I. COMPARING THE CONE AND k_T ALGORITHMS

The following are some comments on the comparison of the usual cone algorithm with the (Soper & Ellis) k_T algorithm, both looked at NLO in perturbation theory. Recall that in the usual Snowmass realization the cone algorithm combines 2 partons (at NLO there are at most 2 partons in a jet) into a single jet if

$$(\eta_i - \eta_{JET})^2 + (\phi_i - \phi_{JET})^2 \leqslant R^2, i = 1,2$$

where

$$\eta_{JET} = \frac{\eta_1 E_{T,1} + \eta_2 E_{T,2}}{E_{T,1} + E_{T,2}},$$
$$\phi_{JET} = \frac{\phi_1 E_{T,1} + \phi_2 E_{T,2}}{E_{T,1} + E_{T,2}}.$$

Since both CDF and DØ make use of seeds (looking for stable cones only where there is initially energy above a defined threshold) and splitting/merging of overlapping cones, a separate phenomenological parameter, R_{sep}, was introduced to simulate these experimental effects. The impact is that the 2 partons are included in a single jet if, along with the previous constraints, the pair also satisfies

$$(\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2 \leqslant R_{sep}^2 \times R^2.$$

In the original Snowmass cone algorithm the default value of this parameter is thus

$$R_{sep}\,(Snowmass) = 2.0.$$

Studies by both CDF (PRD 45, 1448 (1992)) and DØ (FERMILAB-Pub-97/242-E) suggest that a reasonable value for R_{sep} for comparing NLO with data is $R_{sep} = 1.3$, which was also the preferred choice of Ellis, Kunszt & Soper..

In a 1993 paper Soper & Ellis (PRD 48, 3160 (1993)) defined a version of the k_T algorithm that at NLO corresponds to merging the two nearest partons (out of the three available) whenever they are closer in (η, ϕ) than a parameter D. Thus, at NLO, this is equivalent to the cone algorithm after setting

$$R_{sep}\,(k_T) = 1.0$$

and identifying the k_T algorithm resolution parameter D with the cone size R. Thus it is straightforward to perform a NLO comparison of the two algorithms using the same cone algorithm code with the appropriate values of R_{sep} and R. Using the (then popular) HMRS(B) pdfs and the CDF rapidity range $0.1 \leqslant |\eta_{JET}| \leqslant 0.7$, Soper & Ellis found that, for $\mu = E_T/2$ and $\sqrt{s} = 1800$ GeV, the cone algorithm with $R = 0.7$ (and $R_{sep} = 2.0$) agreed with the k_T algorithm with $(R =)D = 1.0$ (and $R_{sep} = 1.0$) to within 10% over the ranges 10 GeV $< E_T < 500$ GeV and $E_T/4 < \mu < E_T$.

This agreement is now a source of discussion given the recent experimental study of the k_T algorithm by DØ. In this note the jet cross sections are recalculated with CTEQ6m pdfs to reproduce the analysis of the 1993 Ellis/Soper paper, including more explicitly the dependence on the R_{sep} parameter. To understand the game consider the cone algorithm NLO jet cross section (averaged over rapidity $0.1 \leqslant |\eta_{JET}| \leqslant 0.7$) as a function of 3 parameters: E_T, R and R_{sep}. While the detailed behavior of this cross section, especially the E_T dependence, is difficult to specify *a priori*, the general dependence on R and R_{sep} is not surprising. Numerical calculations show that the cross section is a monotonically increasing function of both R and R_{sep}. Further, the rate of increase with either variable increasing with the magnitude of the other variable, a feature that is again intuitively reasonable. A crude, but reasonably accurate (a few percent), representation of this behavior, in the ranges $1.0 < R_{sep} < 2.0$, $0.5 < R < 1.1$, is provided by the expression

$$\left.\frac{d\sigma}{d\eta dE_T}\right|_{cone} \approx A\,(E_T) \times \exp\left(0.5R\sqrt{R_{sep}}\right),$$

where the the most important approximation is the factorization of the E_T dependence into the function $A\,(E_T)$, . Thus at NLO the corresponding k_T algorithm form is given by

$$\left.\frac{d\sigma}{d\eta dE_T}\right|_{k_T} \approx A\,(E_T) \times \exp\,(0.5D)\,.$$

