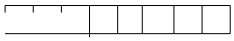

Figure 1: For problem 6. Ladder or zipper model of DNA. In the figure the are 3 open rungs and 5 closed ones, so $n = 3$ and $N = 8$.

PHYSICS 429: Introduction to Biological Physics

April 8 2008 Problem Set 2 These problems are due on Tuesday April 15.

1. In class we showed that the the entropy S is given by the expression

$$S = k_B(\ln Z + \beta \bar{E}).$$

The free energy F is given by the expression $F = \bar{E} - TS$ if we make the reasonable assumption of replacing E by $\bar{E}$. (a) Show that $F = -k_B T \ln Z$.

(b) Show that $\bar{E} = \frac{\partial \beta F}{\partial \beta}$.

(c) Show that $p = -\frac{\partial F}{\partial V}$.

2. Consider a classical gas of N non-interating molecules moving in three dimensions in a volume V at a temperature T with Hamiltonian $H = \frac{p^2}{2M}$.

(a) Compute the free energy F of the system.

(b) Compute the internal energy $\bar{E}$.

(c) The specific heat at constant volume c_V is the ratio between the heat added or removed per mole and the change in temperature. Compute c_V for the present example.

3. Nelson problem 6.5

4. Nelson problem 4.3

5. Nelson Problem 4.5

6. DNA unfolding. DNA unfolds during the transcription process when DNA makes messenger RNA. This problem is concerned with the thermal fluctuations that can cause DNA to partially unfold. Suppose DNA has N links, each of which can be in one of two states: closed of energy $E = 0$ or open $E = \Delta$. Consider DNA to have the structure of a horizontal ladder, with some of the rungs (links) broken (open). DNA runs horizontally from the left to the right. See Fig. 1 above. A link can only be open if all of the links to the left are open. Thus the energy of the system is defined by the number n of open links.

(a) Show that the partition function is given by $Z = \frac{1-e^{-\beta(N+1)\Delta}}{1-e^{-\beta\Delta}}$, where $\beta = 1/(k_B T)$.

(b) Compute the average number, $\bar{n}$, of open links.

(c) Find simplified expressions for $\bar{n}$ in the limits of extremely high and extremely low temperatures.