NAME (Please Print)

PHYSICS 321

Autumn 2007

FINAL EXAM

This is an open book exam. You can use your text, (but no other book), your class notes, and problem set solutions. The value of each question is shown in parentheses. If you have any questions, raise your hand. In my opinion, questions 1,4, and 6, are the easiest, so you might as well do them first.

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Self-consistent field study of the alignment by an electric field of a cylindrical phase of block copolymer

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Self-consistent field theory is applied to a film of cylindrical-forming block copolymer subject to a surface field which tends to align the cylinders parallel to electrical plates, and to an external electric field tending to align them perpendicular to the plates. The Maxwell equations and self-consistent field equations are solved exactly, numerically, in real space. By comparing the free energies of different configurations, we show that for weak surface fields, the phase of cylinders parallel to the plates makes a direct transition to a phase in which the cylinders are aligned with the field throughout the sample. For stronger surface fields, there is an intermediate phase in which cylinders in the interior of the film, aligned with the field, terminate near the plates. For surface fields which favor the minority block, there is a boundary layer of hexagonal symmetry at the plates in which the monomers favored by the surface field occupy a larger area than they would if the cylinders extended to the surface. © *2006 American Institute of Physics.* [DOI: 10.1063/1.2214718]

1) (25) A simple warm-up problem: Consider two infinite plates both parallel to the x, z plane. One is at $y = a$ and has the free charge density σ_0 on it. The other is at $y = -a$ and is grounded ($V = 0$).

a) (5) What is the general solution of Laplace's equation in one dimension?

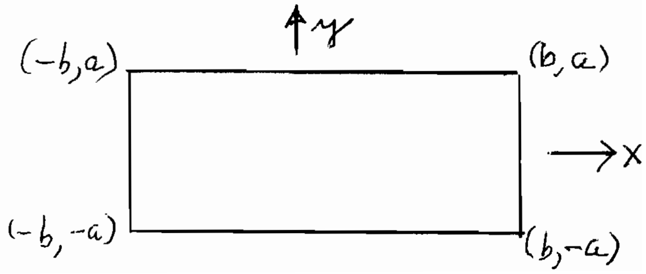
b) (10) For this particular problem, what is the solution of Laplace's equation for the potential, $V(y)$, everywhere between the plates? Please state what two conditions you use to determine coefficients in the general solution.

c) (10) What is the charge density induced on the lower plate, σ_{ind}?

2) (40) Now consider a pipe of rectangular cross-section. The coordinates of its edges are (b, a), $(b, -a)$, $(-b, a)$ and $(-b, -a)$. The bottom and sides of the box are grounded ($V = 0$). There is a free charge per unit area on the top of the box of $\sigma(x) = \sigma_0 \cos(\pi x/2b)$.

a) (25) Determine the potential $V(x, y)$ throughout the box.

b) (15) Determine the charge density, $\sigma_{ind}(x)$ which is induced on the bottom of the box. (Note that, as a check, your result for this problem should go to that of problem 1 in the limit $b \to \infty$.)



3) (40) An idea for an ion channel. Ion channels exist to get Na^+ ions or K^+ ions or Cl^- ions from one side of a membrane to another. To attract a (positive) charge to the top of the channel, (the entrance), I will line it with negative charge. To encourage the positive ion to go out the bottom (the exit), I will line it with negative charge. Let me idealize the situation as follows.

Consider a hollow sphere of radius R cut out of a medium of dielectric constant κ_w. Inside the hollow sphere, the dielectric constant is 1. I use amino acids to paste a free charge per unit are $\sigma(\theta) = -\sigma_0 \cos\theta$ on the sphere (with $\sigma_0 > 0$). The form of the potential is easily seen to be

$$\begin{aligned} V(r,\theta) &= \frac{B}{r^2}\cos\theta \qquad r \geq R \\ &= Ar\cos\theta \qquad r \leq R. \end{aligned}$$

a) (15) There are two unknowns, A and B. State two boundary conditions which will permit you to calculate these two constants.

b) (15) Calculate the two constants so as to determine the potential everywhere.

c) (10) Calculate E_z, the radial component of the electric field, E_r, for $\theta = 0$ everywhere.

