SM (colliders) (Perturbative) QCD & Jets

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Lecture 2: Calculating with QCD –
Hadrons in the Initial State and PDFs
Jets in the Final State

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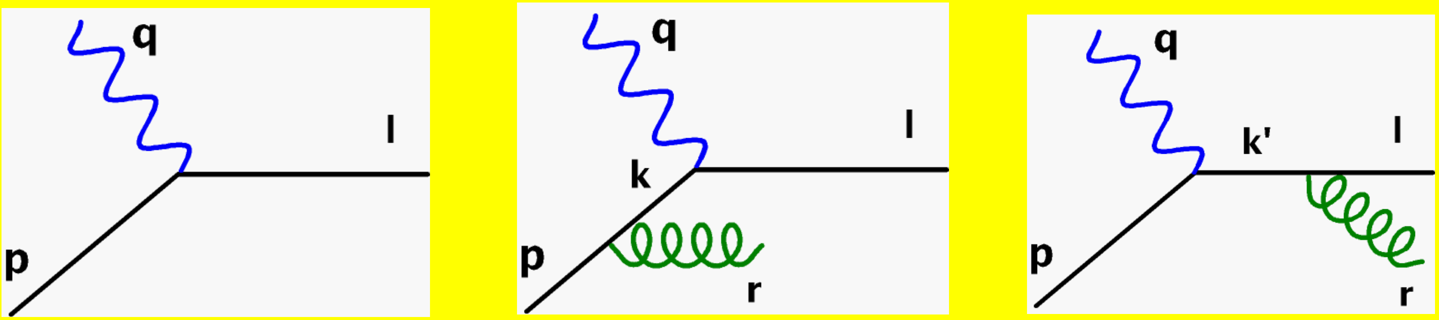
Outline

1. Introduction – The Big Picture
   pQCD - e+e- Physics and Perturbation Theory (the Improved Parton Model);
   pQCD - Hadrons in the Initial State and PDFs
2. **pQCD - Hadrons and Jets in the Final State; Jets at Work**
   (UV$\Rightarrow$ Running Coupling, saw Soft & Collinear in Pert Thy)

pQCD Calculation -

Parton Distribution Functions in QCD

- Revisit DIS – include real gluon emission (massless partons)



LO NLO Real NLO Real

- Singularities arise when the internal propagators go on-shell (collinear and soft gluon emission).

- In the appropriate (light-cone) gauge, the divergent contribution in the middle diagram *. In any case it can be written

$$\hat{F}_2\Big|_{Div} = e_q^2 \frac{\alpha_s}{2\pi} x \hat{P}_{qq}(x) \int_0^{2\nu} \frac{d\left|k_\perp^2\right|}{\left|k_\perp^2\right|}$$

Transverse Momentum

Collinear Fraction x

* This is gauge dependent - only the sum of the middle and right graphs squared is gauge invariant. In the light-cone gauge only the middle graph is singular.

Singular Configurations -

- the $|k^2|$ integral goes all the way up to the kinematic boundary – it is not cutoff at a fixed (small) value as assumed by the parton model (so expect some differences)

- the $|k^2|$ integral is singular at the lower limit – control with a cutoff κ^2 for now (this "long distance" behavior is non-perturbatively controlled by "confinement" in real life)

- the (collinear) singularity is multiplied by a characteristic function of the quark's momentum fraction *x* – the "splitting function" – that tells us how the longitudinal momentum is shared

$$\hat{P}_{qq}(x) = C_F \frac{1+x^2}{1-x}$$

← Singular for soft gluon, $x \to 1$

More on Singular Configurations -

- Put in cutoff and include *all* diagrams above (in standard from) to define for DIS from a quark

$$\hat{F}_{2,q}^{NLO}(x,Q) = e_q^2 x\left[\delta(1-x) + \frac{\alpha_s}{2\pi}\left(\hat{P}_{qq}(x)\ln\left(\frac{Q^2}{\kappa^2}\right) + C(x)\right)\right]$$

where both the collinear term *P*(x) and the non-collinear singular bit *C*(x) are calculable functions in pQCD (i.e., IRS quantities).

⇒ **Conclude! : Naive Scaling is broken** (i.e., the Parton Model) by ln(Q) terms (and we must sum them)!

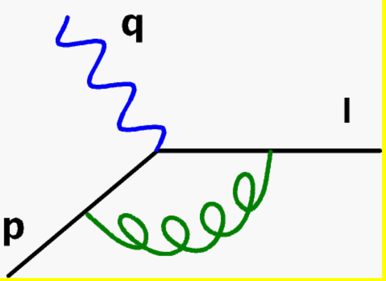
⇒ The distribution of quarks (in a quark) is now (being explicit about the scale μ)

$$q_q\left(x,\frac{Q}{\kappa},\mu\right) = \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi}\left(\hat{P}_{qq}(x)\ln\left(\frac{Q^2}{\kappa^2}\right) + C(x)\right)$$

and quarks are (likely) accompanied by (approximately) collinear gluons

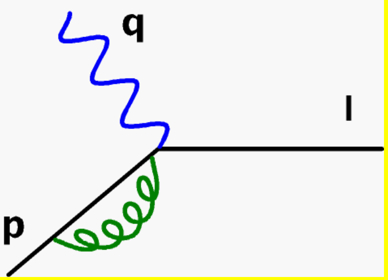
Include virtual graphs – (truly soft gluon ~ no gluon at all)

- ~δ((*p*+*q*)²) - Contribute for *x*=1, → δ(1-x) term + …



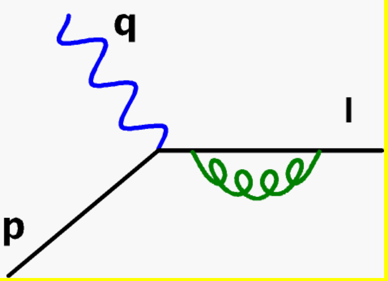
- Quark (baryon) number is conserved*, independent of Q^2

$$\hat{P}_{qq}(x) \to P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] = C_F\left(\frac{1+x^2}{1-x}\right)_+$$



where the "+" distribution is defined by

$$\int_0^1 dx\, f(x)\left(\frac{1+x^2}{1-x}\right)_+ \equiv \int_0^1 dx\left[f(x) - f(1)\right]\left(\frac{1+x^2}{1-x}\right)$$



- With care taken below for the process $g \to q\bar{q}$, this is just (i.e., due to the delta fct, virtual bit)

$$\hat{P}(x) \to \hat{P}(x)_+ = P(x)$$

***Confirm quark number conservation - HW**

Put it together -

- For a quark in a proton, as an intermediate step we introduce a "bare" quark distribution q_0 and convolute with above

$$q\left(x,\frac{Q}{\kappa},\mu\right)=q_0(x)+\frac{\alpha_s(\mu)}{2\pi}\int_x^1\frac{d\xi}{\xi}q_0(\xi)\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\kappa^2}\right)+C_q\left(\frac{x}{\xi}\right)\right]+\ldots$$

- q_0 plays similar role to $\alpha_s(M)$ used earlier – an "unphysical" place to hide infinities. The theory is well behaved but our approach in terms of "bare" objects requires us to follow this round-about path.

- Need to get rid of the "cut-off" κ and the "bare" distribution

Factorization Scale -

Introduce a *factorization* scale μ_F – "absorb" collinear singularities (for $|k^2|<\mu_F^2$) into the bare distribution and obtain the regularized, scale dependent distribution, *i.e.*, the long distance physics is all in the regularized distribution.

- Define $$\ln\left(\frac{Q^2}{\kappa^2}\right)=\ln\left(\frac{Q^2}{\mu_F^2}\right)+\ln\left(\frac{\mu_F^2}{\kappa^2}\right)$$
- Split the non-collinear term in a factorization scheme dependent fashion where the second term will be included in the long distance physics (an arbitrary choice) $$C_q(z)\equiv\bar{C}_q(z)+\tilde{C}_q(z)$$
- Physical quantities are scheme independent and the calculation will be also if <u>all</u> parts are performed in the same scheme!
- E.g., the DIS choice is to absorb everything, $\bar{C}_q^{DIS}=0$

Factorization -

- Finally, choosing the factorization scale to equal the renormalization scale (simplifying but not necessary), $\mu_F^2 = \mu^2$, define

$$q_q\left(x,\mu_F\right) = q_0\left(x\right) + \frac{\alpha_s\left(\mu_F\right)}{2\pi}\int_x^1 \frac{d\xi}{\xi} q_0\left(\xi\right)\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{\mu_F^2}{\kappa^2}\right) + \tilde{C}_q\left(\frac{x}{\xi}\right)\right] + \ldots$$

which formally includes all of the collinear structure, and is thus not calculable in pQCD, but allows us to write

$$q_q\left(x,Q\right) = \int_x^1 \frac{d\xi}{\xi} q_q\left(\frac{x}{\xi},\mu_F\right)\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi}\left\{P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right) + \bar{C}_q\left(\frac{x}{\xi}\right)\right\} + \ldots\right]$$

$$F_{2,q}^{NLO}\left(x,Q^2\right) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} \; q_q\left(\xi,\mu_F\right)$$
$$\times\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s\left(\mu_F\right)}{2\pi}\left\{P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right) + \bar{C}_q\left(\frac{x}{\xi}\right)\right\} + \ldots\right]$$

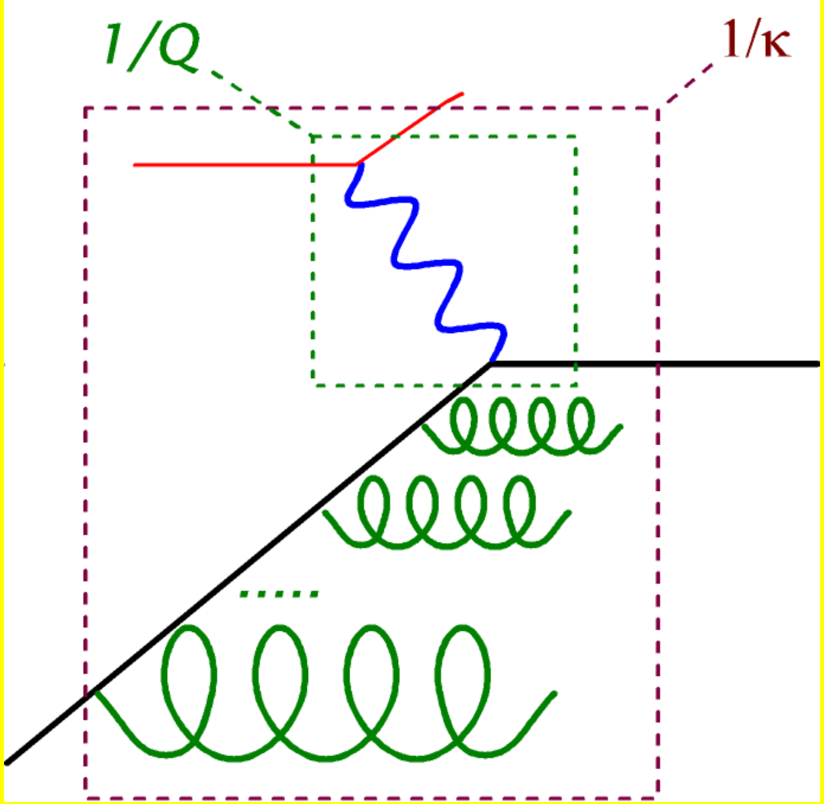
Summary in pictures, first with cut-off

- Order-by-order, we are summing the largest contributions of the emission of multiple gluons

- The change in size (wavelength) of the gluons represents the strong ordering of the transverse momenta (smaller wavelength means larger momentum)

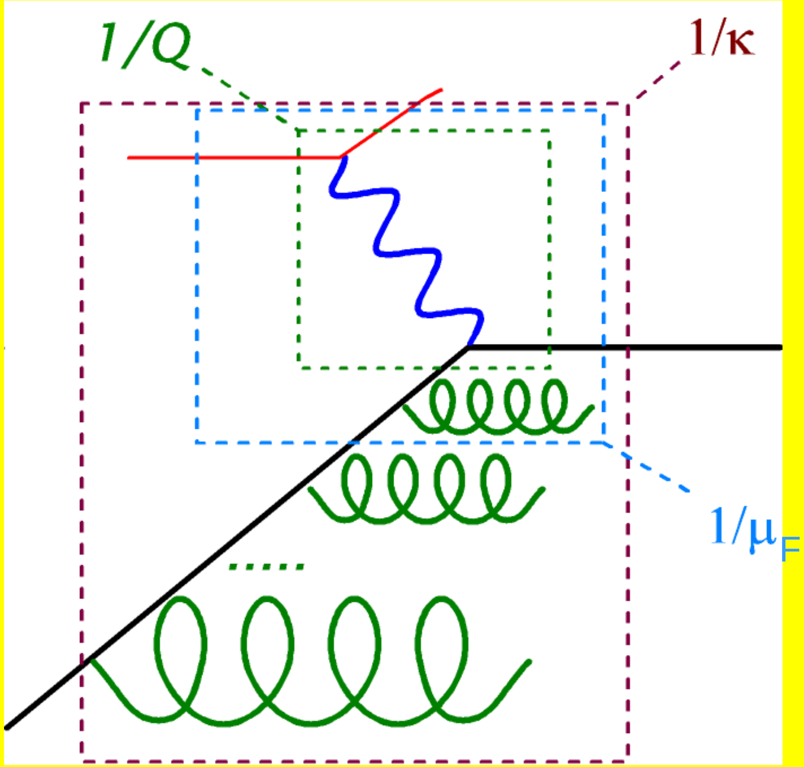
$$k_{T,1}^2 << k_{T,2}^2 << \cdots << k_{T,n}^2$$

With cut-off κ

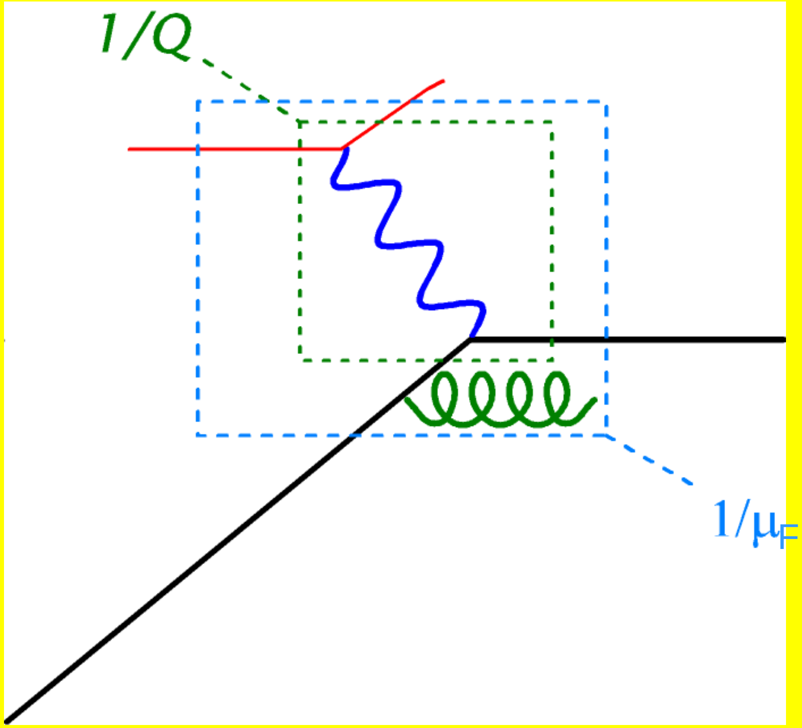


Summary in pictures, with factorization scale

- Separate contributions above and below the factorization scale μ
- And factor scales κ to μ into the renormalized distribution – leaving

