Electrodynamics (PHY 514) : 2006
Assignment 9 :

This problem set is due **Thursday March 9**, at the end of the lecture. Feel free to discuss the problems with others in the class, but you must write your own solutions. Simply writing the answer without showing a derivation will obtain zero credit.

1. A spherical shell of radius a and with uniform surface charge density, σ, is rotating with angular velocity $\omega\ \hat{\mathbf{e}}_z$.
   (a) What is its magnetic moment?
   (b) Draw the magnetic field far from the sphere
   (c) Discuss the distribution of $\mathbf{E}\times\mathbf{B}$ and $\mathbf{E}\cdot\mathbf{B}$ far from the sphere.

2. A circular loop of wire carrying current I_0 is coaxial with an infinite cylinder of permeability μ. The radius of the wire loop is a, and the radius of the cylinder is b, with $b < a$.
   (a) Show that the vector potential due to the presence of the cylinder outside the cylinder is

$$\begin{aligned} A_\phi &= \frac{\mu_0 I a}{\pi}\int_0^\infty dk\ \Psi(k)\ K_1(ka)\ K_1(k\rho)\cos(kz) \\ \Psi(k) &= \frac{(\mu-\mu_0)\ k\ b\ I_0(kb)\ I_1(kb)}{(\mu-\mu_0)\ k\ b\ K_0(kb)\ I_1(kb)\ +\ \mu_0} \quad . \end{aligned} \tag{1}$$

   (b) Find the vector potential inside the cylinder.

3. The plane of a circular loop of wire of radius a, carrying current I_0, is parallel to and a distance b from an infinite slab of material with permeability μ and thickness t.
   (a) Show that the vector potential on the other side of the slab is

$$A_\phi = 2\mu I a\int_0^\infty dk\ \frac{\mu_0^2\ J_1(k\rho)\ J_1(ka)\ e^{-kz}}{(\mu+\mu_0)^2-(\mu-\mu_0)^2 e^{-2kt}} \quad . \tag{2}$$

(b) Find the vector potential in the other two regions.

4. A magnetic dipole $\mathbf{m}$ is embedded at the center of a sphere of radius a and permeability μ.

   (a) What is the magnetic scalar potential ϕ_M everywhere?

   (b) What is the magnetic field $\mathbf{B}$ everywhere?

   (c) What is the $\mathbf{H}$ field everywhere.

   (d) What is the magnetic dipole density $\mathbf{M}$ everywhere?

   (e) What is $\nabla \cdot \mathbf{M}$ everywhere?

   (f) What is $\nabla \times \mathbf{M}$ everywhere?