

## PHYSICS 429: Introduction to Biological Physics

May 28 2008

Problem Set 7 These problems are due on Thursday June 5.

1. Nelson 8.7.

2. Nelson 8.8 a,b

3. Nelson C8.11 on page 593. As discussed during lecture, amino acids exist in various protonation states in solution, mainly because they have carboxyl (-COOH) and/or amino (-NH<sub>2</sub>) groups. For example, at intermediate pH, glycine has both of these groups charged and is therefore overall neutral. At low pH, the carboxyl gains a neutralizing proton with pK=2.35. At high pH, the amino group loses a proton, becoming neutral, with pK=9.78. Find the isoelectric point of glycine, that is, the value of pH at which the average net charge is 0.

4. Nelson C8.10 on page 592. Reaction-diffusion equation. This is: The discussion of Sect 4.61 explored diffusive transport in a long tube, finding a steady state in which the number density of some dissolved substance,  $c(x)$ , is a linear function interpolating between the imposed values at the ends of the tubes. In particular, if the two ends are at the same concentration, then  $c(x)$  is everywhere constant, there is no net flux, and the steady state is one of equilibrium.

Now suppose that some chemical reaction in the tube removes the dissolved substance. Suppose that each dissolved molecule has a probability to disappear in a time  $dt$  equal to  $k dt$ , where  $k$  is a constant. Thus, even if the concentration is spatially uniform (independent of  $x$ ) near a point  $x$ , it will change with time as  $\frac{\partial c}{\partial t} = -kc$ .

(a) Write an equation for  $\frac{\partial c}{\partial t}$  for the case in which  $c$  is not spatially uniform. This equation is said to be the reaction-diffusion equation.

(b) Again imagine a situation where the concentration is constant in the  $y$  and  $z$  directions, but is maintained at a fixed value  $c_0$  at both ends of the container,  $x = \pm L$ . Find the steady-state concentration profile  $c(x)$  and the fluxes at each end of the container. Compare with the case of pure diffusion.

(c) Does the solution you found in (b) correspond to an equilibrium solution? an equilibrium

5. Nelson C4.20 on page 584.

(a) Suppose that a membrane consists of two types of lipids,  $A$  and  $B$ . The  $A$  lipids assemble in large patches that make up a fraction  $f$  of the total membrane area (the rest of the area is composed of  $B$  lipids). Molecule  $X$  has a permeability  $\mathcal{P}_A$  and  $\mathcal{P}_B$  through the two lipids. Find an expression for the permeability  $\mathcal{P}_{AB}$ .

(b) Two membranes  $A$  and  $B$  have permeabilities  $\mathcal{P}_A$  and  $\mathcal{P}_B$ . They are joined together to form a single, two-layered membrane. Derive an expression for the permeability of the two-layered membrane,  $\mathcal{P}_{AB}$ , in terms of  $\mathcal{P}_A$  and  $\mathcal{P}_B$ .