
(name: PLEASE PRINT)

Physics 321 Spring 2009

Second Hour Exam

This is an open-book exam. You may use your notes, anything I have put on the web, your homework, and your text but no other book. If you have any question, raise your hand.

1.) (40) Consider two infinitely long coaxial cylinders. The inner one has radius a and the outer has radius b . Both are coaxial with the z axis. Let the potential on the inner and outer cylinders be even in θ (measured from the x axis) so that, in general $V(s, \theta) = V(s, -\theta)$.

a) (15) Write down the most general solution for the potential *in the region between the cylinders* ($a \leq s \leq b$) in the form of an infinite series.

b) (15) Now assume that $V(a, \theta) = V_0$ and that $V(b, \theta) = V_0 + V_{MAX} \cos \theta$. Impose these boundary conditions on your solution in a) and write down the resulting equations that would determine the unknown coefficients in the series.

c) (10) Solve these equations for the unknown coefficients.

2.) (45) In the 2007 exam, I considered two plates, each parallel to the yz plane, one at $x = d/2$, the other at $x = -d/2$ and each with a uniform charge density σ .

a) (5) Assume first that the plates are in vacuum. What is the magnitude of the force per unit area on the plate at $x = d/2$. Obviously the force is positive (to the right) as the plates repel.

In the exam I considered the plates to be immersed in a salt solution. In the Debye Hückel approximation, I found the potential to be

$$\begin{aligned} V(x) = V(-x) &= \frac{\sigma\lambda}{\epsilon_0} e^{-d/2\lambda} \cosh(x/\lambda) & |x| \leq d/2, \\ &= \frac{\sigma\lambda}{\epsilon_0} \cosh(d/2\lambda) e^{-|x|/\lambda} & |x| \geq d/2 \end{aligned}$$

b) (10) Calculate the electric field for all positive x .

c) (15) Calculate the force per unit area, F/A , exerted on the plate at $x = d/2$ by *all* charges and show that it is positive; that is, the plates repel one another. You will find that the repulsion is weaker than in a).

d) (10) Experimentally, the force between identically charged plates can even be *attractive*. To see how this might come about, assume that all the salt is concentrated between the plates in a uniform slab of thickness d and charge per unit area -2σ (so that the system is neutral). The force per unit area exerted by this slab on the plate at $x = d/2$ is to the left. What is its magnitude?

f) (5) What is the total force on the plate at $x = d/2$, (magnitude and direction)?

3.) (15) Consider two conducting plates, both parallel to the z axis and infinite in that direction. One consists of a piece of metal which lies on the positive x axis and is welded to a piece of metal which lies on the positive y . This plate is grounded. The other plate consists of the curve $xy = L^2$ for positive x . The potential on it is V_0 . The potential in this system is

$$V(x, y) = V_0 \frac{xy}{L^2}$$

as can be easily seen because it satisfies Laplace's equation and the boundary conditions. What is the charge density on the part of the conducting plate along the x axis?