lines and how many to draw is by no means unique. With care, this basic notion of a field lines is of pedagogical and illustrative use but little mathematical utility.

Unfortunately, no completely unique representation is possible. Nevertheless, a good mathematical definition may be based on the concept of flux tubes [1]. We can define flux tubes as follows: take any open surface $S(t)$ and its closed boundary curve $c(t)$, then sweep the boundary curve along a path parallel to the magnetic field lines (See Figure 2). In the limit of infinitesimal diameter, these flux tubes are the physical equivalent of field lines.

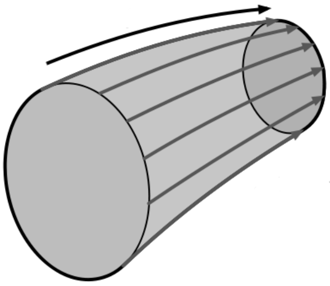

FIG. 2: An example of the surface of a flux tube.The grey lines are the magnetic field, one end of the tube is closed, and the boundary on the right is being swept along the field. Adapted from `http://commons.wikimedia.org/wiki/File:Flux_tube.svg`.

This definition will also help to prove that these flux tubes are “frozen in” for ideal plasmas. Equivalently, the magnetic flux $\phi = \int_{S(t)} \mathbf{B} \cdot d\mathbf{A}$, over the surface $S(t)$ which moves with the plasma, is a conserved quantity of the system. Let $\mathbf{v}(\mathbf{x}, t)$ be the plasma fluid velocity. Recall the familiar form of Ohm’s law,

$$\sigma \mathbf{F} = \mathbf{J}. \tag{1}$$

$\mathbf{F}$ represents the force per unit charge. In an ideal plasma, when the density is low enough that collision rates are negligible, the conductivity σ is infinite. As the only forces are electromagnetic, Eq. 1 tells us that

$$\mathbf{F} = \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0. \tag{2}$$

Combining this with Faraday’s Law,

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \tag{3}$$

we get an expression resembling a continuity equation for the magnetic field:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{4}$$

Consider any open surface $S(t)$ bounded by the closed curve $c(t)$, where the time dependence indicates that the surface moves with the plasma. Application of Stoke’s Theorem to the integral of Eq. 4 over $S(t)$ gives:

$$\int_{S(t)} \partial_t \mathbf{B} \cdot d\mathbf{A} = \oint_{c(t)} (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = -\oint_{c(t)} \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{l}), \tag{5}$$

where the last equality follows from the triple product rule, $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$. Taking the derivative of the flux with respect to time, the product rule gives

$$\frac{d\phi}{dt} = \int_{S(t)} \partial_t \mathbf{B} \cdot d\mathbf{A} + \int_{dS(t)} \mathbf{B} \cdot d\mathbf{A}. \tag{6}$$

The last term describes the flux added by the change of the surface with respect to time. A geometrical argument (see [2] for a nice sketch) shows that the last term is equivalent to

$$\oint_{c(t)} \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{l}) dt = \int_{dS(t)} \mathbf{B} \cdot d\mathbf{A}. \tag{7}$$

Putting Eqs. 5, 6 and the time derivative of Eq. 7 together we finally show that

$$\frac{d\phi}{dt} = -\oint_{c(t)} \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{l}) + \oint_{c(t)} \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{l}) = 0. \tag{8}$$

This is, perhaps, an odd equation to derive in a paper on magnetic reconnection. After all, we are concerned with how magnetic field lines change, and the previous equation shows that such processes cannot happen. However, in order to arrive at this result we assumed the validity of Ohm's Law (Eq. 2). For typical plasmas, Eq. 2 is valid in most of the domain, where the nonideal effects are of negligible magnitude. However, in some regions the introduction of a nonideal term is necessary. Density may be unusually high, or other forces may lead to resistive effects. In this region the nonideal Ohm's Law,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{R} \tag{9}$$

applies, where the resistance term $\mathbf{R}$ stands for a variety of possible nonideal effects in the plasma. In order for reconnection to occur, $\mathbf{R}$ must be non-zero in some region. It is not sufficient for $\mathbf{R}$ to be merely non-zero, however. There are also various forms of $\mathbf{R}$ which can be shown not to allow reconnection. It can be shown that

$$\mathbf{B} \times (\nabla \times \mathbf{R}) = 0. \tag{10}$$

is a necessary and sufficient condition on $\mathbf{R}$ for reconnection not to occur [3].

