Electrodynamics (PHY 514) : 2006

Assignment 1 :

This problem set is due **Thursday January 5**, at the end of the lecture. Feel free to discuss the problems with others in the class, but you must write your own solutions.

1. Consider a modified theory of electrostatics in which the magnitude of the force between particles carrying charges q_1 and q_2 is

$$|\mathbf{F}(\mathbf{r})| = \frac{q_1 q_2}{4\pi\epsilon_0 |\mathbf{r}|^{2+\eta}} \quad , \tag{1}$$

where $\mathbf{r}$ is the vector separating the two particles, and for $\eta = 0$ one recovers the usual electrostatic force. Assume that η is a small but non-zero number, $0 < \eta \ll 1$.

   (a) For a charge density of the form $\rho(\mathbf{r}) = Q\delta^3(\mathbf{r} - \mathbf{r_0})$ compute the potential at the point $\mathbf{R}$.

   (b) Give the charge density, $\rho(r)$, of a spherical conducting (infinitely thin) shell of radius b, with a uniformly distributed charge Q_1.

   (c) Compute the potential both inside and outside this charged shell. Show that the limit $\eta \to 0$ reproduces the potential associated with a Coulomb interaction.

   (d) Consider two concentric conducting shells of radius a and radius b with $b > a$. The two shells are maintained at the same potential, V_0, by a very fine-conducting wire (which you can neglect from this point forward). A charge Q_1 is placed on the outer shell (of radius b). What is the charge on the inner shell (of radius a)? Give your result to lowest-order in an expansion in η.

2. Prove Earnshaw's theorem which states that *a charge, when acted on by electric forces alone, cannot be in stable equilibrium an an electric field.*