Electrodynamics (PHY 514) : 2006

Assignment 4 :

This problem set is due **Thursday January 26**, at the end of the lecture. Feel free to discuss the problems with others in the class, but you must write your own solutions. Simply writing the answer without showing a derivation will obtain zero credit.

1. Consider a system composed of two conducting spheres of radii a and b, with centers separated by a distance $d > a + b$. The sphere of radius a is at potential V_0, while the sphere of radius b is grounded.

   (a) Show that the charge on the sphere of radius a is

   $$q_a = 4\pi\epsilon_0\, a\, b\, V_0 \sinh(\beta) \sum_{n=1}^{\infty} \frac{1}{[b\sinh(n\beta) + a\sinh((n-1)\beta)]} \quad ,$$

   where

   $$\cosh(\beta) = \frac{d^2 - a^2 - b^2}{2ab} \quad .$$

   (b) Consider the situation in which spheres of radius $a = 2$ mm, and $b = 4$ mm are separated by a distance of $d = 1$ cm, and the sphere of radius a is at potential $V_0 = 10$ V. What is the charge on the sphere of radius a determined to 6 significant digits?

2. The walls of an earthed conducting rectangular cavity are located at $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $z = c$. A charge $+q$ is placed at the point x_0, y_0, z_0 inside the cavity. Show that the component of force on the charge in the z-direction is

   $$F_z = -\frac{2q^2}{\epsilon_0 ab} \sum_{n,m=1}^{\infty} \frac{\sinh(A_{mn}(c - 2z_0))}{\sinh(A_{mn}c)} \sin^2\left(\frac{n\pi x_0}{a}\right) \sin^2\left(\frac{m\pi y_0}{b}\right) \quad ,$$

   where $A_{mn} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$. What are the component of force in the x and y directions.

3. An earthed conducting sphere has its center on the axis of a ring of charge Q. Any radial vector $\mathbf{c}$ from the center of the sphere to the ring makes an angle α with the axis. The force on the ring along the axis is of the form

$$F \;=\; \sum_{l=0}^{\infty} g_l \; P_{l+1}(\cos\alpha) \; P_l(\cos\alpha) \quad .$$

Find g_l.

4. The portion of a sphere of radius a that lies between $\theta = \alpha$ and $\theta = \pi - \alpha$ carries uniform charge density σ. Find an expression for the potential at all points outside the sphere.

5. A system is composed of two concentric grounded conducting shells of radius a and c. A point charge is placed a distance b from the center of the spheres, where $a < b < c$. Show that for $a < r < b$, the potential at point r is

$$V(r,\theta) \;=\; \frac{q}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{b^{2n+1} - c^{2n+1}}{b^{n+1}\left[a^{2n+1} - c^{2n+1}\right]} \left(r^n - \frac{a^{2n+1}}{r^{n+1}}\right) P_n(\cos\theta) \quad ,$$

where θ is measured relative to the location of the charge.