Electrodynamics (PHY 514) : 2006

Assignment 7 :

This problem set is due **Thursday February 23**, at the end of the lecture. Feel free to discuss the problems with others in the class, but you must write your own solutions. Simply writing the answer without showing a derivation will obtain zero credit.

1. A cylindrical shell of radius a is co-axial with the z-axis, and extends from $z = -\infty$ to $z = 0$. It is made of a non-conducting material, on which exists a uniform surface charge density σ. The cylinder rotates with angular frequency ω. What is the magnetic field on the z-axis?

2. A fine wire forms an infinite spiral-wound solenoid aligned along the z-axis. The solenoid is invariant under translations in the z-direction by an arbitrary number of displacements by the distance L. When viewed along the z-axis the system has radius R.

   (a) What is the magnetic field at the origin?

   (b) Compare the magnetic field in the limit $L \to \infty$ with that of a straight wire?

   (c) Compare the magnetic field in the limit $L \ll R$ with that of a solenoid.

3. Show that the magnitude of the magnetic field at the center of a loop of fine-wire shaped like a regular plane polygon with N sides, and carrying current I, is

$$|\mathbf{B}| \;=\; \frac{\mu_0 N I}{2\pi a} \sin\left(\frac{\pi}{N}\right) \quad ,$$

   where the distance between parallel sides is $2a$.

4. Consider a cylindrical shell of current flowing in the z-direction, but with angular dependence

$$\mathbf{j} \;=\; I_0 \frac{\delta(r-a)}{2\pi r} \left[\, 1 \;+\; C_1 \cos\theta \,\right] \quad .$$

(a) Find the vector potential everywhere inside the cylindrical shell.

(b) Find the magnetic field everywhere inside the cylindrical shell.

5. Two solid circular cylinders aligned along the z-axis, are located at $x = +c$ and $x = -c$ and each carry a uniformly distributed current $I/2$ in the $+z$-direction. A very large cylindrical shell surrounds these two cylinders and carries a uniformly distributed current I in the $-z$-direction. What is the force per unit length on the cylinder located at $x = +c$?

6. A section of a circular cylinder of length $2c$ and radius a is aligned along the z-axis and is bisected by the xy-plane. A uniformly distributed current I circulates around the cylindrical section. Show that the vector potential outside the cylinder for $\rho > a$ is

$$\mathbf{A} \;=\; \hat{\mathbf{e}}_\phi \, \frac{\mu_0 I a}{\pi c} \int_0^\infty d\log(k)\; I_1(ka)\; K_1(k\rho)\; \cos(kz)\; \sin(kc) \quad .$$