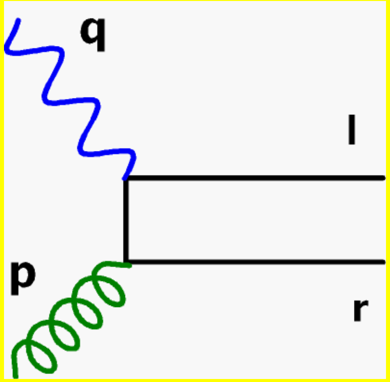


$\Rightarrow$



One more addition -

- At this order we also have (a quark from a gluon) yielding



$$\hat{F}_{2,g}^{NLO}\left(x,Q^2\right)=x\sum_{q,\bar{q}} e_q^2\frac{\alpha_s}{2\pi}\left[P_{qg}\left(x\right)\ln\left(\frac{Q^2}{\kappa^2}\right)+C_g\left(x\right)\right]$$

$$P_{qg}\left(x\right)=T_R\left[x^2+\left(1-x\right)^2\right];\quad T_R=\frac{1}{2}$$

- So we really want a “bare” gluon distribution too -

$$q\left(x,\mu_F\right)=q_0\left(x\right)+\frac{\alpha_s}{2\pi}\int_x^1\frac{d\xi}{\xi}q_0\left(\xi\right)\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{\mu_F^2}{\kappa^2}\right)+\tilde{C}_q\left(\frac{x}{\xi}\right)\right]$$

$$+\frac{\alpha_s}{2\pi}\int_x^1\frac{d\xi}{\xi}g_0\left(\xi\right)\left[P_{qg}\left(\frac{x}{\xi}\right)\ln\left(\frac{\mu_F^2}{\kappa^2}\right)+\tilde{C}_g\left(\frac{x}{\xi}\right)\right]+\ldots$$

DGLAP

- Consider the general version ($\mu^2 = \mu_F^2 \neq Q^2$)

$$F_2\left(x,Q^2\right) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q^{\overline{MS}}\left(\xi,\mu^2\right)$$

$$\times\left[\delta\left(1-\frac{x}{\xi}\right)+\frac{\alpha_s(\mu)}{2\pi}\left\{P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu^2}\right)+\bar{C}_q^{\overline{MS}}\left(\frac{x}{\xi}\right)\right\}+\ldots\right]$$

$$+x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} g^{\overline{MS}}\left(\xi,\mu^2\right)\left[\frac{\alpha_s(\mu)}{2\pi}\left\{P_{qg}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu^2}\right)+\bar{C}_g^{\overline{MS}}\left(\frac{x}{\xi}\right)\right\}+\ldots\right]$$

LHS is μ independent so RHS must be also, order-by-order in pQCD
$\Rightarrow$ DGLAP – (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

DGLAP – Perturbative condition on NONperturbative quantity

$$\mu^2 \frac{\partial}{\partial \mu^2} q(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi,\mu^2)$$

$$= \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dz}{z} P(z) q\left(\frac{x}{z},\mu^2\right)$$

- The splitting function *P* (like the β function) is what is calculable in pQCD.

$$P\left(z,\alpha_s(\mu^2)\right) = P^{(0)}(z) + \frac{\alpha_s(\mu^2)}{2\pi} P^{(1)}(z) + \ldots$$

- The splitting function $P_{ab}(z)$ can be interpreted as the *probability* to find a parton of type *a* in a parton of type *b* with a fraction *z* of its longitudinal momentum and transverse momentum < μ, per unit log k_T

DGLAP -

- We really have a matrix problem ($2n_f+1$ dimensional)

$$\mu^2 \frac{\partial}{\partial \mu^2}\begin{pmatrix} q_i\left(x,\mu^2\right) \\ g\left(x,\mu^2\right)\end{pmatrix} = \frac{\alpha_s\left(\mu^2\right)}{2\pi}\sum_{q_j,\bar{q}_j}\int_x^1 \frac{d\xi}{\xi}\times$$

$$\begin{pmatrix} P_{q_iq_j}\left(\frac{x}{\xi},\alpha_s\left(\mu^2\right)\right) & P_{q_ig}\left(\frac{x}{\xi},\alpha_s\left(\mu^2\right)\right) \\ P_{gq_j}\left(\frac{x}{\xi},\alpha_s\left(\mu^2\right)\right) & P_{gg}\left(\frac{x}{\xi},\alpha_s\left(\mu^2\right)\right)\end{pmatrix}\begin{pmatrix} q_j\left(\xi,\mu^2\right) \\ g\left(\xi,\mu^2\right)\end{pmatrix}$$

- Luckily, symmetries come to our rescue (charge conjugation, SU(n_f),…) –

$$P_{q_iq_j} = P_{\bar{q}_i\bar{q}_j} \quad P_{q_i\bar{q}_j} = P_{\bar{q}_iq_j} \quad P_{q_ig} = P_{\bar{q}_ig} \equiv P_{qg} \quad P_{gq_i} = P_{g\bar{q}_i} \equiv P_{gq}$$

- QCD is flavor blind and, at leading order, is flavor diagonal

So …

- Quark number and momentum conservation means***

$$\int_0^1 dx P_{qq}^{(0)}(x) = 0 \quad \int_0^1 dx\, x\left[P_{qq}^{(0)}(x) + P_{gq}^{(0)}(x)\right] = 0 \quad \int_0^1 dx\, x\left[2n_f P_{qg}^{(0)}(x) + P_{gg}^{(0)}(x)\right] = 0$$

- In summary (LO)

$$P_{qq}^{(0)}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] = C_F\left(\frac{1+x^2}{1-x}\right)_+$$

$$P_{qg}^{(0)}(x) = T_R\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F\left[\frac{1+(1-x)^2}{x}\right]$$

$$P_{gg}^{(0)}(x) = 2C_A\left[\frac{x}{(1-x)_+} + \frac{(1-x)}{x} + x(1-x)\right] + \delta(1-x)\frac{11C_A - 4n_f T_R}{6}$$

*** Verify these sum rules - see HW

DGLAP & Moments (undo convolution with moment) -

- We can explore the DGLAP equation by taking moments – define

$$f\left(j,\mu^2\right)=\int_0^1 dx\, x^{j-1}\, f\left(x,\mu^2\right),\; f=q_i,g$$

- With inverse (contour *C* parallel to the imaginary axis and to the right of all singularities)

$$f\left(x,\mu^2\right)\equiv\frac{1}{2\pi i}\int_C dj\; x^{-j}\, f\left(j,\mu^2\right)$$

- For a non-singlet quark distribution, $q_{NS}=q-\bar{q}$, with evolution controlled by P_{qq}

$$\mu^2\frac{\partial}{\partial\mu^2}q_{NS}\left(j,\mu^2\right)=\frac{\alpha_s\left(\mu^2\right)}{2\pi}\gamma_{qq}\left(j,\alpha_s\left(\mu^2\right)\right)q_{NS}\left(j,\mu^2\right)$$

$$\gamma_{qq}\left(j,\alpha_s\left(\mu^2\right)\right)=\int_0^1 dx\, x^{j-1}P_{qq}\left(x,\alpha_s\left(\mu^2\right)\right)$$

Anomalous dimensions -

- In leading order (1-loop – no μ dependence in *P*) the solution is

$$q_{NS}\left(j,\mu^2\right)=\left(\frac{\ln\left(\mu^2/\Lambda_{QCD}^2\right)}{\ln\left(\mu_0^2/\Lambda_{QCD}^2\right)}\right)^{\frac{2\gamma_{qq}^{(0)}(j)}{\beta_0}} q_{NS}\left(j,\mu_0^2\right)$$

This behavior is characteristic of gauge theories where $\gamma_{qq}^{(0)}\left(j\right)$ is often called the "anomalous dimension" for the j[th] moment.

- ASIDE: If this were a theory with a fixed (not running) coupling, $\alpha_s(\mu)\rightarrow\alpha_0$ we would find

$$q_0\left(j,\mu^2\right)=q_0\left(j,\mu_0^2\right)\left(\frac{\mu^2}{\mu_0^2}\right)^{\alpha_0\gamma_{qq}(j)/2\pi}$$

which makes the label anomalous dimension more clear. In such a theory the evolution is very fast and hard partons are very unlikely!

➢ The falling PDFs ensure that physics happens at the minimum value of $\hat{s}=x_1x_2s$

Singlet Distribution -

- In a similar way we can study the moments of the singlet distribution $\Sigma\left(x,\mu^2\right) \equiv \sum_i \left[q_i\left(x,\mu^2\right) + \bar{q}_i\left(x,\mu^2\right)\right]$,which mixes with the gluon
- Its moments obey a vector/matrix equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \Sigma\left(j,\mu^2\right) \\ g\left(j,\mu^2\right) \end{pmatrix} = \frac{\alpha_s\left(\mu^2\right)}{2\pi} \times$$

$$\begin{pmatrix} \gamma_{qq}\left(j,\alpha_s\left(\mu^2\right)\right) & 2n_f \gamma_{qg}\left(j,\alpha_s\left(\mu^2\right)\right) \\ \gamma_{gq}\left(j,\alpha_s\left(\mu^2\right)\right) & \gamma_{gg}\left(j,\alpha_s\left(\mu^2\right)\right) \end{pmatrix} \begin{pmatrix} \Sigma\left(j,\mu^2\right) \\ g\left(j,\mu^2\right) \end{pmatrix}$$

Anomalous dimensions II -

- The explicit (1-loop) anomalous dimensions are***

$$\gamma_{qq}^{(0)}(j) = C_F\left[-\frac{1}{2}+\frac{1}{j(j+1)}-2\sum_{k=2}^{j}\frac{1}{k}\right]$$

$$\gamma_{qg}^{(0)}(j) = T_R\left[\frac{2+j+j^2}{j(j+1)(j+2)}\right]$$

$$\gamma_{gq}^{(0)}(j) = C_F\left[\frac{2+j+j^2}{j(j^2-1)}\right]$$

$$\gamma_{gg}^{(0)}(j) = 2C_A\left[-\frac{1}{12}+\frac{1}{j(j-1)}+\frac{1}{(j+1)(j+2)}-\sum_{k=2}^{j}\frac{1}{k}\right]-\left(\frac{2}{3}\right)n_f T_R$$

- The moments can be inverted with the inverse Mellin transformation (at least numerically).

*** See HW

Quark Singlet + Gluon System

- For the singlet plus gluon system we must find the corresponding eigenvalues and eigenvectors. With j=2, the momentum sum integral, we have

Eigenvector	Eigenvalue
$\Sigma(2,\mu^2)+g(2,\mu^2)$	0
$\Sigma(2,\mu^2)-(n_f/4C_F)\, g(2,\mu^2)$	$-4C_F/3 - n_f/3$

- The first line confirms that total momentum is conserved during evolution!!

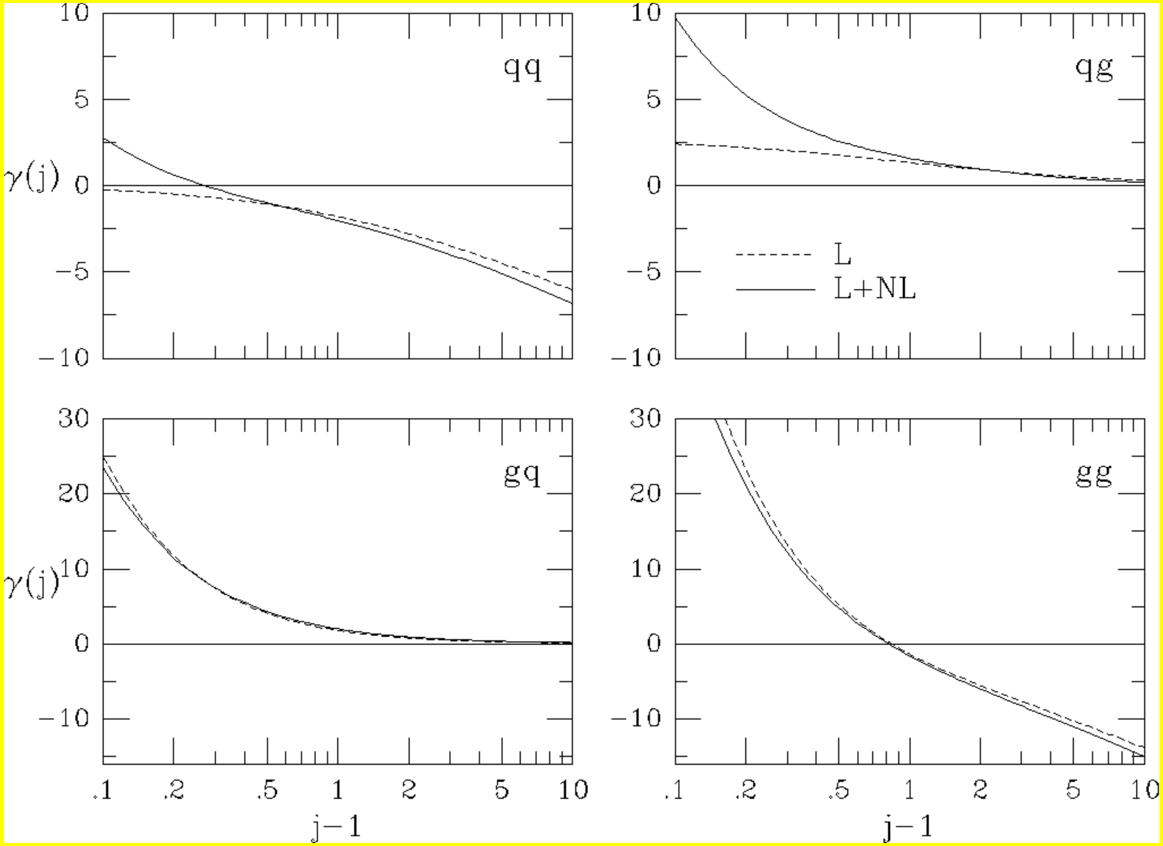
- Since the second eigenvalue is < 0, the second eigenvector vanishes asymptotically (ln $\mu \rightarrow$ infinity)

$$\Sigma\left(2,\mu^2\right)-\frac{n_f}{4C_F}g\left(2,\mu^2\right)\xrightarrow[\ln\mu^2\to\infty]{}0\Rightarrow\frac{\Sigma\left(2,\mu^2\right)}{g\left(2,\mu^2\right)}\xrightarrow[\ln\mu^2\to\infty]{}\frac{n_f}{4C_F}=\frac{3n_f}{16}$$

- Hence the (truly!) asymptotic momentum ratios (at leading order) are

$$f_q = \frac{3n_f}{16+3n_f} \xrightarrow[n_f=6]{} 53\% \qquad f_g = \frac{16}{16+3n_f} \xrightarrow[n_f=6]{} 47\%$$