If this relation holds, we can always account for the variation with respect to R_{sep} (*i.e.*, the difference between the cone and k_T algorithms) by a multiplicative change in R. In particular, a k_T algorithm with $D = \sqrt{R_{sep}} \times R$ will yield approximately the same numerical results (at NLO) as a cone algorithm with R and R_{sep}. The primary source of the violation of the factorization (*e.g.*, the fact that the exponential depends weakly on E_T) is likely the fact that the jets become narrower at larger E_T, an $\alpha_S(\mu)$ effect. Thus for $R = 0.7$ and $R_{sep} = 2.0$, we expect good agreement for $D = \sqrt{2} \times 0.7 = 0.99$ in good agreement with the result of Ellis & Soper.

Figures 1 and 2 illustrate numerical NLO results for this agreement between the inclusive jet cross section for the cone and k_T algorithms in the format (Theory$_1$-Theory$_2$)/Theory$_2$ using Snowmass kinematics (*i.e.*, pseudorapidities and E_T as a scalar sum and in the rapidity range $0.1 \leqslant |\eta_{JET}| \leqslant 0.7$). Note that, as suggested, the factorization is not exact and there is some variation with E_T. The two choices for Theory$_2$ are the cone algorithm ($R = 0.7$) with $R_{sep} = 2.0$ and $R_{sep} = 1.3$, while Theory$_1$ is the k_T algorithm with appropriate choices of the parameter D (and $R_{sep} = 1.0$). Note that in Figure 1 the agreement for the choice $D = 1.0$ is well within the the 10% value suggested by Ellis & Soper and as motivated by the simple analytic expression above. However, the agreement is even better, of order 1%, for the slightly smaller value, $D \simeq 0.925$ (for $R_{sep} = 2.0$). As suggested above, the data suggest the value $R_{sep} = 1.3$ for the cone algorithm and Figure 2 provides a comparison with the k_T algorithm in this case. As expected ($\sqrt{1.3} \times 0.7 \simeq 0.8$) the best agreement is for the smaller D value, $D \simeq 0.835$. For comparison a curve with $D = 1.0$, the value used for comparing the cone and k_T algorithms in the DØ analysis of their data, is also included. As expected the difference between the two algorithms is larger than the optimal value but still within the 10% limit of Ellis & Soper and smaller than observed by D0. Since these jet shape properties are evaluated only at leading order in the overall NLO jet calculation, we should expect nontrivial μ dependence and this is also illustrated in Figure 2. Four of the curves correspond to the choice $\mu = E_T/2$, while the second $D = 1.0$ curve (the top curve) corresponds to $\mu = E_T/4$, the choice suggested by Ellis, Kunszt & Soper for the $R_{sep} = 1.3$ case. The even larger difference (approximately 15 %) observed for this choice of parameters is indicative of the sizeable uncertainty in the NLO prediction and more suggestive of the D0 results.