Infinitesimal flux tubes are not the same as field lines. Mathematically, they correspond to objects of infinitesimal (but non-zero) volume. Field lines, however, are one dimensional. There exists a mathematical formulation of these objects as well. Priest and Forbes [4] define the field lines as the intersection of two flux tubes.

All of the forms given are useful in certain contexts. Conceptually, we can use the idea that two points lie on a field line to derive the "frozen in" condition of the ideal plasma, another statement that reconnection cannot occur. Consider two points, P_1 and P_2 lying very close together on the same field line. Using vector identities on Eq. 2 gives,

$$\partial_t \mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{B} - \mathbf{B}(\nabla \cdot \mathbf{v}). \tag{11}$$

Introducing also the mass continuity equation of the plasma,

$$\partial_t \rho + \mathbf{v} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{v}, \tag{12}$$

in order to eliminate the $\nabla \cdot \mathbf{v}$ term in Eq. 11 gives

$$\partial_t \mathbf{B} + (\mathbf{v} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} + \frac{\mathbf{B}}{\rho}(\partial_t \rho + \mathbf{v} \cdot \nabla \rho). \tag{13}$$

Defining the convective derivate $d/dt = \partial_t + \mathbf{v} \cdot \nabla$, this becomes

$$\frac{d}{dt}\mathbf{B} - \frac{\mathbf{B}}{\rho}\left(\frac{d}{dt}\rho\right) = \rho\left(\frac{1}{\rho}\frac{d}{dt}\mathbf{B} - \frac{\mathbf{B}}{\rho^2}\left(\frac{d}{dt}\rho\right)\right) = \rho\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho}\right) = \rho\left(\frac{\mathbf{B}}{\rho}\cdot\nabla\right)\mathbf{v} \tag{14}$$

or, since ρ is arbitrary,

$$\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho}\right) = \left(\frac{\mathbf{B}}{\rho}\cdot\nabla\right)\mathbf{v}. \tag{15}$$

Now compare this with the movement of the line segment $\mathbf{l}$ connecting P_1 and P_2. Let $\mathbf{v}$ and $\mathbf{v} + \delta\mathbf{v}$ be the plasma velocity at points P_1 and P_2 respectively. Then $\delta\mathbf{v} = (\mathbf{l} \cdot \nabla)\mathbf{v}$. So during an infinitesimal time step dt we have

$$\frac{d\mathbf{l}}{dt} = \delta\mathbf{v} = (\mathbf{l}\cdot\nabla)\mathbf{v}. \tag{16}$$

Because ρ is a scalar, the similarity of Eqs. 15 and 16 implies that if $\mathbf{B}$ and $\mathbf{l}$ are initially parallel (necessary since P_1 and P_2 are on the same field line) then they remain parallel for all time. Thus the two points always remain on the same field line, so the field lines are frozen in to the plasma.