- Numerically the anomalous dimensions look like –



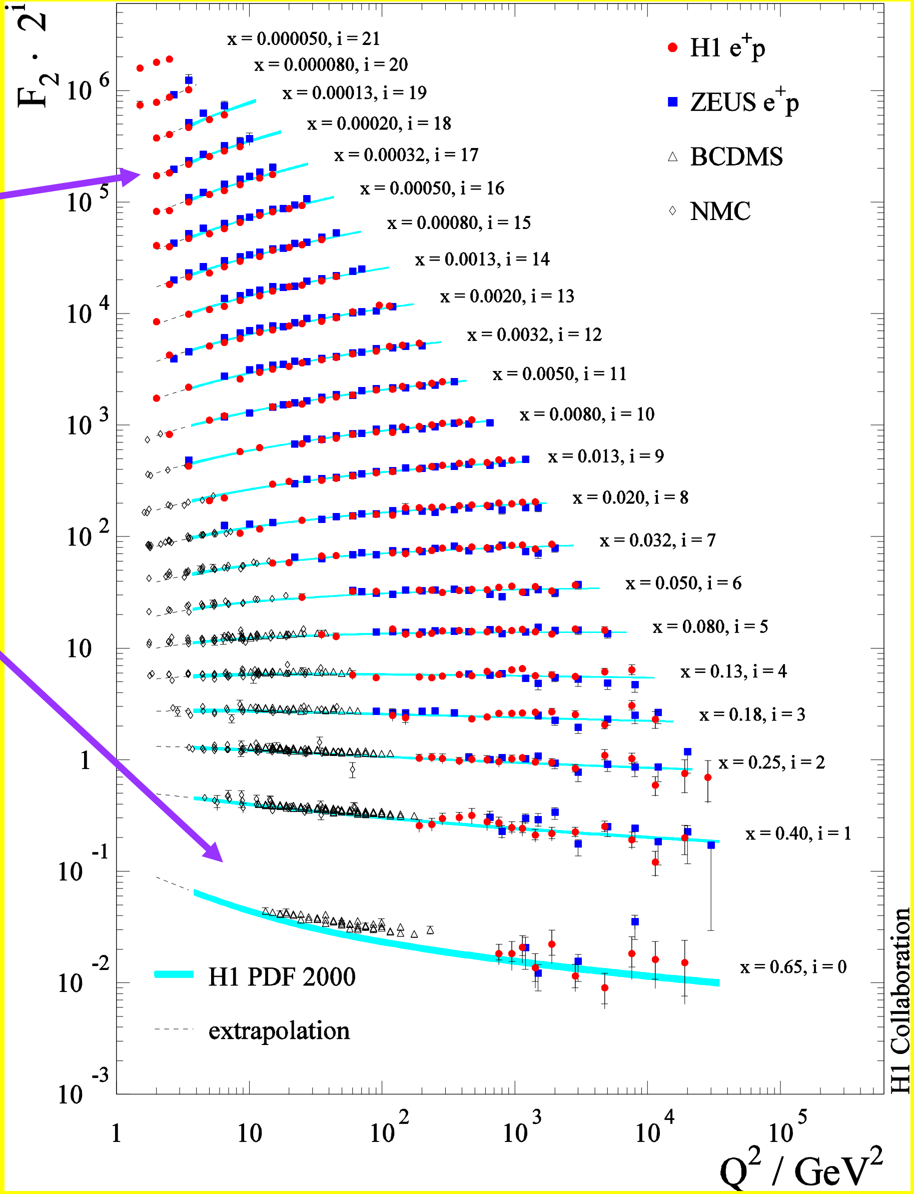
Conclude -

- We expect that the distributions

  increase at small x

  decrease at large x

  as μ = Q increases, and we see this experimentally.

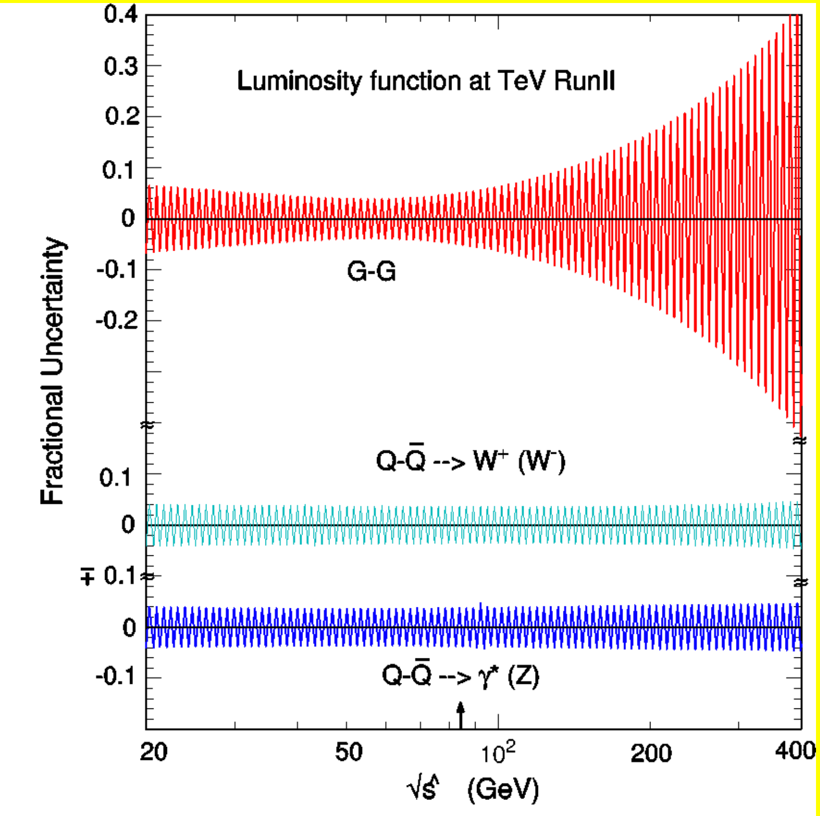


Global Fits -

- pQCD (as will we see) allows us to describe a broad range of experiments in terms of PDFs
- Determine PDFs from GLOBAL fits to a range of data, now including “propagation” of uncertainties in data with range of fits
- CTEQ – http://www.phys.psu.edu/~cteq
- MRST - http://durpdg.dur.ac.uk/HEPDATA/HEPDATA.html
- See also - http://hepforge.cedar.ac.uk/

Current Status

CTEQ



Measures of parton luminosity uncertainties

$$L = \int dx_1 dx_2\, g(x_1,\mu) g(x_2,\mu)\, \delta\left(x_1 x_2 - \hat{s}/s\right)$$

Where $\sqrt{s}$ is the total hadronic energy, and $\sqrt{\hat{s}}$ is the total partonic, hard scattering energy

$\Rightarrow$ Uncertainties < 10% except for large *x* gluons (just where we need them!)

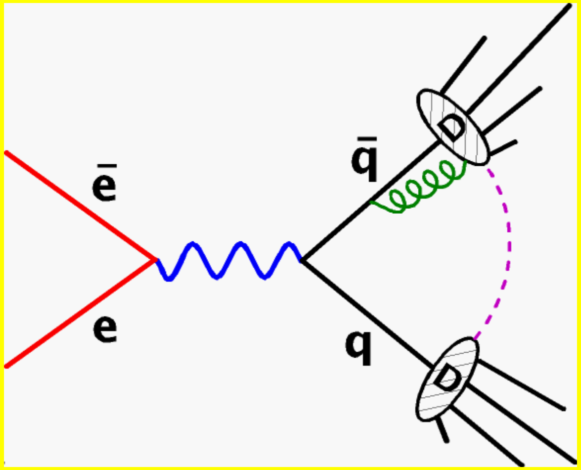
pQCD Calculation: Fragmentation, Hadronization and Jets

- Revisit $e^+e^- \rightarrow$ hadrons – (but note the color connection **- - - -)**

Expect:

$\Rightarrow$ collinear singularities just as for the distribution functions

$\Rightarrow$ Fragmentation functions (recall Lecture 1) acquire μ dependence similarly to the parton distribution functions



Consider first the distribution of hadrons *h* as a function of the fraction of the total energy

$$F^h\left(x,Q^2\right)=\frac{1}{\sigma_{TOT}}\frac{d\sigma}{dx}\left(e^+e^-\rightarrow h+X\right);\, x=\frac{2E_h}{Q}$$

with

$$\left\langle n_h\left(Q^2\right)\right\rangle=\int_0^1 dx\,F^h\left(x,Q^2\right),\quad \sum_h\int_0^1 dx\,x\,F^h\left(x,Q^2\right)=2$$

Fragmentation -

- Naively, this distribution arises from a sum over the contributions from the various primary partons, produced at the short distance scale Q, fragmenting $i \rightarrow h$ (the indices on the fragmentation function D), and described by

$$F^h\left(x,Q^2\right)=\sum_i\int_x^1\frac{dz}{z}C_i\left(Q^2,z,\alpha_s\right)D_i^h\left(\frac{x}{z},Q^2\right)$$

- The (IRS) coefficient function C describes the short distance production of the primary partons

$$C_q \sim e_q^2\left[\delta(1-z)+O\left(\alpha_s\right)+\ldots\right]$$

assuming only photon exchange (no Z's). Gluons will only start to contribute at order α_S.

Fragmentation II -

- The function D^h is *not* calculable perturbatively (hadron formation is intrinsically long distance and non-perturbative).

- The fragmentation (or time-like evolution) of parton $i \to$ parton j is treatable perturbatively. The collinear divergences are factorizable (just as for the parton distribution functions)

$$D_i^j\left(x,\mu^2\right)=\sum_k \int_x^1 dz\, K_i^k\left(z,\mu^2,\mu_0^2\right) D_k^j\left(\frac{x}{z},\mu_0^2\right)$$

where K is calculable (for μ^2 and μ_0^2 large).

Note - Having once factored the collinear singularities into the regularized D, there is no problem doing the same with confinement and setting $j \to h$ again.

Warning!

- This picture suggests that in $e^+e^- \rightarrow$ hadrons each hadron is associated with one specific initial parton
- Thus each hadron is associated with a unique jet
- Analysis of the data has often proceeded with this in mind.

BUT – it ain't so! The soft hadrons (at least) must be associated with the coherent interactions of color singlet *combinations* of partons (the color connection – strings?); the UE for hadron-hadron collisions. Factorization breaks down for the soft hadrons.

Fragmentation III -

In summary - pQCD tells us that

- the regularized fragmentation functions evolve
- the form of the evolution is calculable

$$\mu^2 \frac{\partial}{\partial \mu^2} D_i\left(x,\mu^2\right) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s\left(\mu^2\right)}{2\pi} D_j\left(\frac{x}{z},\mu^2\right) P_{ji}\left(z,\alpha_s\left(\mu^2\right)\right)$$

- like DGLAP except for the different order of the indices (*i* is the initial parton and *j* the final).

$$P_{ji}\left(z,\alpha_s\left(\mu^2\right)\right) = P_{ji}^{(0)} + \frac{\alpha_s\left(\mu^2\right)}{2\pi} P_{ji}^{(1)}\left(z\right) + \ldots$$

- The lowest order splitting functions $P^{(0)}$ are identical to those introduced earlier, the higher order ones are not.

Status – Parton Model + pQCD

- Basic structure of parton model remains valid, but distributions no longer scale precisely - there is a dimensionful quantity, Λ_{QCD}
- QCD coupling is small at short distance, large at large distance (as desired to explain the parton model) due to the short distance (UV) structure of the theory, i.e., physics at scales < $1/\mu$
- Can factor the complicated (hard to calculate) long distance, confining behavior from the short distance perturbative behavior at arbitrary factorization scale μ_F
- Determine the long distance behavior experimentally and evolve to desired scale
- Perturbation theory predicts the form of the evolution and the perturbative factors

pQCD Calculation : Hadron – Hadron scattering

- With our tools in hand we can attack any process that provides a calculable short distance interaction, with the long distance complexity factored into the parton distribution and fragmentation functions.

- Of course, since they are not predicted by QCD, it is best to avoid them – and we can avoid the fragmentation function $\rightarrow$ JETS.

Examples w/o Fragmentation:

$pp \rightarrow \gamma + X$ at large p_T, $\Lambda_{QCD} \ll p_T$

$pp \rightarrow \gamma^*, W, Z, h + X$ $\Lambda_{QCD} \ll$ M, Q

$pp \rightarrow jet + X$ at large p_T with an appropriate (IRS) jet definition (to sum over productions of fragmentation)

Hadron – Hadron scattering II

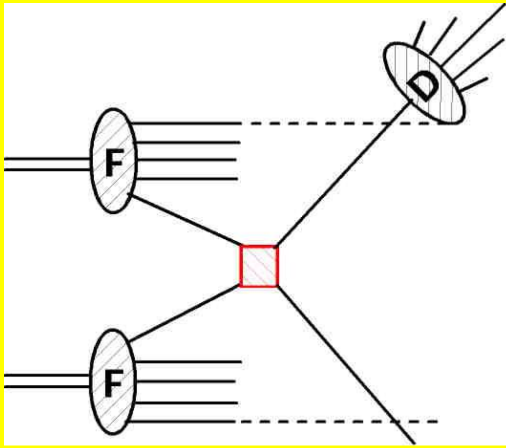
- The jet cross section receives contributions from a vast number of channels, even at LO

$$gg \to gg;\ qq \to qq;\ gg \to q\bar{q};\ q\bar{q} \to q\bar{q};\ gq \to gq;\ etc.$$

- At NLO the bookkeeping issue is even more demanding. (Software on web)

- For inclusive single hadron production, e.g., pp → π + X, we obtain a triple (factorized) convolution (both initial state and final state collinear issues, renormalization and factorization scales).

$$\frac{d\sigma_{pp}^{\pi}}{d\eta dp_T^2} = \iiint F_{a/p}\left(x_a,\mu_F\right) \otimes F_{b/p}\left(x_b,\mu_F\right)$$
$$\otimes\, \hat{\sigma}_{ab\to c}\left(p_T, \frac{s}{p_T^2}, x_1, x_2, z, \frac{p_T}{\mu_F}, \frac{p_T}{\mu}\right) \otimes D_{\pi/c}\left(z,\mu_F\right)$$
$$\times \left\{1 + \mathcal{O}\left(\frac{m^2}{p_T^2}\right)\right\}$$



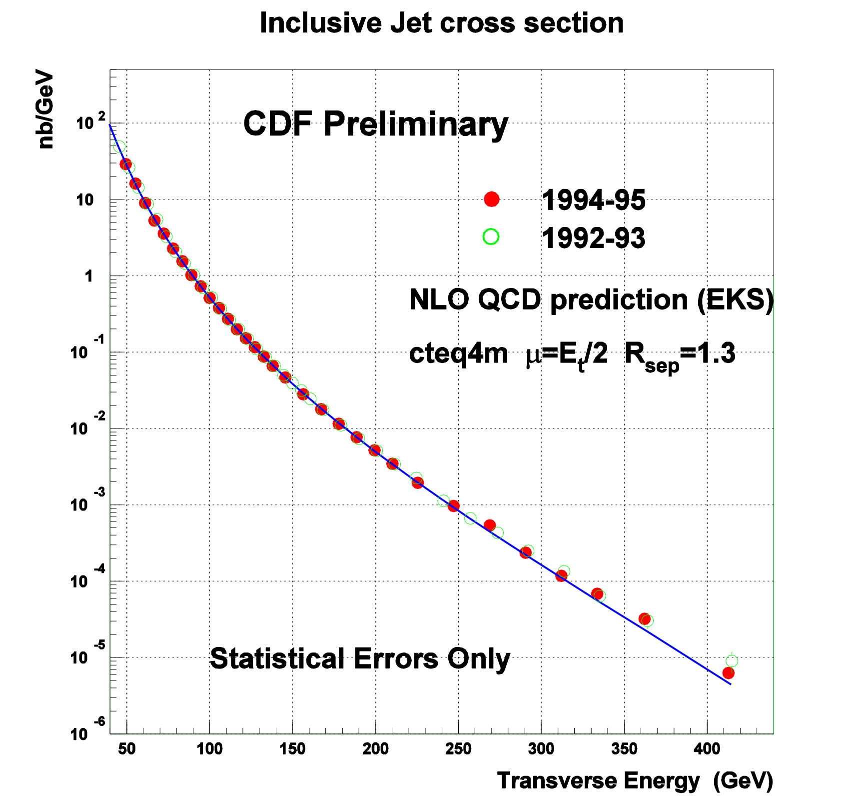
- Sum over fragmentation products
  $\Rightarrow$ JET (set $\mu = \mu_F = E_T/2$)

$$\frac{d\sigma_{pp}^{jet}}{d\eta dp_T^2} = \iiint F_{a/p}\left(x_a,\mu_F\right) \otimes F_{b/p}\left(x_b,\mu_F\right)$$

$$\otimes\, \hat{\sigma}_{ab\to jet}\left(p_T, \frac{s}{p_T^2}, x_1, x_2, z, \frac{p_T}{\mu_F}, \frac{p_T}{\mu}\right)$$

$$\times\left\{1+\mathcal{O}\left(\frac{m^2}{p_T^2}\right)\right\}$$

- Run I Cone jet data - CDF – compared to NLO, note the HUGE dynamic range



Kinematics – jets at hadron colliders

- transverse momentum or scalar transverse energy $E_T = E\sin\theta$
  for a single particle or narrow jet $\left|\vec{P}_T\right| \approx E_T$

