PHYSICS 321: PROBLEM SET 4 (DUE WEDNESDAY, APRIL 29)

FIRST HOUR EXAM FRIDAY MAY 1

1.† I found the section of the text 2.4.4 (a puzzling inconsistency) very interesting. Perhaps the following problem will help clarify it.

Consider a uniformly charged spherical shell with total charge $-Q$ of radius a. It is surrounded by a spherical shell (with the same center) of charge $+Q$ and radius $b > a$.

a) First let us consider the two shells as given, and not count the energy needed to create these shells. Let us only count the energy needed to put one shell in the presence of the other. So either i) use the potential that the outer shell sees due to the inner shell and multiply by the charge of the outer shell or ii) use the potential that the inner shell sees due to the outer shell and multiply by the charge of the inner shell. You will get the same result either way as you should. This energy will certainly be negative. In fact, show that this leads to

$$W_{assemble} = -\frac{Q^2}{4\pi\epsilon_0\ b}$$

b) Now calculate the energy, W_2, needed to create a shell of radius a and charge $(-Q)$ and the energy, W_3, needed to create a shell of radius b and charge Q. Let us call the sum of the two W_{create} as it is the energy needed to create the two shells in isolation from one another.

c) The energy needed to create the shells and then to assemble the system is

$$W_{tot} = W_2 + W_3 + W_{assemble} = W_{create} + W_{assemble} \quad (1)$$

What is this value?

d) Lastly calculate the energy of the system by using the expression

$$W = \frac{\epsilon_0}{2} \int E^2 d^3r \quad (2)$$

and show that this gives the energy W_{tot}. Hence Eq(1) gives you the energy needed to assemble the system of given charges, while Eq(3) gives you the total energy of the system, including that needed to create the charges themselves. If you are only interested in the energy to assemble the system (and not interested in the energy to create the shells themselves), you can use Eq(1) or you can subtract from Eq(3) the energy needed to create the charges in isolation; i.e.

$$W_{assemble} = W_{tot} - W_{create}$$

snd you can calculate both the terms on the right by using Eq(3) appropriately.

2* The energy of a sodium chloride crystal of N atoms can be calculated from

$$U = \frac{Ne^2}{4\pi\epsilon_0 a} S,$$

where a is the lattice constant, and $S = \lim\ S_p$ as $p \to \infty$ with

$$S_p = \frac{1}{2} \sum_{l=-p}^{p} \sum_{m=-p}^{p} \sum_{n=-p}^{p} \frac{f_l^p f_m^p f_n^p}{(l^2 + m^2 + n^2)^{1/2}}.$$

The term with $l = m = n = 0$ is excluded from the sum and $f_n^p = (-1)^n$ for all p. Write some computer program in whatever language you are familiar with and calculate S_p for $1 \leq p \leq 10$ and present your results as a graph of S_p vs. p. You will note that each sucessive cube has charge of magnitude e. You should find that your result oscillates and converges slowly.

3.* For comparison, repeat the above calculation but take each cube to be neutral. This can be done by setting

$$f_n^p = (-1)^n \frac{(2 - \delta_{n,p} - \delta_{n,-p})}{2},$$

with $\delta_{n,q} = 1$ if $n = q$, and zero otherwise. It is not difficult to see that this factor takes care of the partial charge if the charge is on the surface, ($n = \pm p$), of the cube we are looking at. There should be a difference in how fast this series converges when compared to that of problem 2. Again present your results as a graph of S_p vs. p.

4.* a) Given a dipole consisting of two equal and opposite charges of magnitude q a distance d apart in some region of space at which the potential is given by $V(\mathbf{r})$. Calculate the potential energy, U, of the two charges. Then assume that d is a small distance compared to r and expand your result about the point $\mathbf{r}$ and show that the potential energy can be written as $U = -\mathbf{p} \cdot \mathbf{E}(\mathbf{r})$.

b) Verify this result for the case of a dipole a distance r from a point charge Q, a problem we did in class.

5. Griffiths 2.32