III. DEFINING RECONNECTION

Now that various definitions of field lines and the related concept of flux tubes have been discussed, some attempt can be made to define reconnection. Traditionally, reconnection was approached using two dimensional approximations due to computational and analytical simplicity. Defining the phenomenon is also much easier in two dimensions, as there are strong requirements on the field in the vicinity of reconnection processes. When moving to three dimensions, however, definition becomes a more complicated task. A simple definition of reconnection was given by Vasyliunas [5] in 1975: "Magnetic reconnection...is the process whereby plasma flows across a surface separating regions of topologically different field lines". This definition leaves a lot a of open questions, particularly regarding the meaning of topological distinctions, but captures the flavor. Priest and Forbes [4] give a more precise definition of reconnection in two dimensions with four specific criteria. These definitions rely on the concept of separatrices between topologically distinct regions however, and these seperatrices are not resilient against extension into a third dimension (see [1]). Schindler et al ([6]) prefer the more general definition that reconnection is simply a breakdown of the ideal frozen-in field condition described previously. This definition is very intuitive but also includes some processes that are not generally considered reconnective. More restrictive definitions are forced to refer to specific topological features, primarily X-lines (the three dimensional equivalent of the 2-D seperatrix). One striking definition is given in covariant language using four-vectors [7], but this again depends on the presence of an X line feature.

It seems that a single, precise definition is unavailable. While keeping in mind the different requirements given by each of the many possible definitions, the physicist is ultimately left with a slightly refined version of "know it when you see it." However, most of the questionable cases can be dealt with individually as the need arises.

IV. CLASSICAL VS. COLLISIONLESS RECONNECTION

Classical reconnection theory relies on magnetohydrodynamics (MHD) to model plasma dynamics. MHD is a single fluid theory, meaning that it describes a plasma which is electrically neutral (i.e. containing a macroscopically equal density of positive and negative charge carriers). It is applicable only in this macroscopic limit. Reconnection has been described by MHD theory for most of its history. The classical models, however, struggle to provide energy release of the correct magnitude on the right time scales.

The first serious mechanism was developed by Sweet and Parker. This model considers reconnection between two regions with antiparallel magnetic fields and plasma velocities that force the field lines inward towards the connection region (See Figure 3). The reconnection rate is estimated from an analysis of flow rates at the boundaries of a rectangular region with length L to be

$$V_R = \frac{V_A}{\sqrt{S}}, \tag{17}$$

where the Lundquist number S is defined as

$$S = \frac{LV_A}{\eta c/4\pi}. \tag{18}$$

V_A is the inflow velocity which is equal to the Alfvén speed and $\eta = 1/(\mu\sigma)$ is the magnetic diffusivity. This rate is far too small to explain observed phenomena on solar scales, when the length L is typically large and so is S. Petschek proposed a model which requires an external field and shock formation, but greatly reduces the effective length scale of the process. The Petschek reconnection rate, given by

$$V_R < \frac{V_A}{\ln S} \tag{19}$$

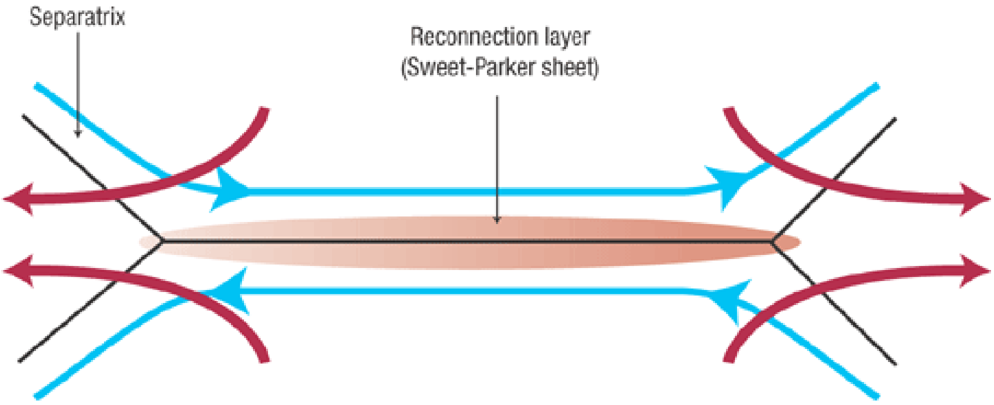

FIG. 3: The Sweet-Parker mechanism. Red lines represent the plasma flow, while the blue lines are the magnetic field. Taken from [9].

is much greater, but the conditions necessary for this rate to be valid seem to be unrealistic. Simulations by Biskamp have given the Sweet-Parker rate rather than the Petscheck rate [8]. The Petschek rate is very controversial, and experiments suggest that it is applicable only when resistivity is non-constant. Reasons for this are not known. Many scientists consider the Sweet-Parker rate to be the maximum allowed by MHD.