- pseudorapidity $\eta_J = -\ln\left(\tan\left(\theta_J/2\right)\right)$

  or true rapidity $y_J = 0.5\ln\left(\left(E_J + P_{z,J}\right)/\left(E_J - P_{z,J}\right)\right)$

  where

  $$E_J = \sqrt{M_J^2 + P_{T,J}^2}\cosh y_J \quad P_{z,J} = \sqrt{M_J^2 + P_{T,J}^2}\sinh y_J$$

  $$\eta \cong y, E_J >> M_J \; ^*$$

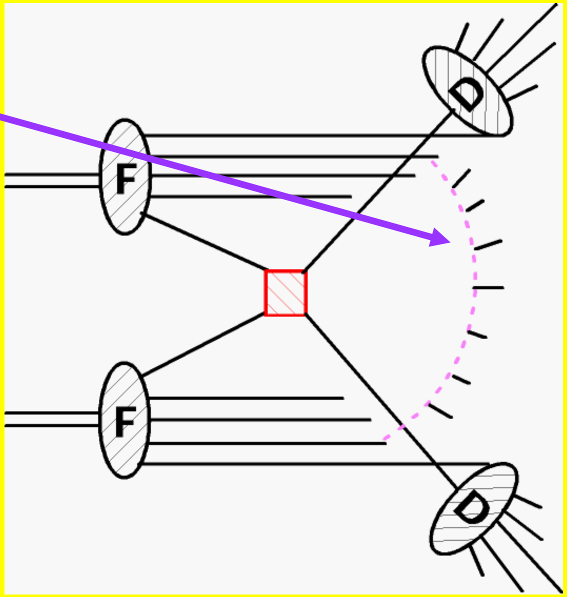
- Without detailed information on masses, *etc*., η has been the variable of choice as it requires only an angle measurement.

***You should verify this limiting result**

Hadron – Hadron scattering III – real life

- In a typical hadron-hadron collision (minimum bias event) final state particles are fairly uniformly distributed in η (an original motivation for the "wee partons" with a *dx/x* ~ dη distribution).

- Even in an event with a "hard" interaction the soft interactions of the spectator partons $\Rightarrow$ Underlying Event ~ Min-Bias event, which can contribute to a jet (Splash In) – not included in pQCD



Evolution In Words:

- Initial long distance – color singlet coherent eigenstates – resolved into colored partons - described by factored PDF (leaving ISR)

- Short distance ($\ll$ 1 fermi) – pQCD (IRS) parton scattering

- Intermediate distances - “Bare” color charges shower (~collinear, final state radiation) simulated in MC, described by Sudakov (double logs)

  Allow showering from exposed remnant colored charges (~ collinear with beam direction, initial state radiation = ISR) simulated in MC (more Sudakov)

  Allow multiple parton-parton interactions to simulate UE in MC

- “long” distance (~ 1 fermi) - associate color singlet sets of partons into hadrons (hadronization)

Dictionary of Hadron Collider Terminology

EVENT

HADRON-HADRON COLLISION

Primary (Hard) Parton-Parton Scattering

Initial-State Radiation (ISR) = Spacelike Showers associated with Hard Scattering

Underlying Event

Multiple Parton-Parton Interactions: Additional parton-parton collisions (in principle with showers etc) in the same hadron-hadron collision.

= Multiple Perturbative Interactions (MPI)

= Spectator Interactions

Fragmentation

Perturbative:

Final-State Radiation (FSR)

= Timelike Showers

= Jet Broadening and Hard Final-State Bremsstrahlung

Non-perturbative:

String / Cluster Hadronization

(Color Reconnections?)

Beam Remnants: Left over hadron remnants from the incoming beams. Colored and hence correlated with the rest of the event →

PILE-UP: Additional hadron-hadron collisions recorded as part of the same event.

From Peter Skands

Defining a Jet -

Use a jet **algorithm** - Based on some measure of the localization of the expected collinear spray of particles

- Start with a list of particles (4-vectors) and/or calorimeter towers (energies and angles)
- End with lists of particles/towers, one list for each jet $\Rightarrow$ Provide an "accurate" measure of kinematics (4-vector) of underlying parton
- And a list of particles/towers not in any jet – the spectators – remnants of the initial hadrons not involved in the short distance physics

Think of the algorithm as a "microscope" for seeing the (colorful) underlying structure -

Defining a Jet II -

Goals of IDEAL ALGORITHM (motherhood)

- *Fully Specified:* including defining in detail any preclustering, merging, and splitting issues
- *Theoretically Well Behaved:* the algorithm should be infrared and collinear safe (and insensitive) with no *ad hoc* clustering parameters (*e.g.*, R_{SEP})
- *Detector Independent*: there should be no dependence on cell type, numbers, or size
- *Order Independent:* The algorithms should behave equally at the parton, particle, and detector levels.
- *Uniformity*: everyone (theory and experiment) uses the same algorithms (to the best possible approximation)

Defining a Jet III -

Theory –

- Boost invariant results – use variables with appropriate boost properties
- Kinematic boundary stability – use variables with appropriate energy conservation to allow resummation calculations

Experiment –

- Minimize resolution smearing and angle biases
- Stability with luminosity – not sensitive to multiple collisions
- Efficient use of computer resources – but do not let this drive problems with physics issues (e.g., seeds and preclustering)

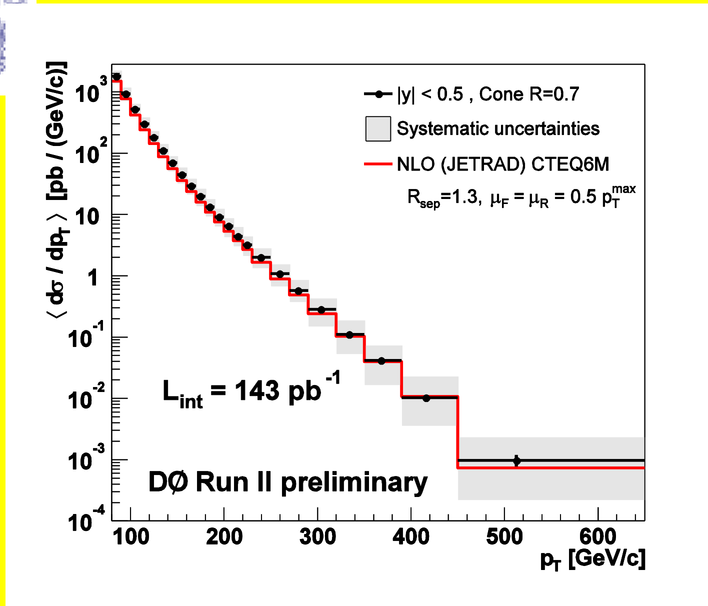
- Easy to calibrate – not so worried about size of corrections as with accuracy of corrections
- Easy to use!

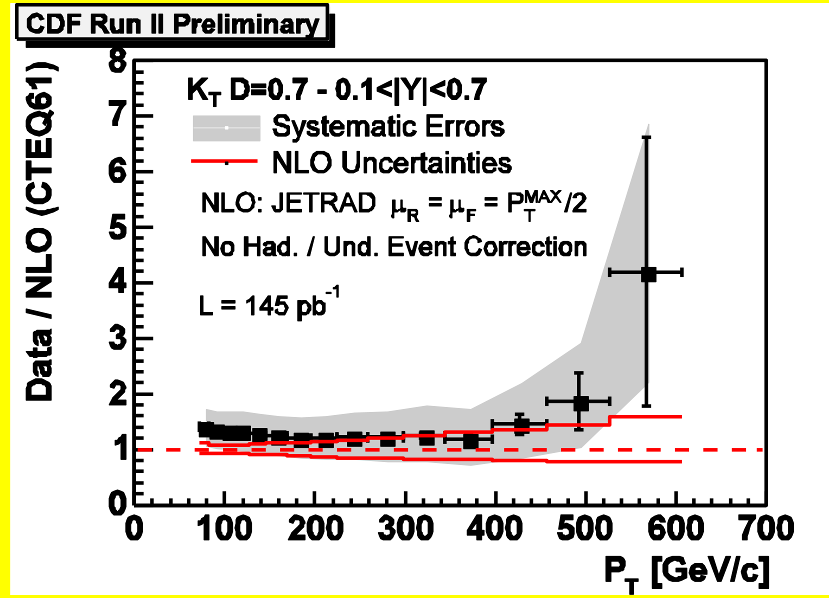
Defining a Jet IV -

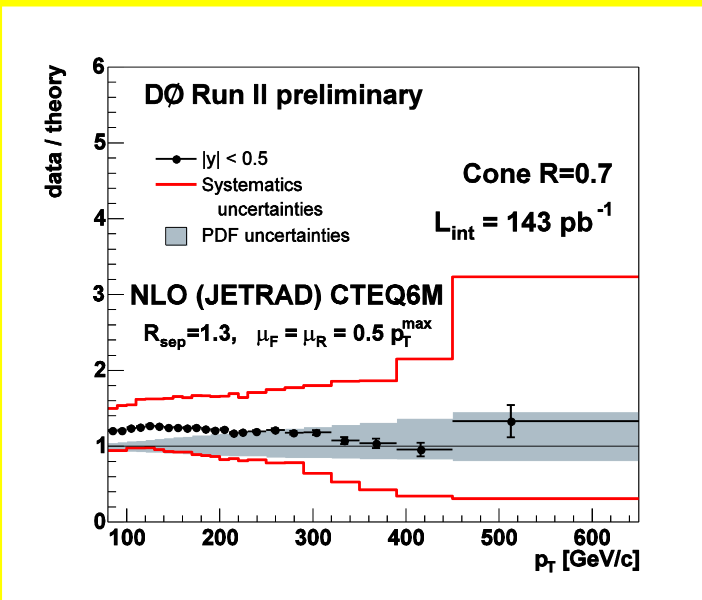
Basic types of Jet Algorithms:

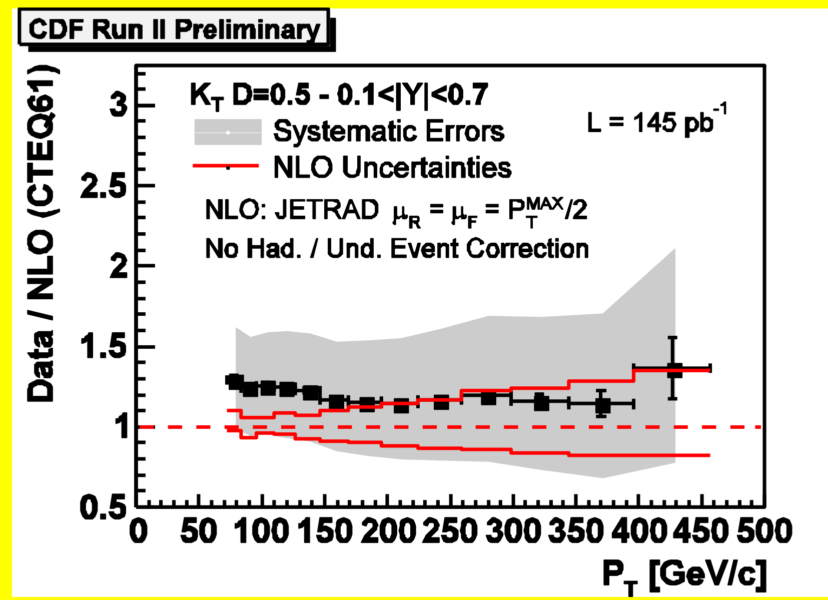
- Fixed Geometric or cone – select particles with momenta nearby in angle (*i.e.*, nearby in the detector) – hadron-hadron
  Simple geometry (in principle), so easy correction for underlying event – Splash-In

- k_T – select particles pair wise nearby in momentum space (small relative transverse moment) – e^+e^-
  Topology event-by-event more complicated – "vacuums-up" underlying event


|y| < 0.5 , Cone R=0.7
Systematic uncertainties
NLO (JETRAD) CTEQ6M
R_{sep}=1.3, $\mu_F = \mu_R = 0.5\, p_T^{max}$
L_{int} = 143 pb^{-1}
DØ Run II preliminary
$\langle d\sigma / dp_T \rangle$ [pb / (GeV/c)]
p_T [GeV/c]


CDF Run II Preliminary
K_T D=0.7 - 0.1<|Y|<0.7
Systematic Errors
NLO Uncertainties
NLO: JETRAD $\mu_R = \mu_F = P_T^{MAX}/2$
No Had. / Und. Event Correction
L = 145 pb^{-1}
Data / NLO (CTEQ61)
P_T [GeV/c]


DØ Run II preliminary
|y| < 0.5
Systematics uncertainties
PDF uncertainties
Cone R=0.7
L_{int} = 143 pb^{-1}
NLO (JETRAD) CTEQ6M
R_{sep}=1.3, $\mu_F = \mu_R = 0.5\, p_T^{max}$
data / theory
p_T [GeV/c]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K_T D=0.5 - 0.1<|Y|<0.7
L = 145 pb^{-1}
Systematic Errors
NLO Uncertainties
NLO: JETRAD $\mu_R = \mu_F = P_T^{MAX}/2$
No Had. / Und. Event Correction
Data / NLO (CTEQ61)
P_T [GeV/c]

In the Beginning (1990) – *Snowmass Cone Algorithm*

- Cone Algorithm – particles, calorimeter towers, partons in cone of size R (single parameter?), defined in angular space, *e.g.*, (η,φ),
- CONE center – (η^C,φ^C)
- CONE i $\in$ *C iff* $\sqrt{\left(\eta^i - \eta^C\right)^2 + \left(\varphi^i - \varphi^C\right)^2} \leq R$
- (Transverse) Energy $E_T^C = \sum_{i \subset C} E_T^i$
- Centroid $\bar{\eta}^C = \sum_{i \subset C} E_T^i * \eta^i / E_T^C \;\; ; \;\; \bar{\varphi}^C = \sum_{i \subset C} E_T^i * \varphi^i / E_T^C$

- Stable cones found by iteration: start with cone anywhere (and, in principle, *everywhere*), calculate the centroid of this cone, put new cone at centroid, iterate until cone stops "flowing", *i.e.*, stable $\Rightarrow$ Proto-jets (prior to split/merge)