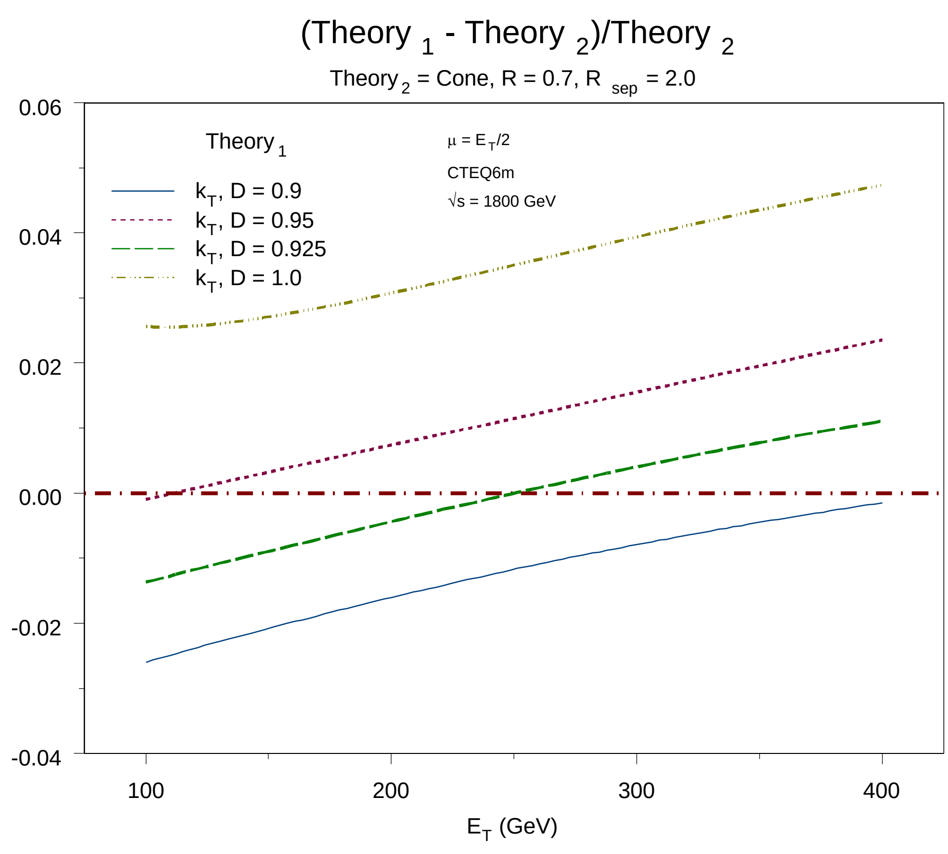

FIG. 1: Snowmass Default, $R_{sep} = 2.0$

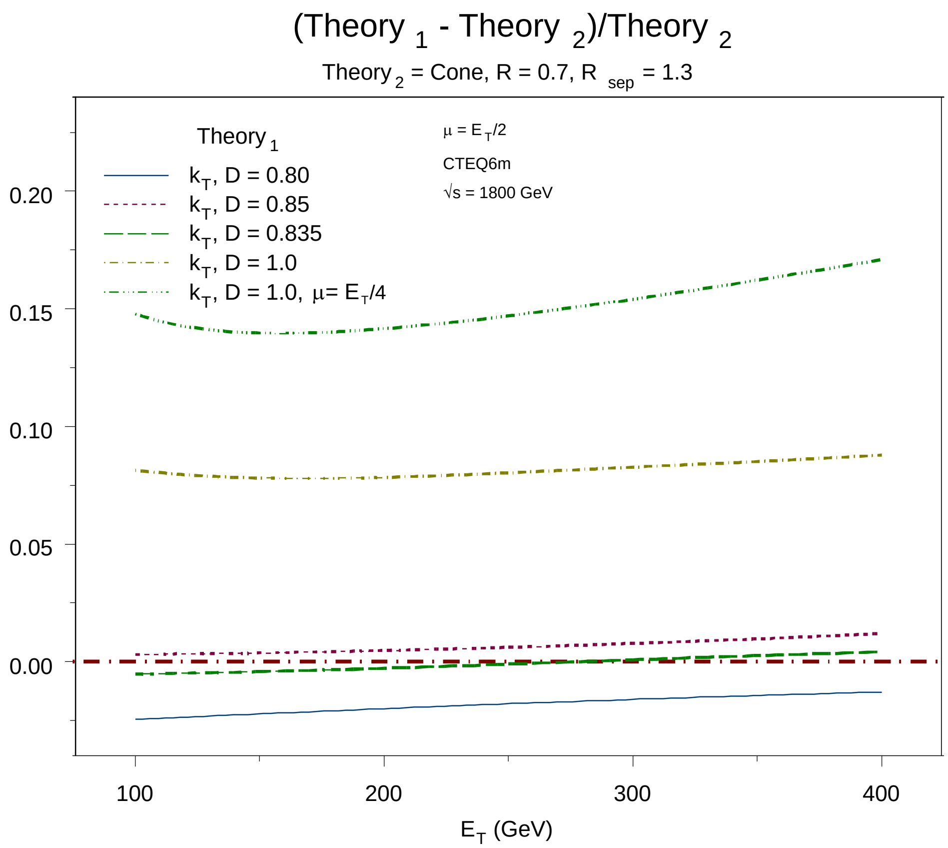

FIG. 2: "Experimental" default, $R_{sep} = 1.3$.

Figures 3 and 4 display similar comparisons when both jet algorithms are defined using true 4-vectors, *i.e.*, the E-scheme, as recommended by the Run II jet working group. Note, in particular, that the jets are binned in terms of the true transverse momentum, $\left|\overrightarrow{P_T}\right|$, rather than E_T. The level of agreement is very similar to the case with Snowmass kinematics, *i.e.*, about 2 %, with similar values for the parameter D, ($D \simeq 0.95$ for $R_{sep} = 2.0$ and $D \simeq 0.83$ for $R_{sep} = 1.3$). The comparison for the choice $R_{sep} = 1.3$ and $D = 1.0$ (with the 2 values of μ) are also very similar to the case of Snowmass kinematics (the differences are only slightly smaller).

In summary, at NLO the k_T algorithm with resolution parameter D is identical to the cone algorithm with cone size $R = D$, and splitting/merging parameter $R_{sep} = 1.0$. The fact that the "experimental" cone algorithm corresponds typically to $R_{sep} = 1.3$ and $R = 0.7$ (and $\mu = E_T/2$), can be compensated for by using $D \simeq 0.83$, *i.e.*, at NLO the inclusive jet cross section using the cone algorithm with $R_{sep} = 1.3$ and $R = 0.7$ agrees to within a few % with the cross section using k_T algorithm with $D \simeq 0.83$ (for a cone algorithm with $R_{sep} = 2.0$ and $R = 0.7$ the best agreement occurs for $D \simeq 0.925$). These conclusions are essentially independent of whether the kinematics are Snowmass style or 4-vector. To the extent that NLO with $R_{sep} = 1.3$ and $\mu = E_T/4$ provides the best description of the data, the NLO perturbative results presented here are at least suggestive of the results (*i.e.*, sizeable differences between the cone and k_T cross sections) reported by D0, recognizing that there is considerable uncertainty in the NLO results.

