PROBLEM SET 7: DUE WEDNESDAY MAY 27

1.† Consider the insulated, hollow, spherical shell of radius R. Given azimuthal symmetry, the potential could be written

$$\begin{aligned} V(r,\theta) &= \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \qquad r \leq R, \\ &= \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \qquad r \geq R, \\ B_l &= A_l R^{2l+1} \end{aligned}$$

Suppose that you are given the charge per unit area on the sphere, $\sigma(\theta)$.

a) Show that the coefficients A_l are given by

$$A_l = \frac{1}{2\epsilon_0 R^{l-1}} \int_0^{\pi} \sigma(\theta) P_l(\cos\theta) \sin\theta \; d\theta.$$

b) Check this for the trivial case $\sigma(\theta) = Q/4\pi R^2$.

c) Determine the potential everywhere if $\sigma(\theta) = 3\epsilon_0 V_{MAX} \cos\theta / R$.

2.* Suppose the potential on the surface of the sphere is

$$V(R,\theta) = V_{MAX} \cos^2\theta,$$

determine the potential everywhere. (Hint: express $\cos^2\theta$ in terms of the Legendre polynomials.)

3.* Griffiths 3.22