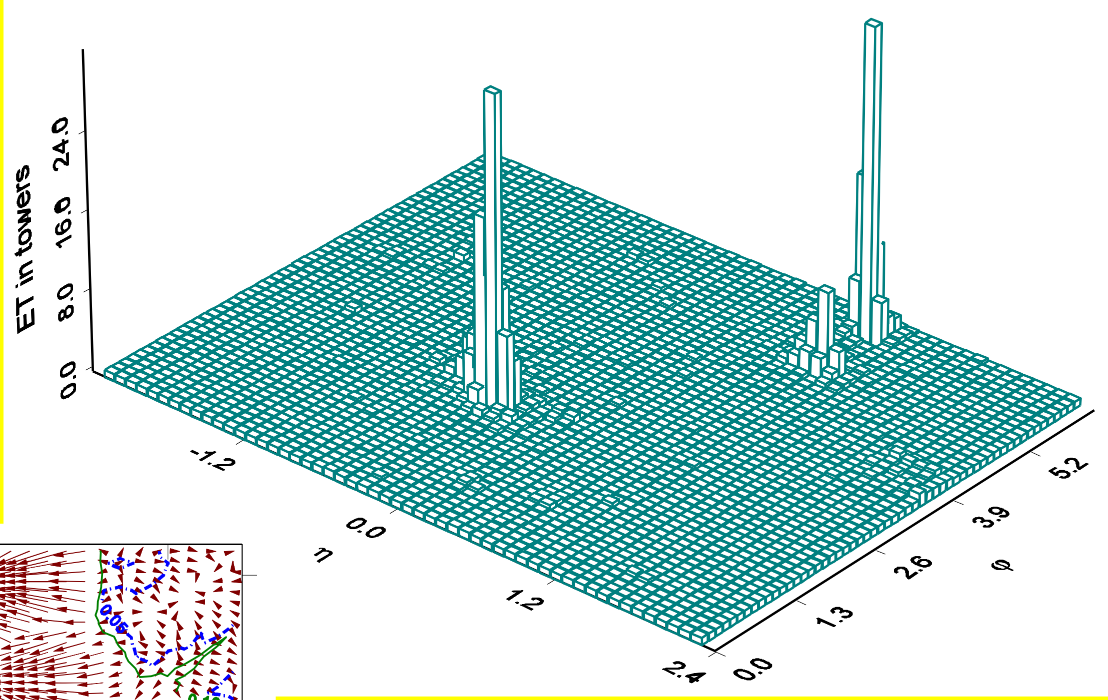
- *"Flow vector"* $\vec{F}^C = \left(\bar{\eta}^C - \eta^C, \bar{\varphi}^C - \varphi^C\right)$

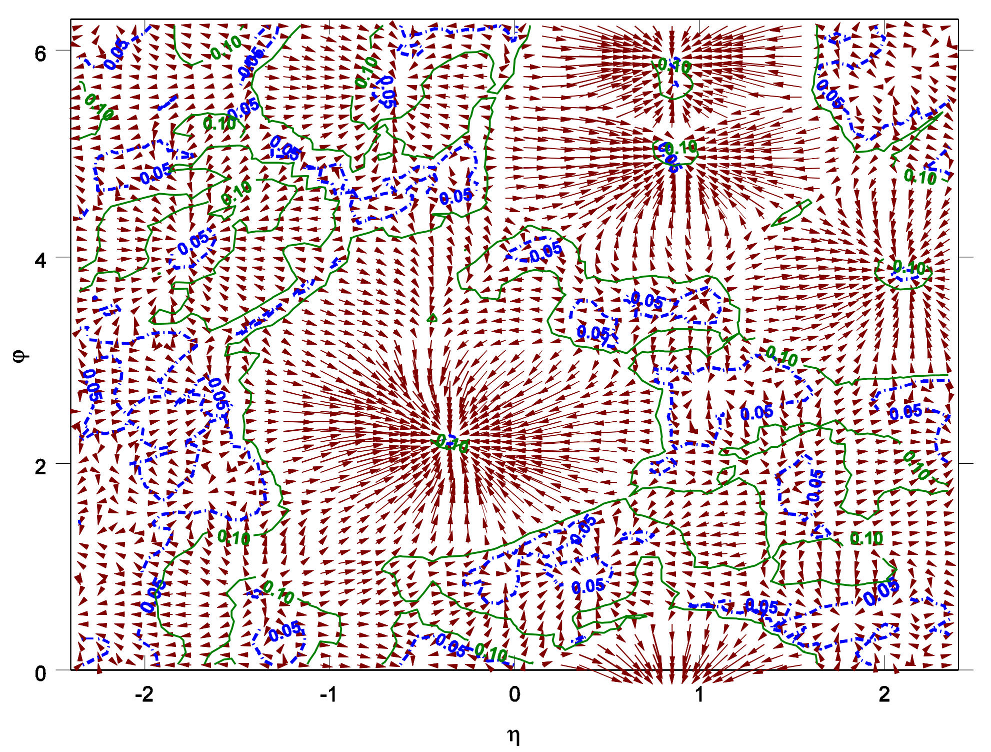
- Jet is defined by "stable" cone

$$\eta^J = \eta^C = \bar{\eta}^C \;\; ; \;\; \varphi^J = \varphi^C = \bar{\varphi}^C \;\; ; \;\; \vec{F}^C = 0$$

$\Rightarrow$ *unique, discrete jets event-by-event* (at least in principle) with a single parameter *R*

Example Lego & Flow





The k_T Algorithm

- Merge partons, particles or towers pair-wise based on “closeness” defined by minimum value of

$$d_{ij}^2 \equiv \mathrm{Min}\left[p_{T,i}^2, p_{T,j}^2 \right] \frac{\left(y_i - y_j\right)^2 + \left(\phi_i - \phi_j\right)^2}{D^2}, d_i^2 = p_{T,i}^2$$

  If d_{ij}^2 is the minimum, merge pair and redo list;
  If d_i^2 is the minimum -> *i* is a jet!
  (no more merging for *i*), 1 parameter *D* (?), [NLO $R = 0.7$, $R_{sep} = 1.3 \Leftrightarrow D = 0.83$]

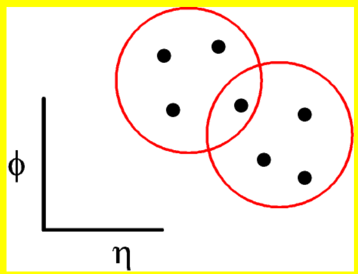
- Jet identification is unique – no merge/split stage (see cone problem below) 👍

- Resulting jets are more amorphous, energy calibration difficult (subtraction for UE?), and analysis can be very computer intensive (time grows like N^3, recalculate list after each merge) 👎
  But new version goes like N ln N (only recalculate nearest neighbors) 👍

Cone Issues:

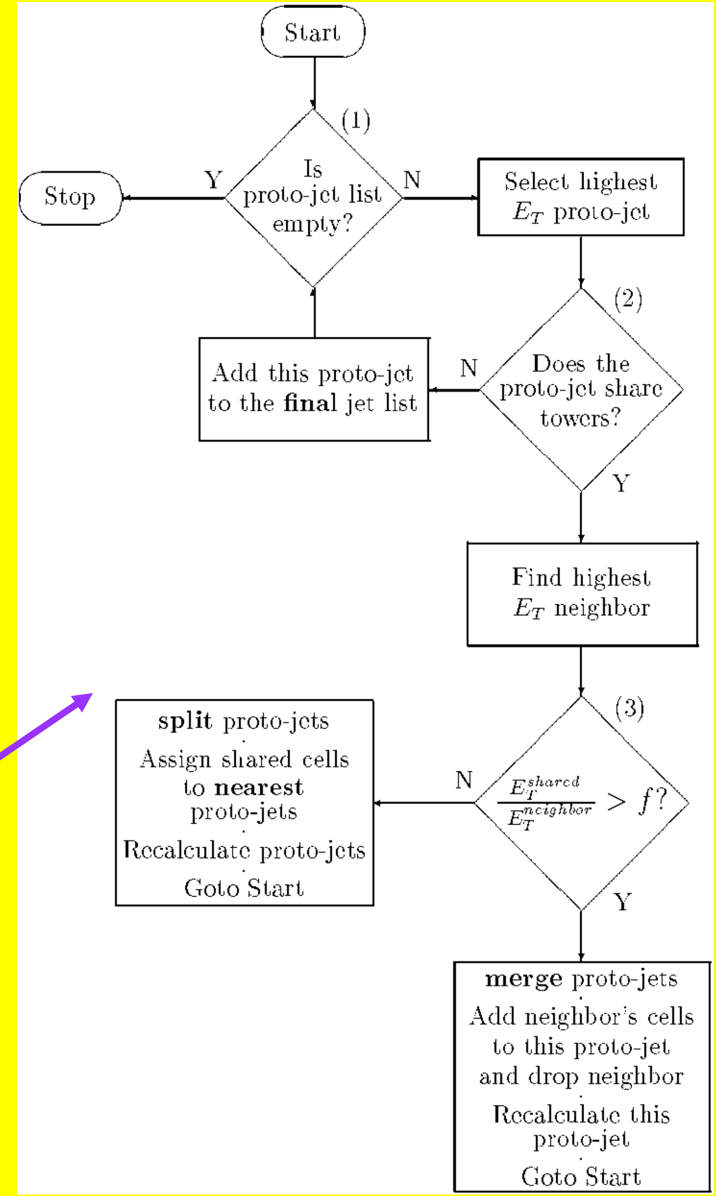
1) Stable Cones can Overlap

- Stable cones can and do overlap! Need rules for merging and splitting (protojet = stable cone)



- Typical split/merge algorithm
  ⇒ New parameter f_{merge}

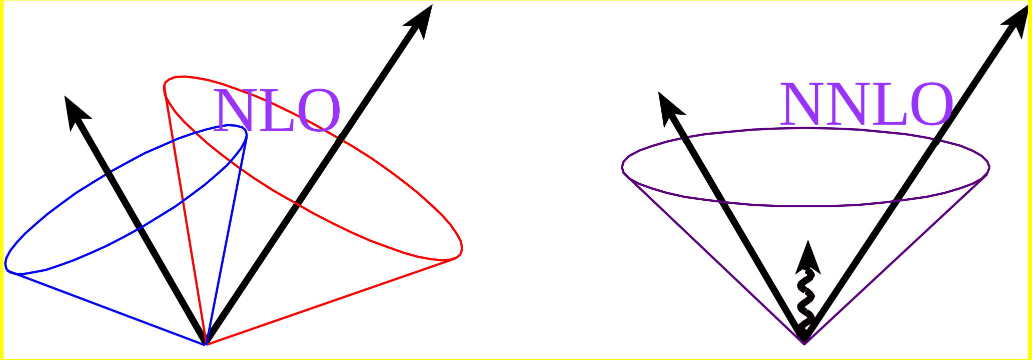
  Not the same for D0 and CDF



Cone Issues:

2) <u>Seeds</u> – experiments only look for jets under brightest street lights, *i.e.*, near very active regions (save computer time)

$\Rightarrow$ problem for theory, IR sensitive (Unsafe?) at NNLO



$\Rightarrow$ Don't find "possible" central jet between two well separated proto-jets (partons)

This is a BIG deal for theory (1 more seed really matters)

– but not a big deal numerically for data (many seeds, ~2%)

To understand this last issue consider Snowmass "Potential"

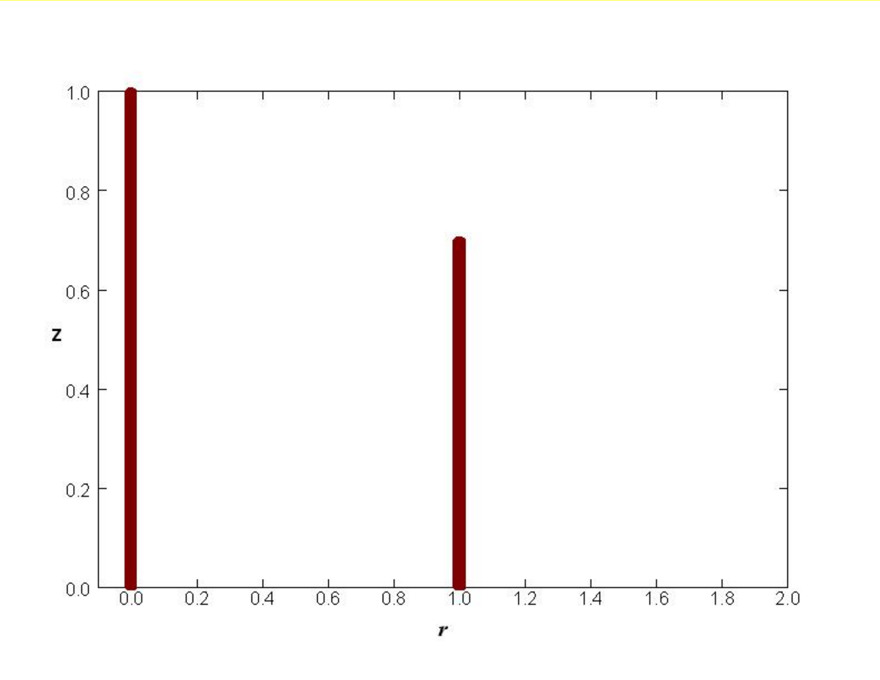
- In terms of 2-D vector $\vec{r}=(\eta,\varphi)$ or (y,φ) define a "potential"

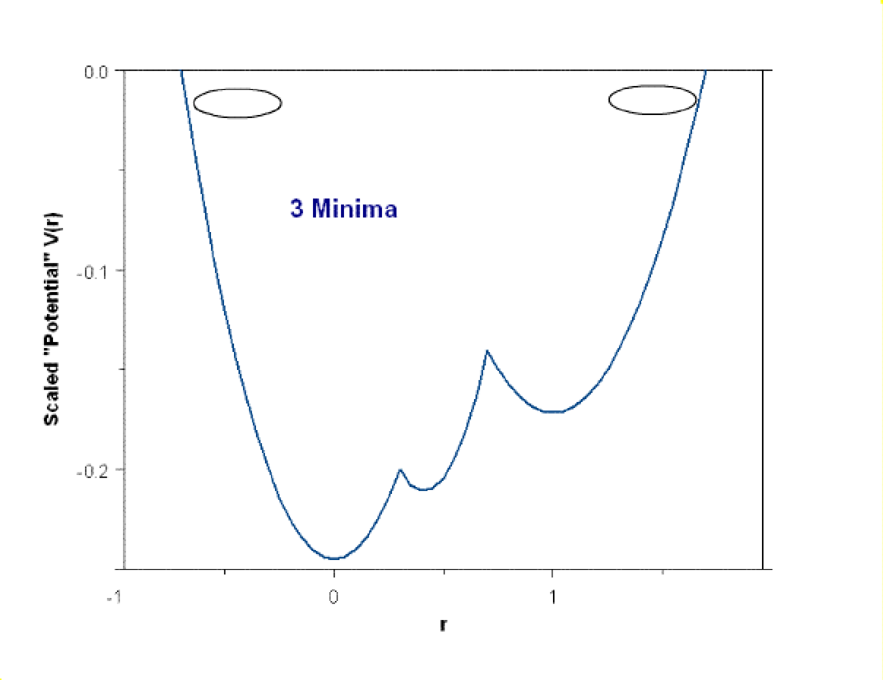
$$V(\vec{r}) \equiv -\frac{1}{2}\sum_i E_T^i \left(R^2-\left(\vec{r}^{\,i}-\vec{r}\right)^2\right)\Theta\left(R^2-\left(\vec{r}^{\,i}-\vec{r}\right)^2\right)$$

- Extrema are the positions of the stable cones; gradient is "force" that pushes trial cone to the stable cone, *i.e.*, the flow vector

$$\vec{F}(\vec{r}) = -\vec{\nabla}V(\vec{r}) = \sum_i E_T^i \left(\vec{r}^{\,i}-\vec{r}\right)\Theta\left(R^2-\left(\vec{r}^{\,i}-\vec{r}\right)^2\right)$$

(THE) Simple Theory Model - 2 partons (separated by < 2R): yield potential with 3 minima – trial cones will migrate to minima from seeds near original partons ⇒ miss central minimum

