The incredible many facets of a unitary Fermi system

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(Slides will be posted on my webpage)

Outline:

- **Setting up the playground**
- **Thermodynamics of unitary Fermi gas**
- **Imbalanced Fermi gas and induced P-wave superfluidity**
- **A totally unexpected realization of Larkin-Ovchinnikov superfluidity (Fulde-Ferrell-Larkin-Ovchinnikov)**
- **The surprising properties of Higgs-like excitation modes**
- **Lessons learned**

Why would one want to study this system?

The Unitary Fermi System

One reason:

(for the nerds, I mean the hard-core theorists,
not for the phenomenologists)

Bertsch's Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Let me consider as an example the hydrogen atom.

The ground state energy could only be a function of:

- ✓ **Electron charge**
- ✓ **Electron mass**
- ✓ **Planck's constant**

and then trivial dimensional arguments lead to

$$E_{gs} = -\frac{e^4 m}{\hbar^2} \times \frac{1}{2}$$

Only the factor ½ requires some hard work.

Let me now turn to dilute fermion matter

The ground state energy is given by a function:

$$E_{gs} = f(N, V, \hbar, m, a, r_0)$$

Taking the scattering length to infinity and the range of the interaction to zero, we are left with:

$$E_{gs} = F(N, V, \hbar, m) = \frac{3}{5}\varepsilon_F N \times \xi$$

$$\frac{N}{V} = \frac{k_F^3}{3\pi^2}, \qquad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

Pure number

Essentially what George Bertsch asked for in 1999 is:

Tell me the value of ξ !

But he wished to know the properties of the system as well:

The system turned out to be superfluid !

$$E_{gs} = \frac{3}{5}\varepsilon_F N \times \xi \qquad \Delta = \varepsilon_F \times \varsigma$$

$$\xi = 0.40(1), \qquad \zeta = 0.50(1)$$

The answers proved to be a bit unexpected.

✓ **The energy reads almost like that of *a non-interacting system*!**
(no other dimensional parameters in the problem)

✓ **The system has *a huge pairing gap*!**

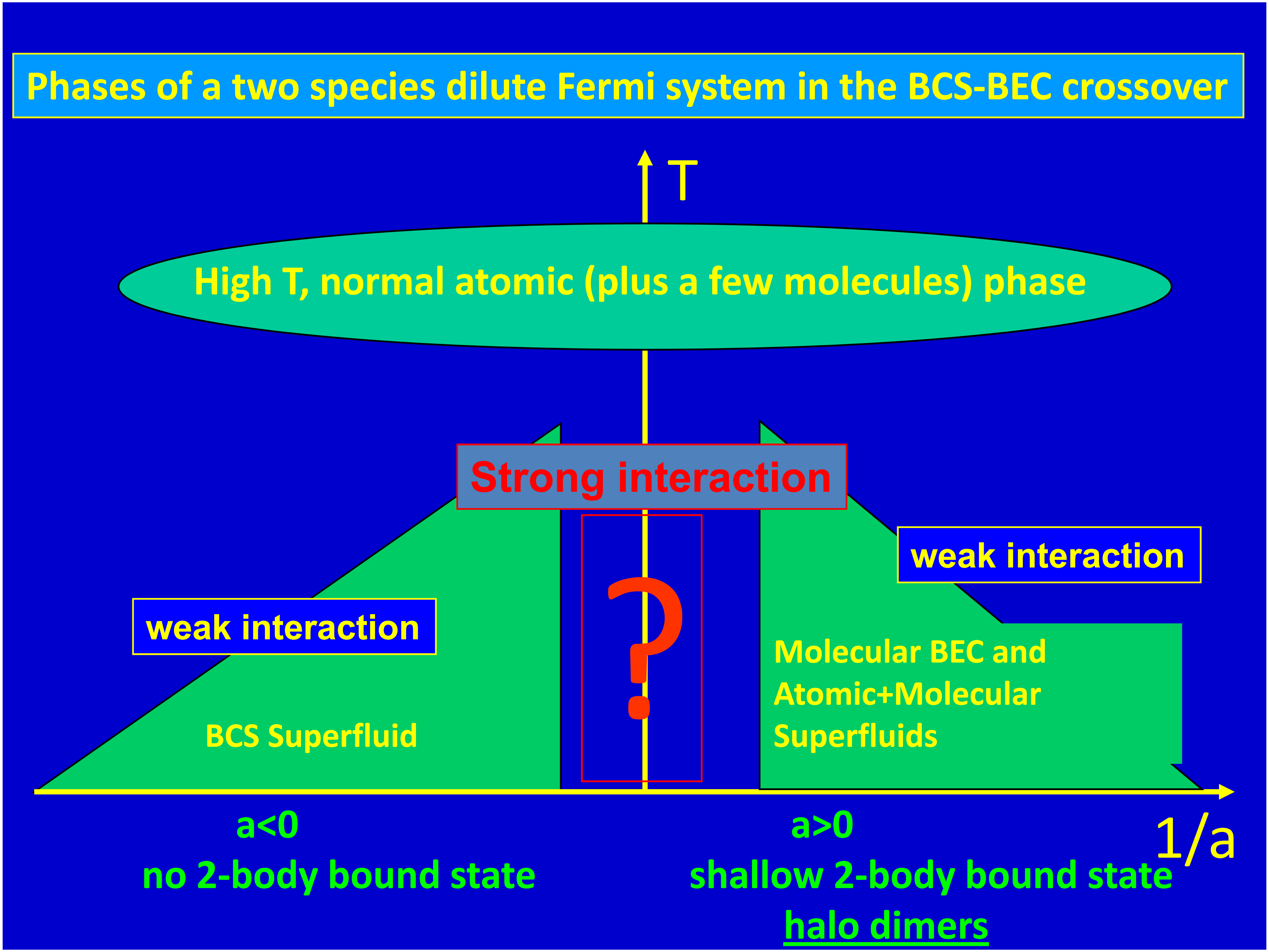
✓ **This system is a very strongly interacting one, since the *elementary cross section is infinite*!**

