Electrodynamics (PHY 514) : 2006

Assignment 2 :

This problem set is due **Thursday January 12**, at the end of the lecture. Feel free to discuss the problems with others in the class, but you must write your own solutions. Simply writing the answer without showing a derivation will obtain zero credit.

1. Consider a system composed of two conducting spheres of radius a_1 and a_2 separated by a distance d with $d \gg a_i$.

   (a) Show that the capacitance matrix for this system is

   $$\mathbf{C} = \frac{4\pi\epsilon_0\, d^2}{d^2 - a_1 a_2} \begin{pmatrix} a_1 & -\frac{a_1 a_2}{d} \\ -\frac{a_1 a_2}{d} & a_2 \end{pmatrix} \tag{1}$$

   (b) If the sphere of radius a_1 carries charge Q_1 and the sphere of radius a_2 carries charge Q_2, what is the energy in the electric field?

   (c) By placing charge $+Q$ on one sphere and $-Q$ on the other, what is the capacitance of the system as conventionally defined.

2. Consider a system composed of three identical conducting spheres of radius a, all lying on the z-axis. Sphere 1 is located at the origin, sphere 2 is a distance r_1 from the origin, and sphere 3 is located a distance $r_1 + r_2$ from the origin, with $r_{1,2} > 0$ (sphere 3 is a distance r_2 from sphere 2). As in problem 1, we assume that $r_i >> a$. Initially, the charge on sphere 2 is Q, while sphere 1 and sphere 3 carry no charge.

   (a) A wire is used to put sphere 1 and sphere 2 in electrical contact, and then removed. What is the charge on sphere 1 and sphere 2?

   (b) Subsequently, a wire is used to put sphere 2 and sphere 3 in electrical contact and then removed. What is the charge in sphere 3?

3. A ring of uniformly distributed charge $Q_1 = 1\mu C$ and radius of $a =$ 1 cm is centered on the z-axis and lies in the xy-plane. A second ring of uniformly distributed charge $Q_2 = 7.3\mu C$ and radius of $b = 1.3$ cm

is centered on the z-axis, lies in the xy-plane but is displaced by $d = 4.5$ cm above the first ring. What is the force, in Newtons, between the two rings?

4. A charge q, when placed on the axis of a thin earthed conducting ring of radius a at a distance d from the center, is found to induce a charge $-Q$ on the ring. From this information, find the capacitance of the ring.

5. Consider a conducting and infinite cylindrical shell of radius R aligned along the z-axis.

   (a) Find the Greens function inside the shell.

   (b) Write the relation between the potential at any point in space and the Greens function when Dirichlet boundary conditions are specified on the cylinder.

   (c) From this relation show that when the potential is specfied on the cylinder, the potential inside is

$$V(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} V(R,\phi)\frac{R^2-r^2}{R^2+r^2-2Rr\cos(\phi-\theta)} \quad . \quad (2)$$

   (d) Use the above result to compute the potential inside a cylinder that is divided longitudinally into quarters with one quarter at potential $+V_0$ and the one diagonally opposite at potential $-V_0$, while the other two are grounded.