NAME (Please Print)

PHYSICS 321

Autumn 2007

SECOND EXAM

This is an open book exam. You can use your text, (but no other book), your class notes, and problem set solutions. The value of each question is shown in parentheses. If you have any questions, raise your hand.

1) (25) Consider a coaxial cable consisting of a cylinder of radius a and charge per unit length λ surrounded by another coaxial cylinder of radius $b > a$ carrying charge per unit length $-\lambda$.

a) (10) Calculate the magnitude of the electric field, $E(s)$, for $b > s > a$.

b) (15) Calculate the energy per unit length, u, of this system.

2. (35) Consider a pipe aligned along the z axis having a square cross-section. The corners have coordinates (a, a), $(a, -a)$, $(-a, a)$ and $(-a, -a)$. The potentials on the sides are $V(x, -a) = V(x, a) = 0$, and $V(a, y) = -V(-a, y) = 100$. Taking account of the obvious symmetry $V(x, y) = V(x, -y) = -V(-x, y)$, we can write an analytic solution of Laplace's equation in the form

$$V(x, y) = \sum_k G_k \frac{\sinh(kx)}{\sinh(ka)} \frac{\cos(ky)}{\sqrt{a}},$$

where the allowed values of k and the coefficients G_k are to be determined by the boundary conditions.

a) (13) Use the boundary conditions to determine the allowed values of k.

b) (22) Use the boundary conditions to determine the unknown coefficients G_k.

3. (40) Consider two identical non-conducting plates each with a charge per unit area of σ. Both are oriented parallel to the y, z plane with one located at $x = d/2$ and the other at $x = -d/2$. Were they in vacuum, the electric field between them would vanish, and would be of magnitude σ/ϵ_0 outside of them.

Now let the two plates be in a salt solution at temperature T. Within the Debye-Hückel approximation, the Poisson equation reads

$$\frac{d^2V(x)}{dx^2} = \frac{V(x)}{\lambda^2}$$

where λ is the Debye length.

a) (8) Write down the solution of the above equation for $x \geq d/2$. Impose the boundary condition that, as $x \to \infty$, $V(x) \to 0$.

b) (8) By symmetry the potential is an even function of x. So, write down the solution of the above equation for $|x| \leq d/2$.

c) (8) You should have two unknown amplitudes, one in the answer to a), the other in the answer to b). Impose the condition that the potential is continuous at $x = d/2$. This eliminates one unknown amplitude. Solve for the unknown coefficient in the solution of a) in terms of the unknown coefficient in the solution to b).

d) (12) Obtain the electric field from your potentials. Use the fact that the discontinuity in the electric field at $x = d/2$ is related to the charge density to obtain the one remaining unknown coefficient.

e) (4) Write your solution for the electric field for $0 \leq |x| < d/2$, and also for $x > d/2$.