Problem Set 2 Physics 321

Problems marked with an asterisk will be graded and are worth 10 points. Those marked with a dagger will be graded and are worth 20 points.

1.* Consider the vector field

$$\mathbf{v}(\mathbf{r}) = \frac{\hat{\mathbf{i}} \times \mathbf{r}}{\mathbf{y^2} + \mathbf{z^2}}.$$

Show that $\nabla \times \mathbf{v} = 0$ everywhere except on the x axis. Find its value there by use of Stokes' theorem to end up with

$$\nabla \times \mathbf{v} = 2\pi\delta(y)\delta(z)\hat{\mathbf{i}}$$

2.* Evaluate the following integrals:

$$a) \quad \int_{-3}^{1} (x^3 - 3x^2 + 2x - 1)\delta(x+2)dx,$$
$$b) \quad \int_{0}^{\infty} [\cos(3x) + 2]\delta(x - \pi)dx.$$

c) Use Eq. 1.94 to show that

$$\delta(x^2 - a^2) = \frac{1}{2|a|}[\delta(x-a) + \delta(x+a)]$$

3. Griffiths Problem 1.47

4.* Consider a line of charge extending from $x = -1,\ y = -1$ to $x = -1,\ y = 1$ with a charge per unit length λ. There is a point charge of magnitude -2λ at $x = 1,\ y = 0$.

a) The dipole moment of a collection of charges is

$$\mathbf{p} = \sum_i q_i \mathbf{r}_i,$$

which, for a continuos distribution, becomes

$$\mathbf{p} = \int \mathbf{r}\lambda(\mathbf{r})d\ell.$$

What is the dipole moment, **p**, of the given distribution?

b) Calculate the magnitude and direction of the electric field along the x axis for $x > 1$. Show that for large values of x that the field reduces to that of a dipole with the appropriate dipole moment.

5.* Griffiths 2.7 The whole point of this problem is to do the vector integral. Do **not** use Gauss' law which, other than demonstrating the power of Gauss' law, would render the problem a trivial one.