PHYSICS 321

Spring 2009

PROBLEM SET 8

No problems will be graded, but are assigned to give you some experience in problems on dielectric materials.

1. Griffiths Problem 4.15

2. Griffiths Problem 4.18

3. This problem came up in my research. I want to show you that an electric field can be used to align a system of slabs like those in Fig 4.24, something one might want to do to make a practical device.

Consider the two slabs of Fig 4.24. Let the dielectric constants of the two slabs be κ_1 and κ_2. I have a parallel plate capacitor with plates in the x,y plane of size L_x by L_y. The plates are separated by a distance d. I have a whole bunch of the sandwiches shown in Fig 4.24, and want to know if the capacitance is greater if I put the slabs so that they are parallel to the plates (so that each set of slabs is in series with the others) or put them in so that they are standing up side by side in the y,z plane. In this case, the capacitors are in parallel. The capacitors are connected to a battery of constant voltage V. The larger capacitance situation will have the lower energy $-CV^2/2$.

a) Slabs oriented parallel to plates. Area of slabs in $L_x \times L_y$. Show that the capacitance of the slab of Fig 4.24 in this orientation is

$$C = \frac{L_x L_y \epsilon_0}{a} \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}$$

To fill the capacitor I need $d/2a$ such sandwiches. Show that the total capacitance is

$$C_{series} = \frac{L_x L_y \epsilon_0}{d} \frac{2\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}$$

b) Now consider the sandwich of Fig. 4.24 standing upright in the yz plane so that it has a length d in the z direction and length L_y in the y direction. Show that the capacitance of the sandwich in this orientation is

$$C = \frac{L_y a \epsilon_0}{d} (\kappa_1 + \kappa_2).$$

To fill my capacitor I need $L_x/2a$ such sandwiches. Show that the capacitance is now

$$C_{parallel} = \frac{L_x L_y \epsilon_0}{d} \frac{\kappa_1 + \kappa_2}{2}$$

It is now simple to show that $C_{parallel} > C_{series}$. Thus if the slabs were originally in the xy plane, application of an electric field in the z direction would make them stand up to lower their energy.

4. Consider a parallel plate capacitor with plates of area A, and distance between plates d. Let the length of the plates be L. A dielectric, of dielectric constant κ is partially inserted into it (see Fig. 4.30) so that the length not occupied by dielectric is x.

(a) Show that the capacitance is

$$C(x) = C(L)\left(1 + (\kappa - 1)\left[1 - \frac{x}{L}\right]\right),$$

where $C(L) = \epsilon_0 A/d$.

(b) Now consider inserting the dielectric into a capacitor in which the charge $Q = Q_0 = C(L)V_0$ is held fixed. As your book notes in Eq. 4.64, the force on the dielectric is

$$F_Q(x) = \frac{1}{2}V^2\frac{dC(x)}{dx} = \frac{1}{2}\frac{Q_0^2}{C^2(x)}\frac{dC(x)}{dx}.$$

If the voltage on the plates is held fixed at the initial value $V = V_0$ above, then the force is given by Eq. 4.67

$$F_V(x) = \frac{1}{2}V_0^2\frac{dC(x)}{dx} = \frac{1}{2}\frac{Q_0^2}{C^2(L)}\frac{dC(x)}{dx}.$$

Note that, contrary to your book's protestations, these are *different* functions of x.

Plot $F_Q(x)$ and $F_V(x)$ (in units of $Q_0^2/LC(L)$) as functions of x/L for a value of $\kappa = 3$.