
(name: PLEASE PRINT)

Physics 321 Spring 2009

Final Exam

This is an open-book exam. You may use your notes, anything I have put on the web, your homework, and your text but no other book. If you have any question, raise your hand.

1. (35) Another, and simpler, pass at an ion channel. According to an article in SCIENCE, (280, 69 (1998)) the potassium channel has a “selectivity filter” which is like a hollow cylinder of radius R and length L , dielectric constant $\kappa = 1$. Assume that the walls of the cylinder have a uniform charge per unit area σ which is negative and attracts the K^+ ion.

a) (20) Let the axis of the cylinder be the z axis, and let the cylinder extend from $z' = -L/2$ to $z' = L/2$. What is the magnitude of the electric field on the z axis for arbitrary z ?

b) (5) If the K^+ ion is on the axis of the cylinder at $z = L/2$ what is the magnitude of the force pulling it into the cylinder?

Why isn't the ion trapped in the filter by this attraction? The article says that when a second ion enters the filter at $z = -L/2$, the attraction of the first ion for the filter is now perfectly balanced by the repulsion between the two ions, which are a distance L apart “...and this is what allows conduction to occur.” Assume that this is true. What I want to know is what is the magnitude of the charge density σ on the cylinder. In particular, as the area of the cylinder is $2\pi RL$,

c) (5) I want to know what is the value of

$$\frac{\sigma}{e/2\pi RL}$$

which is a pure number. It is the number of unit charges (or fraction of such charges) which are spread over the surface of the filter.

d) (5) Evaluate your result for the numbers they give for the size of the filter; $R = 1.5 \times 10^{-10}\text{m}$, $L = 7.5 \times 10^{-10}\text{m}$.

2. (30) Electrostatic clamps are used for holding workpieces while they are being machined, for holding silicon wafers during electron beam microfabrication, etc. They consist of a conducting plate maintained at a potential, V , of several thousand volts which is covered with a thin insulating sheet. The sheet is a linear dielectric of dielectric constant κ and of thickness d . The workpiece, or the wafer, rest on the sheet and is grounded. The workpiece and the conducting plate form a parallel plate capacitor with plates of area A .

a) (15) What is the total energy of the system? (Recall that the battery which holds the potential constant is part of the system.)

b) (8) What is the force per unit area, (i.e. the pressure), which holds the workpiece in place?

c) (7) One particular type operates at 3000 volts and exerts a pressure of 2 atmospheres, which is 2×10^5 Pascals, the Pascal being the MKS unit of pressure. If the insulator is Mylar, with dielectric constant $\kappa = 3.2$, what is the thickness d of the Mylar?

3. (30) A model of an axon: Consider a cylindrical tube of inner radius R_I and outer radius R_O . The insulating walls of the tube have a dielectric constant κ .
- a) (15) If the potential on the inner wall is V_I and that on the outer wall is V_O , what is the electric field in the region $R_I < s < R_O$?
- b) (15) Calculate the capacitance per unit area of this tube, and evaluate it (in Farads/m²) for a typical axon for which $R_O \approx R_I \gg R_O - R_I \approx 5 \times 10^{-10}$ m, and $\kappa = 2$.

4. (30) Why is water such a good solvent? Consider two non-conducting spheres, each of radius R , which carry charge q and $-q$ respectively. Assume that they are just touching. This is my model of a molecule.

a) (8) How much energy, W_{vac} , does it take to separate these charges in vacuum? (i.e separate them by an infinite distance?)

b) (8) Now consider the same two spheres in a dielectric medium of dielectric constant κ . How much energy, W_{wat} , does it take to separate these charges now?

c) (7) Evaluate your result in b) to one place accuracy for the case $q = e = 1.6 \times 10^{-19}$ C, $\kappa = 80$ (water), and $R = 1 \times 10^{-10}$ m, a typical atomic size. (Recall that $1/4\pi\epsilon_0 = 9 \times 10^9 \text{Nm}^2/\text{C}^2$.)

d) (7) At temperature T , the fraction of ionized molecules in water can be shown to be $f_{wat} = \exp(-W_{wat}/k_B T)$ while the fraction ionized in vacuum is $f_{vac} = \exp(-W_{vac}/k_B T)$. Show that $f_{vac} = (f_{wat})^\kappa$

(The fraction of NaCl which is ionized in water is measured to be $f_{wat} = 0.1$. Therefore the fraction ionized in vacuum would only be $(.1)^{80} = 10^{-80}$, a vanishingly small fraction. Thus it is the large dielectric constant of water which makes it such a good solvent.)

5. (35) Consider two, concentric, insulating shells of radius R_1 and $R_2 > R_1$. The potentials on the shells are

$$V(R_1, \theta) = V_0 \cos \theta,$$

$$V(R_2, \theta) = -V_0 \cos \theta.$$

- a) (10) Calculate the potential, $V(r, \theta)$, for $r \leq R_1$.
- b) (10) Calculate the potential, $V(r, \theta)$, for $r \geq R_2$
- c) (15) Write down the form of the potential for $R_2 \geq r \geq R_1$ and the equations which would enable you to calculate unknown coefficients. Do not solve the equations.

6. (40) Consider a hollow, square, conducting pipe of length a on each side. The corners are at $(a/2, a/2)$, $(a/2, -a/2)$, $(-a/2, a/2)$, and $(-a/2, -a/2)$. The top and bottom plates are both at potential $V = 1$, and the two plates on the side are each at potential zero. The potential within the pipe is clearly an even function of x and of y ; $V(x, y) = V(-x, y) = V(x, -y)$.
- a) (25) Write down the solution for the potential $V(x, y)$ in the form of an infinite series. You need not evaluate the coefficients in the expansion at this point.
- b) (15) Now evaluate the coefficients in the expansion.