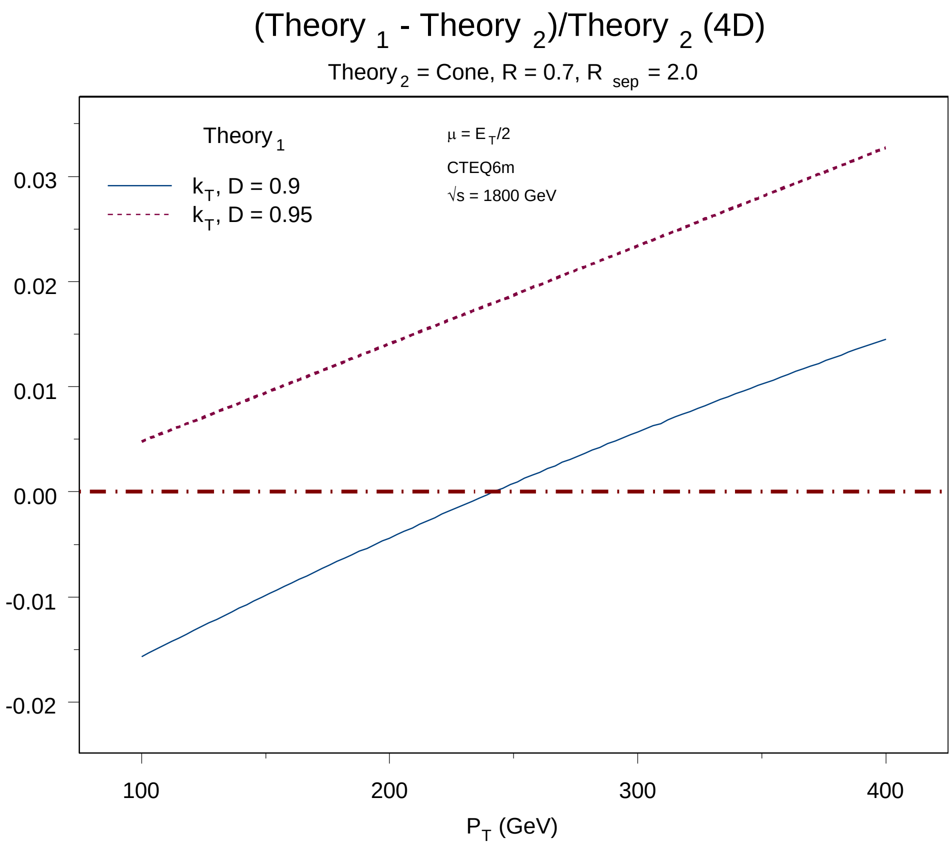

FIG. 3: Snowmass Default, $R_{sep} = 2.0$ with 4-D kinematics

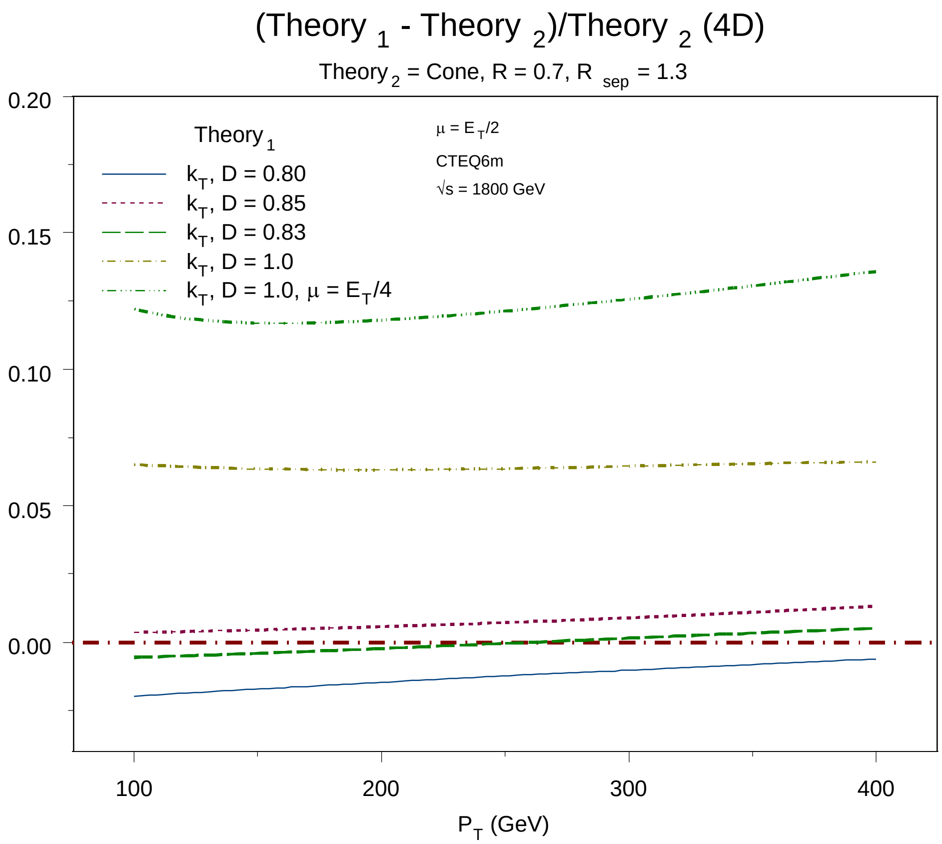

FIG. 4: "Experimental" default, $R_{sep} = 1.3$ with 4-D kinematics