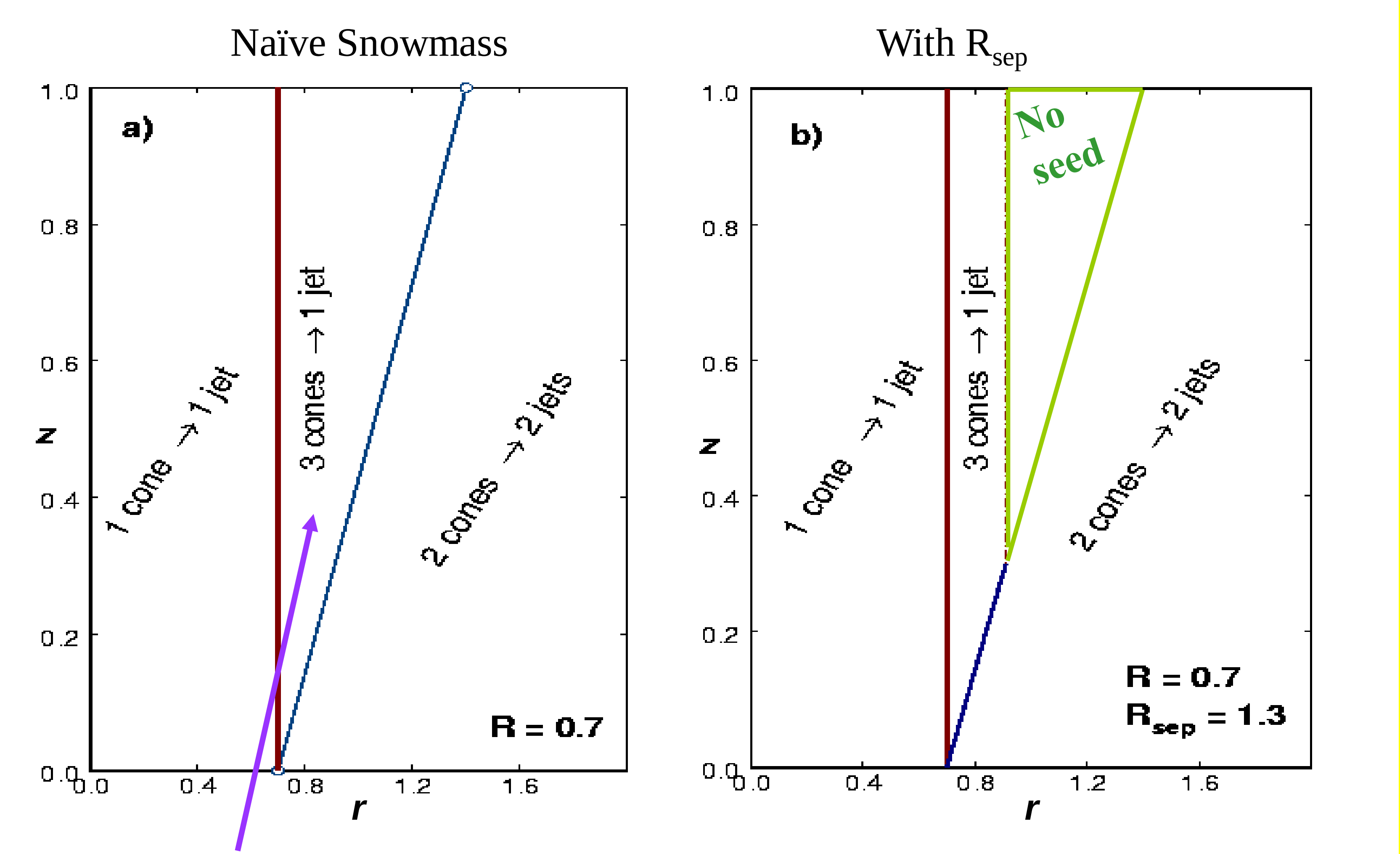




$z = p_{\min} / p_{\max}$, r = separation

Smearing of order R

NLO Perturbation Theory – r = parton separation, $z = p_2/p_1$
Simulate the missed middle cones with R_{sep}



~10% of cross section here

Cone Issues:

3) Kinematic variables:
$E_{T,Snow} \neq E_{T,CDF} \neq E_{T,4D} = p_T$

Different in different experiments and in theory

4) Other details –

- Energy Cut on towers kept in analysis (*e.g.*, to avoid noise)
- (Pre)Clustering to find seeds (and distribute "negative energy")
- Energy Cut on precluster towers
- Energy cut on clusters
- Energy cut on seeds kept

5) Starting with seeds find stable cones by iteration, but in JETCLU (CDF), "once in a seed cone, always in a cone", the "ratchet" effect

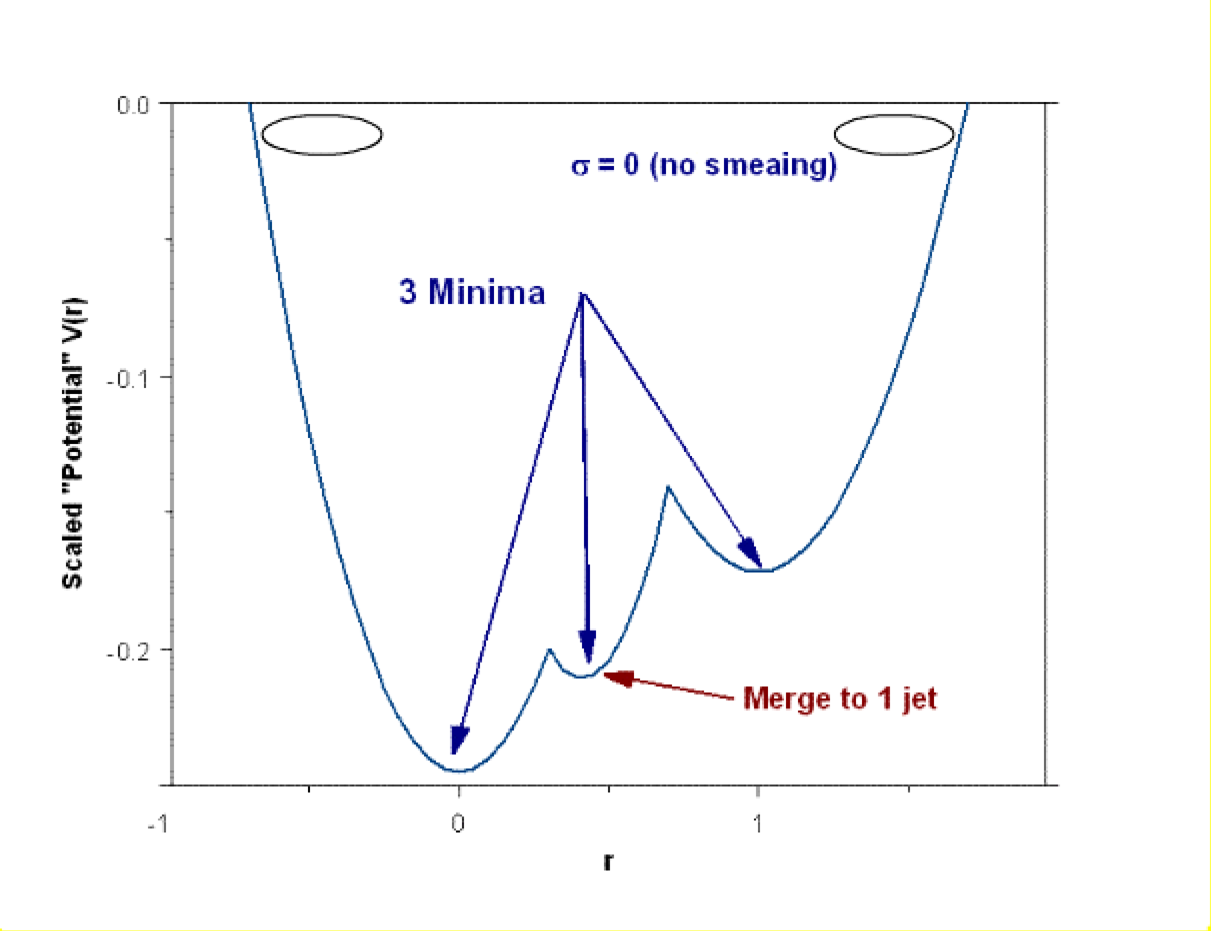
To address these issues, in the Tevatron Run II

Both experiments use

- (legacy) Midpoint Algorithm – always look for stable cone at midpoint between found cones
- Seedless Algorithm (i.e., put seeds on regular grid, not well studied)
- k_T Algorithms
- Use identical versions except for issues required by physical differences (in preclustering??)
- Use (4-vector) E-scheme variables for jet ID and recombination (y instead η, p_T instead of E_T)

Likely similar at the LHC

Consider the corresponding “potential” with 3 minima, expect via MidPoint or Seedless to find middle stable cone



MidPoint Fix -

- Find stable merged solution, if present (see next)
- Remove seed-driven issue of IRS (sensitivity to seed p_T threshold) at NNLO (but not NNNLO)

A NEW issue for Midpoint & Seedless Cone Algorithms – DARK TOWERS

- Compare jets found by JETCLU (with ratcheting) to those found by MidPoint and Seedless Algorithms

- "Missed Energy" – when energy is smeared by showering/hadronization do not always find stable cones expected from perturbation theory

  $\Rightarrow$ 2 partons in 1 cone solutions
  $\Rightarrow$ or even second cone

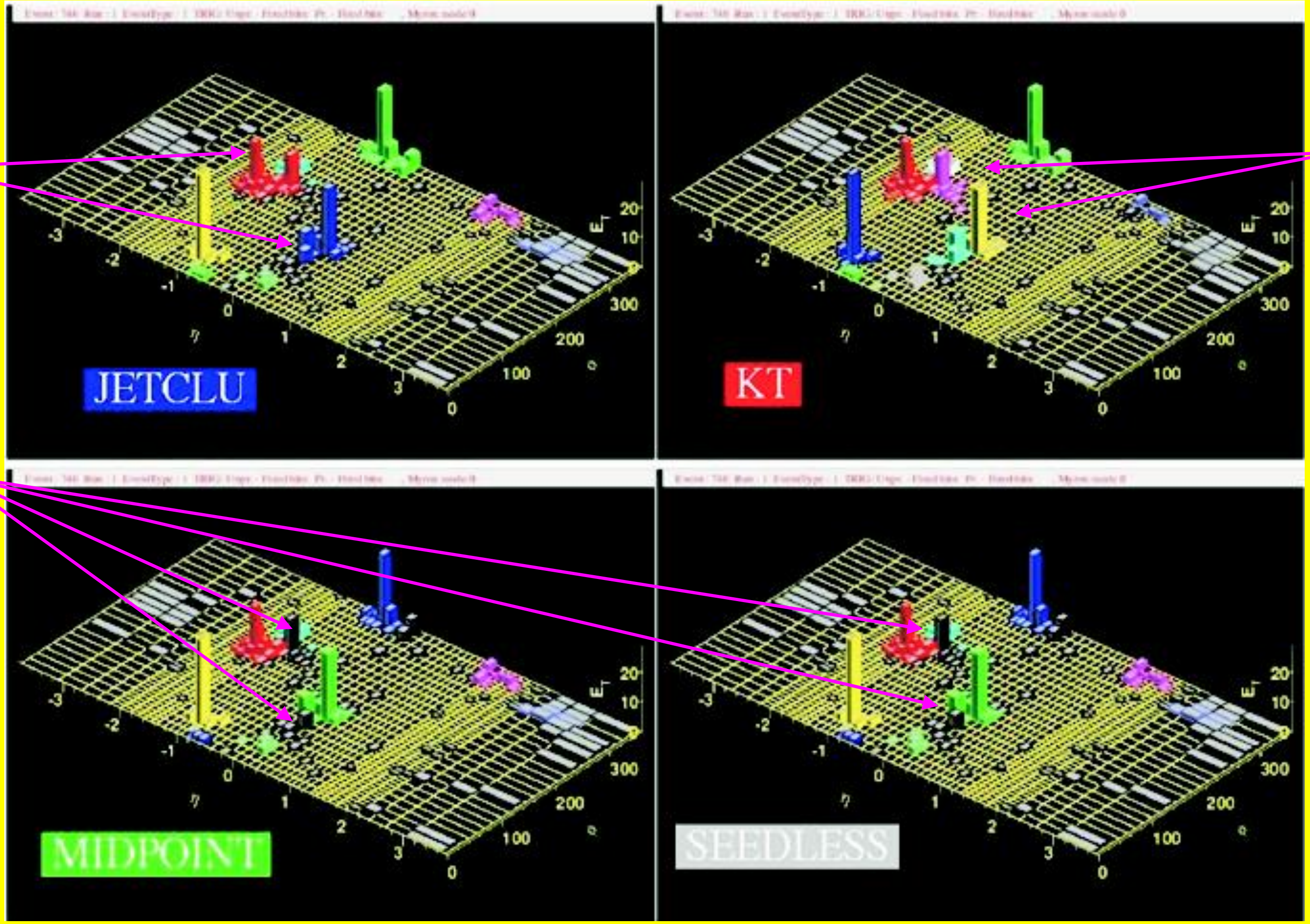
  Under-estimate E_T – new kind of Splashout

