PHYSICS 429: Introduction to Biological Physics

April 28 2008 Problem Set 4 Solution These problems are due on Tuesday May 6.

1. Nelson 7.5

(a) When two large objects are closer than 20 nm apart, the concentration of proteins between them is forced to be zero and they feel an attractive depletion force.

(b) The volume fraction is the product of the number density and the volume of each individual protein. The concentration is then

$c = \frac{0.3}{\frac{4}{3}\pi(10 \text{ nm}^3} = 7.2 \times 10^{22} m^{-3} = 0.12$ mM.

The osmotic pressure is then $ckT = (7.2 \times 10^{22} m^{-3}\ 4.1 \times 10^{-21}$ J ≈ 290 Pa. When the surfaces join, the volume changes by ΔV so that the change in free energy ΔF is given by $\Delta F = c\, kT\, \Delta V = (290 J/m^3)(2 \times 10 \text{ nm} \times 10 \mu m^2) = 5.7 \times 10^{-17}\ J$. This reduction in free energy is very much larger than the thermal energy kT.

2. This is Hobbie-Roth 5.5. The definition of osmolarity is the number of osmoles per liter of solution. An osmole is the equivalent of a mole of solute particles. Thus the term osmolarity of 0.3 osmole is 0.3 moles of solute per liter.

An understanding of osmotic pressure is important in medicine. Consider the case reported in Steinmuller (1998) in the New England Journal of Medicine 338,1226. A 5% solution of albumin was needed to infuse into a patient with kidney disease (renal insufficiency). No 5% solution was available, so the hospital pharmacy used 25% albumin diluted 1:4 with pure water. Injection of the solution into the patient caused renal failure. The albumin in a 25% albumin solution has an osmolarity of about 36 mosmole. Typically, such a solution also contains about 300 mosmole =.3 osmole of other ions.

(a) Calculate the osmolarity of the solution injected into the patient.

The concentration goes down by a factor of five so the osmolarity is given by

$\frac{(300+36) \text{ mosmol}}{5} = 67.2$ mosmole.

(b) Calculate the osmolarity of the solution if the pharmacy had properly used isotonic saline, which also has a osmolarity of 300 mosmole, instead of pure water to perform the 1:4 dilution. Saline solution has the same osmolarity as blood.

We now have $\frac{336 \text{ mosmol} + 4(300) \text{ mosmol}}{5} = 307$ mosmole.

3. This is HR 5.6. Articular cartilage covers the ends of bones in joints and allows the bones to move smoothly against each other. It contains a network of collagen fibers that can exert a mechanical tensile stress to resist tissue swelling, resulting in a pressure P_c within the cartilage. The collagen fibers do not withstand compression. The cartilage also contains proteoglycan molecules that cause tissue swelling because of their osmotic pressure, π_{PG}. Once can determine P_c by placing the cartilage in a polyethylene glycol solution with osmotic pressure π_{PEG}, measuring π_{PEG} and π_{PG}, using the relationship $P_c = \pi_{PG} - \pi_{PEG}$. Typical data are

π_{PEG} (atm)	π_{PG}
0.0	4.0
2.5	5.5
5.0	7.0
7.5	8.5
10.0	10.0

(a) Determine the excess pressure P_c exerted by the collagen fibers under normal conditions ($\pi_{PEG} = 0$.)

$P_c = \pi_{PG} = 4$ atm.

(b) Determine the value of $\pi_{PEG} = 0.$ for which the collagen fibers exert no tensile stress (become limp).

$P_c = 0$ if $\pi_{PG} = \pi_{PEG} = 10$ atm.

(c) Find a linear equation for P_c as a function of $\pi_{PEG} = 0$.

$P_c = -0.4\pi_{PEG} + 4$ atm.

(d) Osteoarthritis is thought to occur when the collagen fibers are weakened. If the collagen in an arthritic joint can only exert a pressure of 2 atm when $\pi_{PEG} = 0$, by how much will the tissue swell (by what percent will its volume change?

If the osmotic pressure of the proteoglycan molecules is greater than the tensile stress in the collagen fibers, then water will enter and the cartilage swells. Eventually the swelling will lower the concentration of proteoglycans (more water same number of PG molecules), until the osmotic pressure is reduced to 2 atm. This is half of the normal value of P_c, so the cartilage will swell to twice its normal volume.

In (b) (d) assume that only the proteoglycans cause osmotic pressure and that their number does not change, but the tissue volume increases as the tissue swells with water. This problem is based on the work of Basser et al (1998) Arch. Biochem. Biophys. **351**:207.

3. Nelson problem 7.7

For small values of $\bar{V}$, $\sinh(\bar{V}) = 0 + \bar{V} = \bar{V}$, so that 7.34 becomes

$\frac{d^2\bar{V}}{x^2} = (\lambda_D)^{-2}\bar{V}$

which means that

$\bar{V} = \text{const } e^{-\lambda_D x}$.

The exponential fall off is known as Debye shielding.

4. Nelson problem 7.8

The use of Eq. (6.36) on page 233 gives

$S/k = -\int d^3x\ (c\ln c - c\ln c_*)$,

where c_* is a constant. The second term is just a constant times the total number of solute molecules, also a constant. So drop the second term. The the time derivative of S is given by

$\frac{dS}{dt} = -k\int d^3x\ \frac{\partial c}{\partial t}(1 + \ln c)$.

Now we use the diffusion equation $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$ to find

$\frac{dS}{dt} = -kD\int d^3x\ \frac{\partial^2 c}{\partial x^2}(1 + \ln c)$.

The first term is the first partial derivative of c with respect to x, evaluated at infinity where it vanishes. Use integration by parts to write the remaining term so that

$\frac{dS}{dt} = +kD\int d^3x\ (\frac{\partial c}{\partial x})^2\frac{1}{c}$.

This quantity is always positive:Entropy increases with time.