SOLUTION TO EXAM 2 PHYSICS 321

1.a) Because the potential is even, the series solution will only contain $\cos m\theta$ so the general solution in the region $a \le s \le b$ can be written

$$V(s,\theta) = A_0 + B_0 \ln s + \sum_{m=1}^{\infty}(A_m r^m + \frac{B_m}{r^m})\frac{\cos m\theta}{\sqrt{\pi}}.$$

b)

$$\begin{aligned}
V(a,\theta) &= V_0 = A_0 + B_0 \ln a + \sum_{m=1}^{\infty}(A_m a^m + \frac{B_m}{a^m})\frac{\cos m\theta}{\sqrt{\pi}} \qquad \text{so that} && (1)\\
A_0 + B_0 \ln a &= \frac{1}{2\pi}\int_0^{2\pi} V_0 d\theta = V_0, && (2)\\
A_1 a + \frac{B_1}{a} &= \int_0^{2\pi} d\theta V_0 \frac{\cos\theta}{\sqrt{\pi}} = 0, && (3)\\
A_m a^m + \frac{B_m}{a^m} &= \int_0^{2\pi} d\theta V_0 \frac{\cos m\theta}{\sqrt{\pi}} = 0 \qquad m \ge 2, && (4)\\
V(b,\theta) &= V_0 + V_{MAX}\cos\theta = A_0 + B_0 \ln b + \sum_{m=1}^{\infty}(A_m b^m + \frac{B_m}{b^m})\frac{\cos m\theta}{\sqrt{\pi}} \qquad \text{so that} && (5)\\
A_0 + B_0 \ln b &= \frac{1}{2\pi}\int_0^{2\pi} d\theta[V_0 + V_{MAX}\cos\theta] = V_0, && (6)\\
A_1 b + \frac{B_1}{b} &= \int_0^{2\pi} d\theta[V_0 + V_{MAX}\cos\theta]\frac{\cos\theta}{\sqrt{\pi}} = V_{MAX}\sqrt{\pi}, && (7)\\
A_m b^m + \frac{B_m}{b^m} &= \int_0^{2\pi}[V_0 + V_{MAX}\cos\theta]\frac{\cos m\theta}{\sqrt{\pi}} = 0 \qquad m \ge 2. && (8)
\end{aligned}$$

c) From Eqs (2) and (6), $A_0 = V_0$, $B_0 = 0$, from Eqs. (3) and (7),

$$A_1 = \frac{V_{MAX}\sqrt{\pi}b}{b^2 - a^2} \qquad B_1 = -\frac{V_{MAX}\sqrt{\pi}ba^2}{(b^2 - a^2)}$$

From Eqs(4) and (8), $A_m = B_m = 0,\ m \ge 2$, so that

$$V(s,\theta) = V_0 + \frac{V_{MAX}ba}{b^2 - a^2}\left[\frac{r}{a} - \frac{a}{r}\right]\cos\theta \qquad a \le s \le b$$

2.a) The field from the left plate at the right plate is $\mathbf{E} = \sigma/2\epsilon_0\hat{\mathbf{i}}$ so that the force per unit area is $F/A = \sigma^2/2\epsilon_0$ to the right.

$$b) \qquad E_x = -\frac{\partial V(x)}{\partial x} = -\frac{\sigma}{\epsilon_0} e^{-d/2\lambda} \sinh(x/\lambda), \qquad |x| < d/2,$$
$$= \frac{\sigma}{\epsilon_0} \cosh(d/2\lambda) e^{-x/\lambda} \qquad x > d/2.$$

c) The force per unit area on the plate is $F/a = \sigma[E(x \to d/2+) + E(x \to d/2-)]/2$ so

$$F/A = \frac{\sigma\epsilon_0}{2}[\cosh(d/2\lambda)e^{-d/2\lambda} - e^{-d/2\lambda}\sinh(d/2\lambda)],$$
$$= \frac{\sigma}{2\epsilon_0} e^{-d/2\lambda}.$$

d) Apply Gauss' law to the slab to find that its electric field at $x = d/2$ is $E_x = -\sigma/\epsilon_0$, so it exerts a force per unit area to the left of magnitude σ^2/ϵ_0.

e) In this case the total force per unit area is

$$\frac{\mathbf{F}}{A} = \frac{\sigma^2}{2\epsilon_0}\hat{\mathbf{i}} - \frac{\sigma^2}{\epsilon_0}\hat{\mathbf{i}},$$
$$= -\frac{\sigma^2}{2\epsilon_0}\hat{\mathbf{i}},$$

so if the salts were distributed like this the plates would certainly attract one another.

3. Because the plate is conducting, the electric field just outside the plate is normal to it and is related to the charge density by

$$\sigma(x) = \epsilon_0 E_y(x, 0) = -\left(\frac{\partial V(x, y)}{\partial y}\right)_{y=0},$$
$$= -\frac{\epsilon_0 V_0 x}{L^2}.$$