PHYSICS 429: Introduction to Biological Physics

May 5 2008 Problem Set 5 These problems are due on Tuesday May 13.

1. Nelson 7.6

2. Nelson 7.10

3. Nelson 7.17. This is on page 591, additional problems.

4. This is Hobbie-Roth 9.10. A collection of molecular electric dipoles of dipole moment vector **p** are in thermal equilibrium at temperature T. If the dipoles experience an electric field of strength E, then determine the average value of $\cos\theta$ where θ is the angle between the dipole and the electric field. Hint: consider the Boltzmann distribution. Show that if $pE \ll k_BT$ the average of $\cos\theta$ is proportional to E, but if $pE \gg k_BT$ the average of $\cos\theta$ approaches unity. Interpret this last result physically.

5. Consider the system of two identical plates at $x = \pm a$, each with a surface charge density σ that we studied in class. Starting with the Poisson-Boltzmann equation:

$$\frac{d^2\bar{V}}{dx^2} = -\frac{e^2 c(x)}{k_BT\epsilon}, \qquad c(x) = c(0)\exp(-\bar{V})$$

we obtained

$$\frac{1}{2}\left(\frac{d\bar{V}}{dx}\right)^2 - \frac{e^2 c(x)}{\epsilon k_BT} = \text{const.}$$

This can be written

$$\frac{1}{2}\left(\frac{d\bar{V}}{dx}(x_1)\right)^2 - \frac{e^2 c(x_1)}{\epsilon k_BT} = \frac{1}{2}\left(\frac{d\bar{V}}{dx}(x_2)\right)^2 - \frac{e^2 c(x_2)}{\epsilon k_BT}$$

for two positions x_1, x_2.

(a) Take $x_2 = 0$ and $x_1 = a$ because you know the electric field at these two places. Show that

$$c(a) = c(0) + \frac{\sigma^2}{2\epsilon k_BT}.$$

Note that the concentration of counterions at the surface, $c(a)$ never falls below the limiting value $\frac{\sigma^2}{2\epsilon k_BT}$ because if the surfaces are far apart $c(0) \to 0$.

(b) Consider $\sigma = 0.2\text{C/m}^2$ which is typical for a fully ionized surface and which corresponds to one elementary charge per 0.8 nm^2 or 1.25×10^{18} charges per square meter. Given, σ, obtain the limiting value of the concentration $\frac{\sigma^2}{2\epsilon k_BT}$ at the surface. If these counterions are considered to occupy a layer of thickness $\delta = 0.2$ nm, what the charge per unit area, $\frac{\sigma^2\delta}{2\epsilon k_BT}$ does this correspond to? You should find that it is almost the same as the charge density itself. This is an interesting result, for it shows that regardless of the charge density away from the surface, most of the counterions that effectively balance the surface charge are located in the first few angstroms from the surface. (For lower surface charge densities, the diffuse layer extends well beyond the surface.)