Current situation for Algorithms at the Tevatron

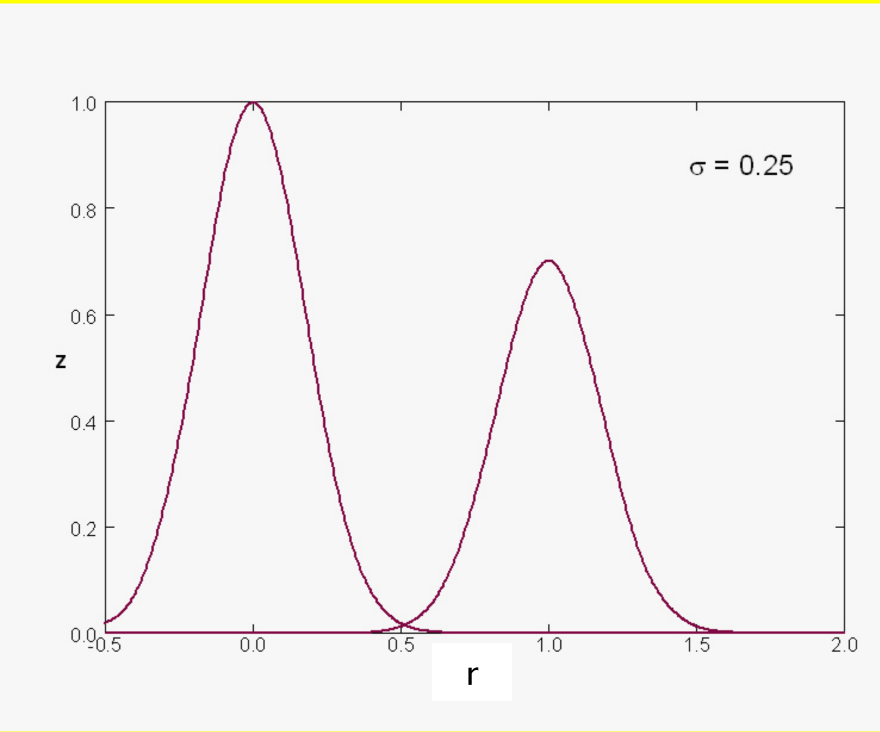
Merged jets

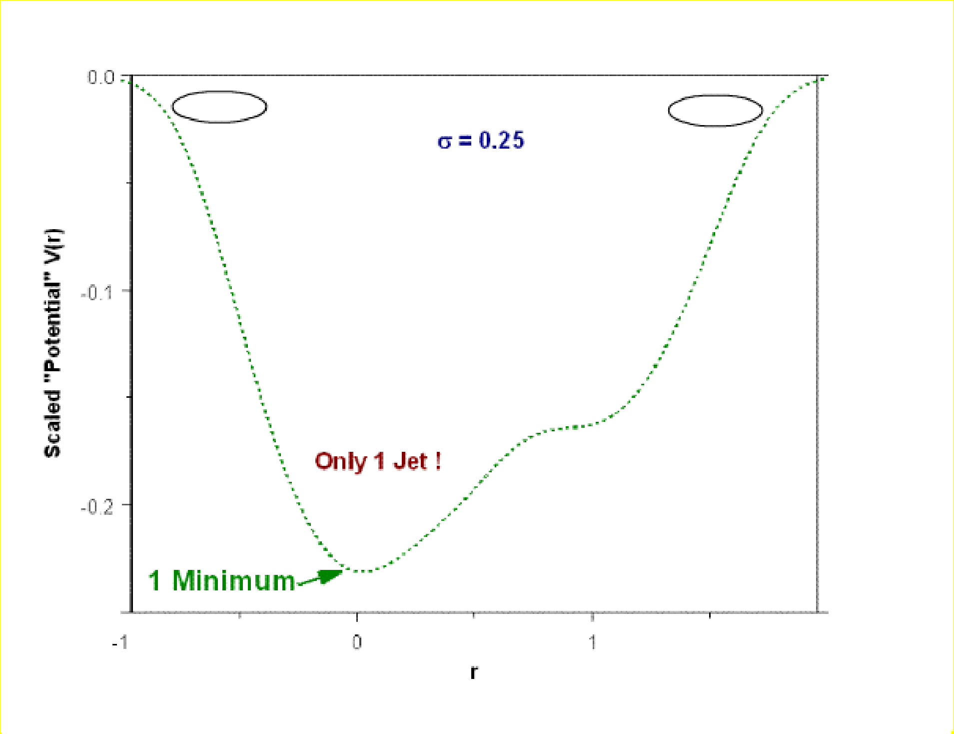
UN Merged jets

Dark towers

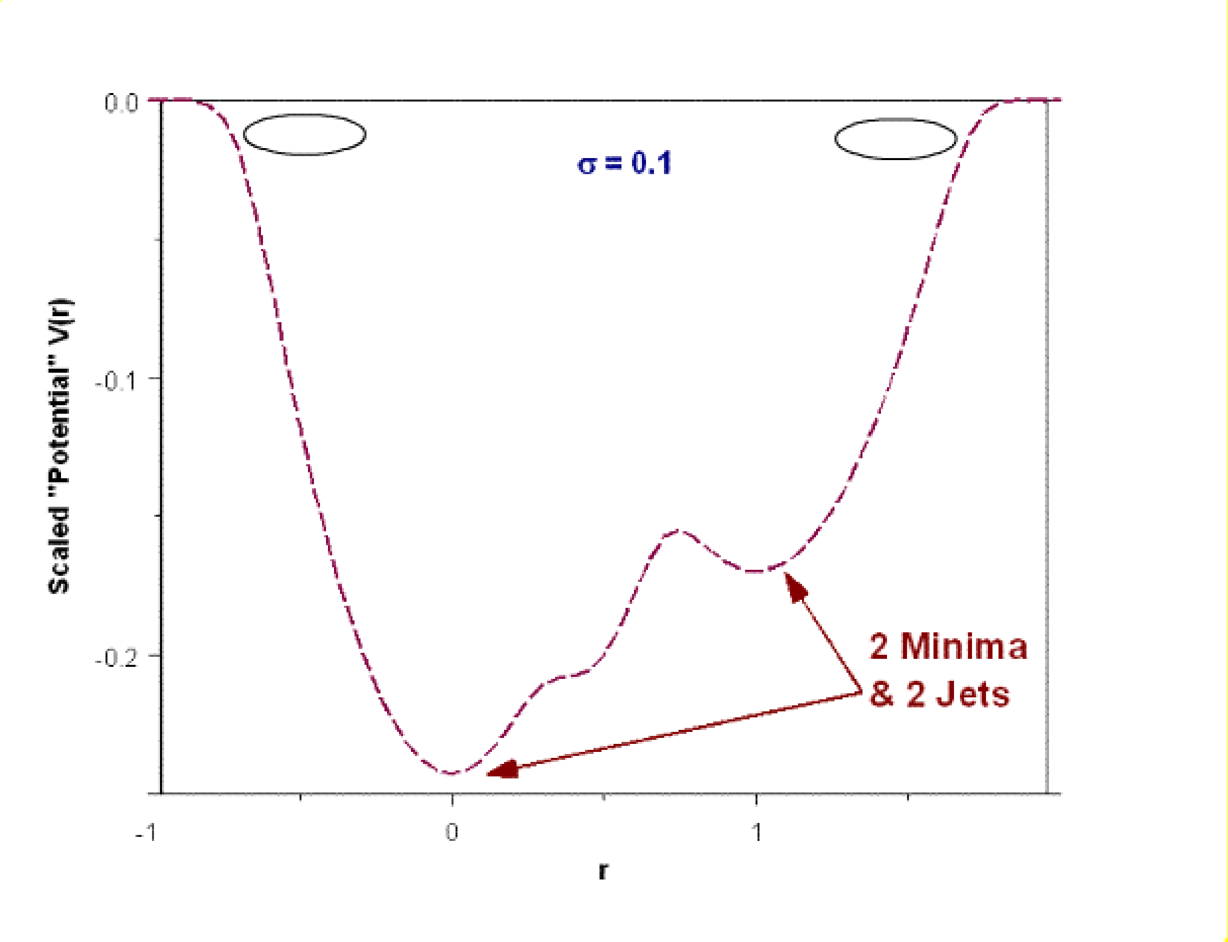


Why Dark towers? Include smearing (~ showering & hadronization) in simple picture, find only 1 stable cone





NOTE: Even if 2 stable cones, central cone can be lost to smearing



Jet Summary:

- Seeds & pQCD are a bad mix (not IRS). **It would be better to correct for seeds in the data (a small correction) and compare to theory w/o seeds (so no IRS issue) !!**
- Dark towers are a real 5 - 10% effect, but the search cone fix aggravates the IRS issue (a low p_T cone stable with radius R' < R, but not R, can trigger the merger of 2 well separated energetic partons) – need a better solution, or recognize as a correction
- Need serious phenomenology study of the k_T algorithm (happening)
- These issues will be relevant at the LHC, where the masses of the jets will play a larger role

pQCD Summary -

- A reliable tool for phenomenology, with well understood limitations

- Progress being made in areas of

  Wide range of NNLO analyses (using improved tools)

  MC@NLO – matching NLO pQCD to MC event generators while avoiding double counting

  Summing logs in a variety of processes leading to more thorough understanding of boundary with non-perturbative dynamics

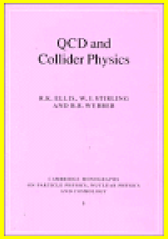
- Basis for studies of BSM physics

Extra Detail Slides

ASIDE: Some calculational details -

- chose the following vectors for the incident quark, light-like gauge fixing vector and virtual photon –

see Chapter 4 in



$$p^{\mu} = \left(P,0,0,P\right);\, n^{\mu} = \left(\frac{1}{2P},0,0,\frac{-1}{2P}\right);\, q^{\mu} = \nu n^{\mu} + q_T^{\mu}$$

- such that $p^2 = n^2 = q_T \cdot n = q_T \cdot p = 0\,;\, n \cdot p = 1\,;\, q \cdot p = \nu$

$$q_T^2 = -q^2 = Q^2\,;\, x = Q^2 / 2\nu$$

- If the emitted gluon has momentum and polarization , we require (conserved current and gauge choice) $\varepsilon \cdot r = \varepsilon \cdot n = 0$

- The momentum of the internal quark leg can be written in terms of a transverse vector k_T (similar to q_T)

$$k^{\mu} = \xi p^{\mu} + \frac{k_T^2 - \left|k^2\right|}{2\xi} n^{\mu} + k_T^{\mu}\,;\, d^4k = \frac{d\xi}{2\xi} dk^2 d^2k_T$$

More details -

- The appropriately summed, averaged (spin and color) and projected matrix element is

$$\frac{1}{4\pi} n^{\alpha} n^{\beta} \overline{\Sigma} \left| M \right|^2_{\alpha\beta} = \frac{8 e_q^2 \alpha_s}{\left| k^2 \right|} \xi P(\xi)$$ with P(ξ=*x*) above!

- The 2-body phase space in these variables is

$$d\Phi_2 = \frac{1}{16 \nu \pi^2} \int d\xi \, dk^2 dk_T^2 d\theta \, \delta\left(k_T^2 - (1-\xi)\left| k^2 \right| \right) \delta\left(\xi - x - \frac{\left| k^2 \right| + 2 q_T \cdot k_T}{2\nu} \right)$$

- Performing all the integrals ($0<\theta<\pi$) except $d\xi$ yields the result above (the δ fcts put the outgoing q and g on-shell).

Coefficient Functions

$$\bar{C}_q^{\overline{MS}}(z) = C_F\left[2\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{3}{2}\left(\frac{1}{1-z}\right)_+ - (1+z)\ln(1-z)\right.$$

$$\left. - \frac{1+z^2}{1-z}\ln z + 3 + 2z - \left(\frac{\pi^2}{3} + \frac{9}{2}\right)\delta(1-z)\right],$$

$$\bar{C}_g^{\overline{MS}}(z) = T_R\left[\left((1-z)^2 + z^2\right)\left(\frac{\ln(1-z)}{z}\right) - 8z^2 + 8z - 1\right].$$

Finally for DIS proton in $\overline{MS}(\mu_F = Q)$

$$F_2\left(x,Q^2\right) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi,Q)\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s(Q)}{2\pi} C_q^{\overline{MS}}\left(\frac{x}{\xi}\right) + \ldots\right] +$$

$$+ x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} g(\xi,Q)\left[\frac{\alpha_s(Q)}{2\pi} C_q^{\overline{MS}}\left(\frac{x}{\xi}\right) + \ldots\right]$$

- As with the renormalized, running coupling, pQCD does *not* tell us about the full, running parton distributions. These must be determined experimentally.

- pQCD does tell us how they *evolve* with the scale μ

General Structure of Convolution - Factorization

- Consider the general version ($\mu^2 \neq \mu_F^2 \neq Q^2$)

$$F_2^{\gamma N}\left(x,Q^2\right) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q^{\overline{MS}}\left(\xi,\mu_F,\alpha_s(\mu)\right)$$

$$\times\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s(\mu)}{2\pi}\left\{P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right) + \bar{C}_q^{\overline{MS}}\left(\frac{x}{\xi}\right)\right\} + \ldots\right]$$

$$+ x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} g^{\overline{MS}}\left(\xi,\mu_F,\alpha_s(\mu)\right) \left[\frac{\alpha_s(\mu)}{2\pi}\left\{P_{qg}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right) + \bar{C}_g^{\overline{MS}}\left(\frac{x}{\xi}\right)\right\} + \ldots\right]$$

$$\equiv \sum_{a=q,\bar{q},g} C_2^{\gamma a}\left(\frac{x}{\xi},\frac{Q}{\mu_F},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \otimes f_{a/N}\left(\xi,\mu_F,\alpha_s(\mu)\right)$$

- Convolution of (non-perturbative) long distance physics with short-distance (perturbative) IRS physics (defined by factorization scale) – μ^2 and μ_F^2 dependence must be matched between the 2 components - **The General Structure of pQCD** -

Examples/Conclusions -

- Since $d_{qq}(1)=0$, the number of valence quarks does not evolve – flavor is conserved by QCD

- Since $d_{qq}(j \geq 2)<0$, the non-singlet quark distribution evolves by decreasing at large *x* and increasing at small *x* – *as expected as the quark emits gluons*

- Next note that $$\int_0^1 dx\, x^{j-1}\frac{1}{x}=\frac{1}{j-1}$$

$$\int dx\, x^{j-1}\frac{1}{(1-x)_+}=-\int_0^1 dx\frac{x^{j-1}-1}{x-1}\sim \ln j \quad j>>1$$

The pole at *j=1* means the fixed order analysis is unreliable for the limit of small x

More -

- The large x behavior can be inferred from the ln *j* behavior of the anomalous dimensions and the fact that

$$f\left(x,\mu^2\right)\xrightarrow[x\to 1]{}(1-x)^{a\left(\mu^2\right)} \Rightarrow f\left(j,\mu^2\right)\xrightarrow[j>>1]{} j^{-a\left(\mu^2\right)}$$

- Thus we find

$$\gamma_{qq}^{(0)}(j>>1)\sim -4C_F \ln j \quad \Rightarrow$$

$$j^{-a\left(\mu_0^2\right)}\left(\frac{\ln\left(\mu^2/\Lambda_{QCD}^2\right)}{\ln\left(\mu_0^2/\Lambda_{QCD}^2\right)}\right)^{-4C_F \ln j/\beta_0} = j^{-a\left(\mu_0^2\right)-4C_F \ln\left(\ln\left(\mu^2/\Lambda_{QCD}^2\right)/\ln\left(\mu_0^2/\Lambda_{QCD}^2\right)\right)/\beta_0}$$

$$\Rightarrow q_{NS}\left(x,\mu^2\right)\sim(1-x)^{a\left(\mu_0^2\right)+4C_F \ln\left(\ln\left(\mu^2/\Lambda_{QCD}^2\right)/\ln\left(\mu_0^2/\Lambda_{QCD}^2\right)\right)/\beta_0}$$

Current PDF issues

- More precision for the Gluons
- Flavor, charge asymmetries, e.g., s vs $\bar{s}$
- Heavy flavors (*c*,*b*)
  experimental determination
  include mass effects, defining thresholds
  role of nonperturbative effects (i.e., besides perturbative gluon splitting)
- Do we need NNLO fits? (global data probably not that good yet)

ASIDE: Sudakov Form Factor -

- Consider the function $$\Delta\left(\mu^2,\mu_0^2\right)\equiv\exp\left[-\int_{\mu_0^2}^{\mu^2}\frac{d\kappa^2}{\kappa^2}\int^1 dz\,\frac{\alpha_s\left(\kappa^2\right)}{2\pi}\hat{P}\left(z\right)\right]$$

  which involves the unregulated version of the splitting function but, in a sense, contains the information about the regulation of the soft singularity ($z\to 1$).

- This is the bare* version of the Sudakov Form Factor mentioned earlier.

- Using $P\left(z\right)=\hat{P}\left(z\right)_+$, and $P\left(z\right)=0$ outside of $0\leq z\leq 1$, we can write

$$\mu^2\frac{\partial}{\partial\mu^2}q\left(x,\mu^2\right)=\frac{\alpha_s\left(\mu^2\right)}{2\pi}\int_x^1\frac{dz}{z}\hat{P}\left(z\right)q\left(\frac{x}{z},\mu^2\right)+\frac{q\left(x,\mu^2\right)}{\Delta\left(\mu^2,\mu_0^2\right)}\mu^2\frac{\partial}{\partial\mu^2}\Delta\left(\mu^2,\mu_0^2\right)$$

* In physical applications the physics will control the soft singularity as was displayed earlier.