Use of anomalous resistivity models also allows reasonable reconnection rates to be achieved. The reasoning is that in the reconnection region particles are undergoing more turbulence, collisions, etc. and resistivity may increase there. A good theoretical understanding of this is not available, although there is some experimental evidence that it may be a real effect.

While MHD provides the framework for the classical models described above, it may break down on the small scales at which reconnection occurs. Ion and electron motions decouple in this regime, with the electron flow rate becoming much greater than the ion rate and driving so-called whistler waves (see Figure 4). Whistler waves are dispersive; their name comes from the sound made on World War II radios as the high-frequency waves passed first, followed by lower frequencies (whistler waves can be generated naturally by lightning, among other things). This electron driven mechanism gives a much faster reconnection rate. Whistler driven reconnection creates a strong out-of-plane magnetic field, which has been observed in Earth's magnetosphere [10].

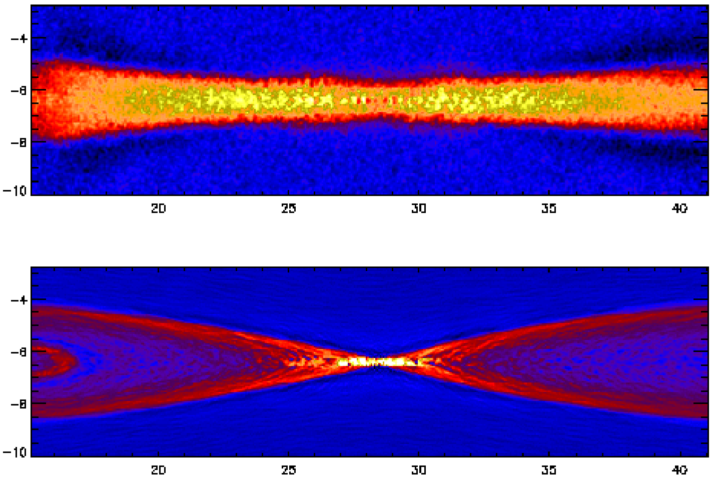

FIG. 4: Ion (top) and electron (bottom) currents during whistler driven collisionless reconnection. The decoupling is clearly strong in this region. Taken from `http://terpconnect.umd.edu/~drake/`.

V. CONCLUSION

As a theory, magnetic reconnection is still undergoing revolutionary changes. Because plasma systems are so complicated, computer simulations in three dimensions are only now coming into their own. But these are driving new understanding, proving revolutionary ideas and changing the way physicists think about reconnection. Whistler driven reconnection is very promising, and is quite probably a very good description of the process for certain conditions. There is still much to be learned however, and certainly more experimental study of reconnection in natural processes is necessary.

---

[1] D. Biskamp, *Magnetic Reconnection in Plasmas* (Cambridge University Press, 2000).

[2] R. J. Goldstone and P. H. Rutherford, *Introduction to Plasma Physics* (CRC Press, 1995).

[3] M. Hesse and K. Schindler, J. Geophys. Res. **93**, 5559 (1988).

[4] E. Priest and T. Forbes, *Magnetic Reconnection: MHD Theory and Applications* (Cambridge University Press, 2000).

[5] V. M. Vasyliunas, Rev. Geophys. **13**, 303 (1975).

[6] K. Schindler, M. Hesse, and J. Birn, J. Geophys. Res. **93**, 5547 (1988).

[7] G. Hornig and L. Rastatter, Physica Scripta **T74**, 34 (1998).

[8] D. Biskamp, Phys. Fluids **29** (1986).

[9] J. M. Finn, Nature Physics **2**, 445 (2006).

[10] X. H. Deng and H. Matsumoto, Nature (2001).