PROBLEM SET 3 DUE WEDNESDAY APRIL 22

1. Griffiths 2.6

2.[†] Consider two circular capacitor plates of radius R a distance d apart, one with uniform charge per unit area σ is at $z = d/2$, the other with uniform charge per unit area $-\sigma$ is at $z = -d/2$. Find the electric field on the axis of the capacitor, $E_z(z)$, at an arbitrary value of z. What is the value of the field just outside of the upper plate, that is, as $z \to d/2$ from above? If the plates were infinite in extent, this value would be zero. You should find that the field is of order d/R for $d/R << 1$.

3.[*] Consider a sphere of radius a with a charge density $\rho(r) = A/r$ for $r \leq a$.

a) If the total charge on the sphere is Q, calculate the constant A.

b) Calculate the electric field inside and outside the sphere.

4.[*] In class we showed that the magnitude of the electric a distance s from an infinitely long wire with charge per unit length λ is $E(s) = \lambda/2\pi\epsilon_0 s$. From this, calulate the electric potential at a distance s from the axis. Take the zero of potential to be such that $V(s_0) = 0$.

5. [†] The potential of a line of charge. Consider a line of charge on the z axis from $-a/2$ to $a/2$ with a charge per unit length of λ. Calculate the potential $V(s, z)$ at the point (s, z) in cylindrical coordinates. One needs to be acreful with the integral $\int dy/(y^2 + s^2)^{1/2}$. The integrand is even in y so the resultant integral should be odd in y. Many tables of integrals are cavalier about this, and you can get into trouble if you use a result which is not odd. The correct integral is

$$\int \frac{dy}{\sqrt{y^2 + s^2}} = \frac{1}{2} \ln \left[\frac{\sqrt{y^2 + s^2} + y}{\sqrt{y^2 + s^2} - y} \right]$$

a) Verify that your result is finite for $s \to 0$ if $|z| > a/2$ as it should be. (It diverges as $s \to 0$ if $|z| < a/2$ as one expects.)

b) Let us also make another simple check. The general result is a bit simpler at $z = 0$. Verify that for $a/2s << 1$ (i.e. when you are far from the line), the potential in the $z = 0$ plane, $V(s, 0)$, reduces to that of a point charge of magnitude λa, i.e. $V(s, 0) \to (\lambda a/4\pi\epsilon_0 s)$.