In Detail

$$\mu^2 \frac{\partial}{\partial \mu^2} q\left(x,\mu^2\right) = \frac{\alpha_s\left(\mu^2\right)}{2\pi} \int_x^1 \frac{dz}{z} P(z) q\left(\frac{x}{z},\mu^2\right) = \frac{\alpha_s\left(\mu^2\right)}{2\pi} \int_x^1 \frac{dz}{z} \hat{P}(z)_+ q\left(\frac{x}{z},\mu^2\right)$$

$$= \frac{\alpha_s\left(\mu^2\right)}{2\pi} \int_x^1 \frac{dz}{z} \hat{P}(z) q\left(\frac{x}{z},\mu^2\right) - \left[\frac{\alpha_s\left(\mu^2\right)}{2\pi z} q\left(\frac{x}{z},\mu^2\right)\right]_{z=1} \int_0^1 dz \hat{P}(z)$$

$$= \frac{\alpha_s\left(\mu^2\right)}{2\pi} \int_x^1 \frac{dz}{z} \hat{P}(z) q\left(\frac{x}{z},\mu^2\right) - \frac{\alpha_s\left(\mu^2\right)}{2\pi} q\left(x,\mu^2\right) \int_0^1 dz \hat{P}(z)$$

$$= \frac{\alpha_s\left(\mu^2\right)}{2\pi} \int_x^1 \frac{dz}{z} \hat{P}(z) q\left(\frac{x}{z},\mu^2\right) + \frac{q\left(x,\mu^2\right)}{\Delta\left(\mu^2,\mu_0^2\right)} \mu^2 \frac{\partial}{\partial \mu^2} \Delta\left(\mu^2,\mu_0^2\right)$$

Cont'd

- Or, more compactly,

$$\mu^2 \frac{\partial}{\partial \mu^2}\left(\frac{q\left(x,\mu^2\right)}{\Delta\left(\mu^2,\mu_0^2\right)}\right) = \frac{1}{\Delta\left(\mu^2,\mu_0^2\right)} \frac{\alpha_s\left(\mu^2\right)}{2\pi}\int_x^1 \frac{dz}{z}\hat{P}(z)\,q\left(\frac{x}{z},\mu^2\right)$$

- The solution can be written

$$q\left(x,\mu^2\right) = \Delta\left(\mu^2,\mu_0^2\right)q\left(x,\mu_0^2\right) + \int_{\mu_0^2}^{\mu^2}\frac{d\kappa^2}{\kappa^2}\frac{\Delta\left(\mu^2,\mu_0^2\right)}{\Delta\left(\kappa^2,\mu_0^2\right)}\frac{\alpha_s\left(\kappa^2\right)}{2\pi}\int_x^1\frac{dz}{z}\hat{P}(z)\,q\left(\frac{x}{z},\kappa^2\right)$$

- So we interpret

$\Delta\left(\mu^2,\mu_0^2\right)$ as the probability to evolve without splitting $\mu_0^2 \to \mu^2$

$\Delta\left(\mu^2,\mu_0^2\right)/\Delta\left(\kappa^2,\mu_0^2\right)$ as the probability to evolve $\kappa^2 \to \mu^2$, with an emission at κ^2.

- This interpretation will be helpful when thinking about time-like evolution and parton showering

Fragmentation IV -

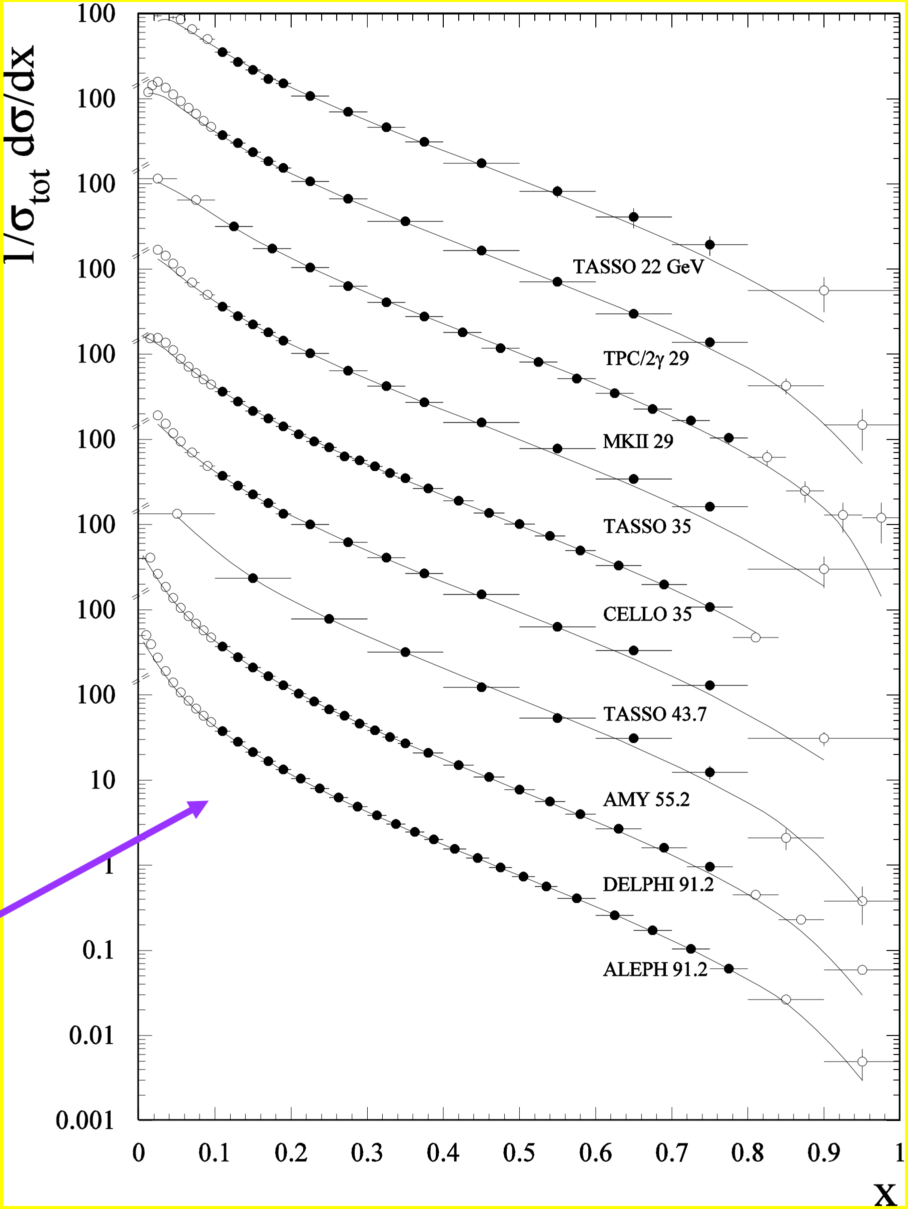
- Fragmentation functions have scaling violations just like parton distribution function

  – as μ increases

- the fragmentation function decreases at large *z*

- the fragmentation function increases at small *z*

ALEPH comparison of NLO scaling violations (and a simple 1/Q correction) yields pretty good agreement (if somewhat uncertain) !



Evolution In Words:

- Initial long distance – color singlet coherent eigenstates – described by factored PDF

- Short distance ($\ll$ 1 fermi) – pQCD (IRS) parton scattering

- Intermediate distances - “Bare” color charges shower (~collinear, final state radiation) simulated in MC, described by Sudakov

  Allow showering from exposed remnant colored charges (~ collinear with beam direction, initial state radiation) simulated in MC (more Sudakov)

  Allow multiple parton-parton interactions to simulate UE in MC

- “long” distance (~ 1 fermi) - associate color singlet sets of partons into hadrons (hadronization)

Dictionary of Hadron Collider Terminology

EVENT

HADRON-HADRON COLLISION

Primary (Hard) Parton-Parton Scattering

Initial-State Radiation (ISR) = Spacelike Showers associated with Hard Scattering

Underlying Event

Multiple Parton-Parton Interactions: Additional parton-parton collisions (in principle with showers etc) in the same hadron-hadron collision.

= Multiple Perturbative Interactions (MPI)

= Spectator Interactions

Fragmentation

Perturbative:

Final-State Radiation (FSR)

= Timelike Showers

= Jet Broadening and Hard Final-State Bremsstrahlung

Non-perturbative:

String / Cluster Hadronization

(Color Reconnections?)

Beam Remnants: Left over hadron remnants from the incoming beams. Colored and hence correlated with the rest of the event →

PILE-UP: Additional hadron-hadron collisions recorded as part of the same event.

From Peter Skands

Jets?

- What can we learn about (so far ill-defined*) jets, especially quarks versus gluons?

- Since the (coupling)^2 of a gluon to a gluon is $\frac{C_A}{C_F} = \frac{9}{4}$ times stronger than a gluon to a quark, we (naively) expect 9/4 times more radiation in a gluon initiated jet than a quark initiated one.

- We see exactly this result in the ratio $\frac{\gamma_{gg}^{(0)}(j)}{\gamma_{gq}^{(0)}(j)} \xrightarrow[j \to 1]{} \frac{C_A}{C_F}$ which we (naively) expect to control the particle multiplicity ratio**

$$\frac{\langle n_X \rangle_g}{\langle n_X \rangle_q} \xrightarrow[\mu^2 \to \infty]{} \frac{C_A}{C_F} \quad X = h, q, g$$

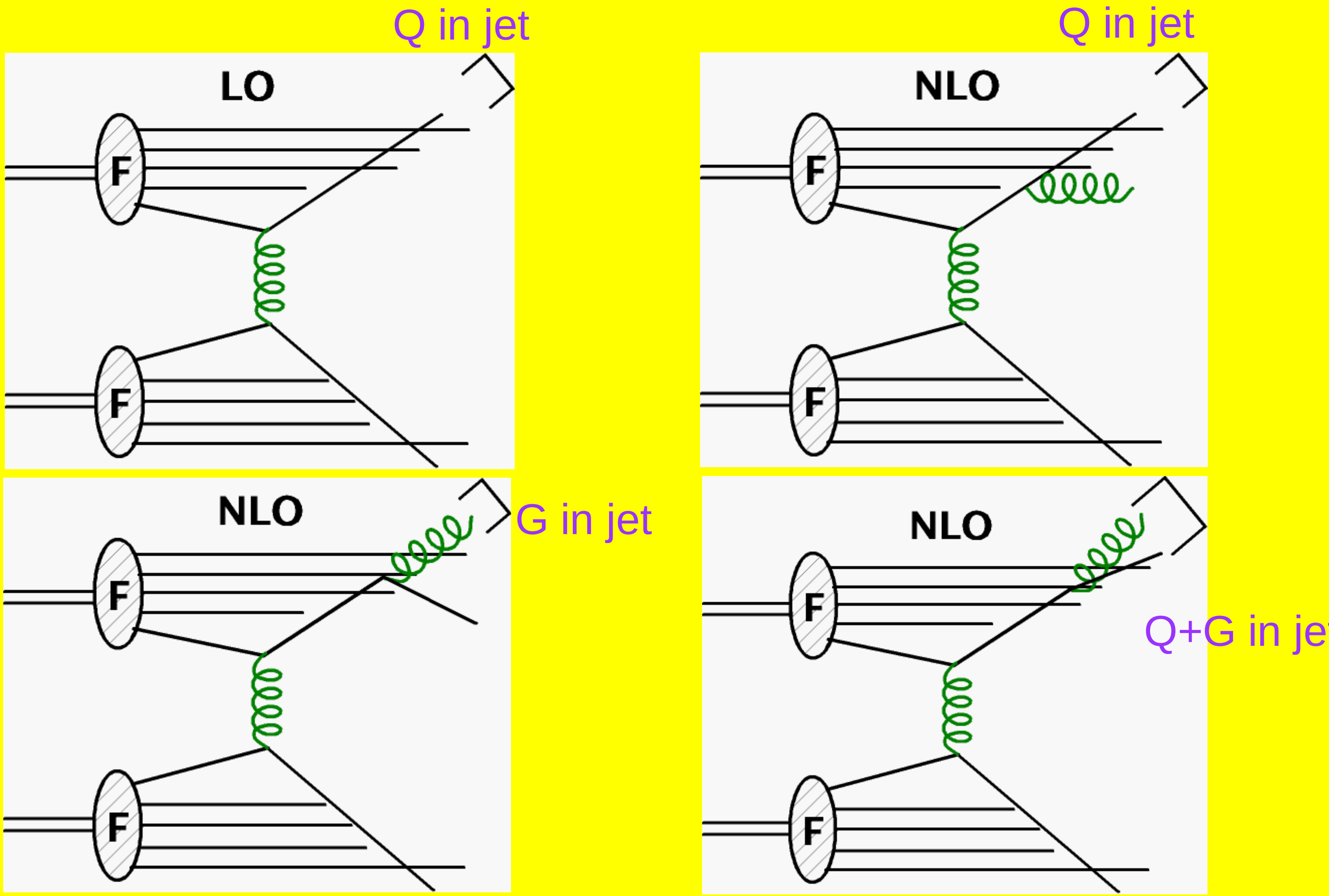
Gluon jets – more, softer hadrons

*We will define jets more carefully shortly.

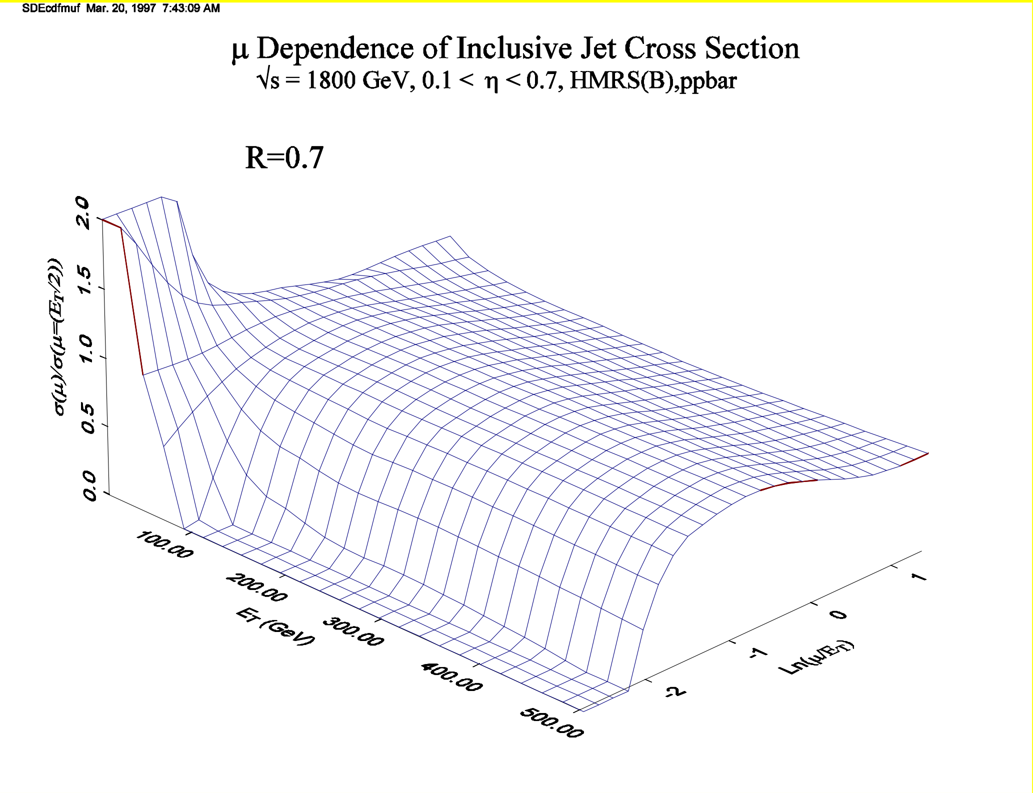
**Since both anomalous dimensions are singular at j=1, the analysis of the full multiplicity distributions is more complicated than discussed here.

Jets at NLO

- sample real emission graphs



- + … + Virtual ⇒ Improved accuracy (smaller μ dependence) and gives structure to the jet. For E_T > 100 GeV there is a region where μ is small, μ ~ $E_T/2$



$$\frac{d\sigma_{pp}^{jet}}{d\eta dE_T} = f_{a/p}\left(x_a, \mu, \mu_F\right) \otimes \hat{\sigma}_{ab\to c}^{jet}\left(x_a, x_b, z, p_T, \mu, \mu_F\right)$$

$$\otimes f_{b/p}\left(x_b, \mu, \mu_F\right)$$