4) (30) Origin of the van der Waals interaction. Given two neutral atoms, each of polarizability α. The energy of the system is zero. The system will always lower its energy as follows.

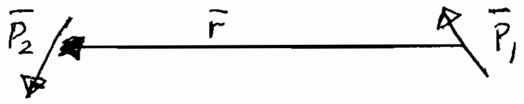
Suppose by some fluctuation, a dipole moment $\mathbf{p}_1$ appears on atom 1. This dipole moment produces an electric field $\mathbf{E}_D(\mathbf{r})$ at atom 2 which induces a dipole moment $\mathbf{p}_2$ at atom 2.

(a) (5) Given the field $\mathbf{E}_D(\mathbf{r})$, what is the energy of dipole $\mathbf{p}_2$?

(b) (7) What is the value of the dipole moment $\mathbf{p}_2$ in terms of the electric field $\mathbf{E}_D(\mathbf{r})$?

(c) (9) What is the electric field $\mathbf{E}_D(\mathbf{r})$ at atom 2 in terms of $\mathbf{p}_1$ and the displacement $\mathbf{r}$ of atom 2 with respect to atom 1?

(d) (9) Finally, express the energy of the system in terms of $\mathbf{r}$ and $\mathbf{p}_1$. The energy is negative, lower than that of the atoms without induced dipoles, and is stable therefore. This is the primary origin of the van der Waals interaction between neutral atoms.



5) (40) Almost Griffiths 4.28

Two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a tank of dielectric oil (dielectric constant κ, mass density ρ_m). The oil rises a height h in the space between the tubes.

a) (25) What is the capacitance of this system?

b) (15) The inner tube is maintained at potential V, and the outer one is grounded. Assume that there are no forces other than the electrostatic one pulling the oil upward and the gravitational one pulling it downward, and calculate the height h to which the oil rises in the space between the tubes. (Note: In general, there are other forces, like the van der Waals force, which tends to make the fluid rise. As capacitance can be measured with great precision, this kind of experiment is a common way of determining the magnitude of the forces that are acting on the fluid.)

6) (25) Almost Griffiths 4.40 Suppose a molecule has a permanent dipole moment of magnitude p. If we turn on an electric field E_0 in the z direction, the energy of the dipole is $u = -p_z E_z = -p\cos\theta E_0$ where θ is the angle between the direction of the dipole and the z axis. In contact with a thermal bath, the average magnitude of the dipole moment is

$$< p_z > = \frac{\int_0^\pi e^{-u/kT}(p\cos\theta)\sin\theta d\theta}{\int_0^\pi e^{-u/kT}\sin\theta d\theta}$$

$$= p\frac{\int_{-1}^{1} e^{ax} x\, dx}{\int_{-1}^{1} e^{ax}\, dx} = p\frac{d}{da}\ln\int_{-1}^{1} e^{ax} dx, \qquad a \equiv pE_0/kT$$

a) (7) Carry out the integral explicitly to obtain

$$< p_z > = p\left[\coth a - \frac{1}{a}\right]$$

b) (5) For normal fields and temperatures, $pE_0/kT << 1$. Use the Taylor series $\coth x = 1/x + x/3 + ..+$ to show that this contribution to the polarizability, α, of the molecule is, neglecting higher orders, $\alpha = p^2/3k_bT$.

c) (13) For a dilute gas, it is probably reasonable to assume that the effect of all other molecules on the polarization is negligible, in which case the dielectric constant is related to the polarizability by $\kappa - 1 \approx N\alpha/\epsilon_0$, where N is the number of molecules per unit volume. Check this for water vapor. Table 4.2 gives $\kappa - 1 = 6 \times 10^{-3}$ for water vapor at $T = 373K$. The dipole moment of the water molecule is $p = 6.1 \times 10^{-30}$, Boltzman's constant $k = 1.38 \times 10^{-23}$, $\epsilon_0 = 8.85 \times 10^{-12}$. Take the density, N, to be that of an ideal gas which is Avagadro's number per 22.4 liters or $N = 2.7 \times 10^{25} m^{-3}$. I find that the agreement is pretty good. (Note: I also tried to apply the Claussius-Mossotti relation to *liquid* water, and it is a complete disaster. I find that water should be a ferroelectric; i.e. $N\alpha/3\epsilon_0 > 1$!)