Physics 321: Problem Set 5 (due Wednesday May 11)

1.* Use the relaxattion method to calculate the potential within a square with sides of length $L = 1$ ($0 \leq x, y \leq 1$). The potentials on the boundaries are fixed at $V(0, y) = V(x, 0) = 0$, and $V(1, y) = V(x, 1) = 240$. Set the potential $V(0, 1) = V(1, 0) = 120$. Use an 12 by 12 grid of points with the first and last row and column fixed by the boundary conditions. Stop the iterations after the root mean square change in potential from one iteration to the next is less than 0.001, that is (note that there are 100 values that change during the calculation)

$$\left(\frac{1}{100} \sum_{x,y} [V_{new}(x, y) - V_{old}(x, y)]^2 \right)^{1/2} \leq 0.001$$

2. Griffiths 2.36

3.* The ordinary planar capacitor has a capacitance $C = \epsilon_0 A/d$ or a capacitance per unit area of $\mathcal{C} \equiv C/A = \epsilon_0/d$. Calculate the capacitance per unit length of coaxial cylinders of radius R_{inner} and R_{outer} which carry equal and outer charge per unit length of magnitude λ. Define the distance $d \equiv R_{outer} - R_{inner}$. For an axon (a cable which carries nerve impulses in the body), d is much smaller than either R_{inner} or R_{outer}. Show that in this case the capacitance per unit area, $\mathcal{C}$ of cable is given to a good approximation by the planar result ϵ_0/d. Evaluate this (in F/m^2) for a reasonable $d = 1$nm$=1 \times 10^{-9}$m.

4. Griffiths 2.48

5.* In class I said that the induced charge density on the inner surface of the grounded sphere with the charge q within it was

$$\begin{aligned} \sigma(\theta) &= -\frac{q}{2\pi a^2} f(\theta, d/a), \\ f(\theta, \epsilon) &\equiv \frac{1}{2} \frac{1 - \epsilon^2}{[1 - 2\epsilon \cos\theta + \epsilon^2]^{3/2}} \end{aligned}$$

a) Show that the total charge induced on the sphere is equalt to $-q$ as expected.

b) Plot the function $f(\theta, \epsilon)$ from $0 \leq \theta \leq \pi$ for three values of ϵ: 0.1, 0.5, and 0.9.

6.* Suppose instead of a grounded sphere, the conducting sphere is not grounded, and is therefore electrically neutral. What is the potential now in all regions of space? (Hint: This problem is actually very easy. Because Laplace's equation is linear, you can always add two different solutions together to get a third solution in order to satisfy some boundary condition. In this case you want to ensure that the total charge on the sphere is zero.)