NAME (Please Print)

PHYSICS 321

Autumn 2007

FIRST EXAM

This is an open book exam. You can use your text, (but no other book), your class notes, and problem set solutions. The value of each question is shown in parentheses. If you have any questions, raise your hand.

1. (25) Consider an infinitely long cylinder of radius R that carries a uniform charge per unit length λ. Calculate the potential difference $V(s) - V(R)$ between a point a distance $s > R$ from the axis and a point on the surface of the cylinder, $s = R$.

2. (25) Consider a spherical shell of radius R. Outside the shell, the potential is (in spherical coordinates)

$$V(\theta) = \frac{V_M R^2}{r^2} \cos\theta \qquad r \geq R,$$

while inside

$$V(\theta) = \frac{V_M r}{R} \cos\theta \qquad r \leq R.$$

a) (15) Calculate the electric field $\mathbf{E}$ both inside and outside, and express it in spherical components.

b) (10) What is the charge per unit area on the sphere, $\sigma(\theta, \phi)$?

3. (30) We often treat capacitors as if their plates were infinite in extent, in which case the electric field outside of them is zero. Let's examine a more realistic model of one.

(a) (23) First let us consider one plate. Show that the magnitude of the electric field $E(\ell)$, a distance ℓ on the axis of a circular disc of radius R which has a uniform charge density per unit area σ is

$$E(\ell) = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{\ell}{R(1+\ell^2/R^2)^{1/2}}\right].$$

(b) (7) Now let us make a capacitor out of two such plates. Consider a capacitor with two plates in the form of discs of radius R separated by a distance d as shown. The magnitude of the charge density on each plate is σ. Calculate the electric field just to the right (outside) of the right-hand (positively charged) plate on the axis. Again, this would be zero if the plates were infinite in extent.

4. (20) Consider a membrane, like a cell wall. The concentration of a given ion, like potassium, K^+, is different on one side of the membrane (call it the "inside") than on the other side (the "outside") As a result, the electric potentials there, V_{in} and V_{out} are different. The potassium ion has a charge q, and the cell is in contact with a thermal reservoir of absolute temperature T. Assume that the system is in equilibrium.

a) (15) What is the probability of finding a potassium ion on the inside, P_{in}, relative to finding it on the outside, P_{out}; i.e. what is P_{in}/P_{out} in terms of the potentials etc.?

b) (5) The relative concentrations of potassium inside and outside the membrane is equal to the relative probabilities of finding them there; i.e. $c_{in}/c_{out} = P_{in}/P_{out}$. From your result in a), express the potential difference $V_{in} - V_{out}$ in terms of the relative concentrations c_{in}/c_{out}. The result is called the Nernst equation.