Superconductivity and Superfluidity in Fermi Systems

About 20 orders of magnitude over a century of (low temperature) physics

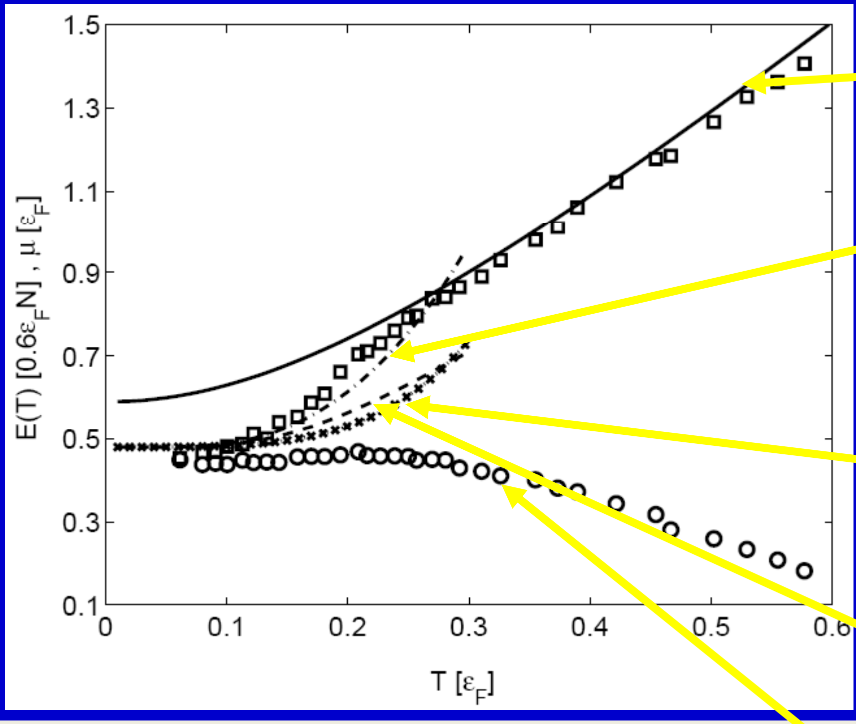
- ✓ Dilute atomic Fermi gases — $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ Liquid ^{3}He — $T_c \approx 10^{-7}$ eV
- ✓ Metals, composite materials — $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ Nuclei, neutron stars — $T_c \approx 10^{5} - 10^{6}$ eV
- • QCD color superconductivity — $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)


Phases of a two species dilute Fermi system in the BCS-BEC crossover
T
High T, normal atomic (plus a few molecules) phase
Strong interaction
weak interaction
weak interaction
?
Molecular BEC and
Atomic+Molecular
Superfluids
BCS Superfluid
a<0
no 2-body bound state
a>0
shallow 2-body bound state
halo dimers
1/a

a = ±∞

Bulgac, Drut, and Magierski
Phys. Rev. Lett. 96, 090404 (2006)



Normal Fermi Gas (with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution (dot-dashed line)

Bogoliubov-Anderson phonons contribution only

Quasi-particles contribution only (dashed line)

μ - chemical potential (circles)

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

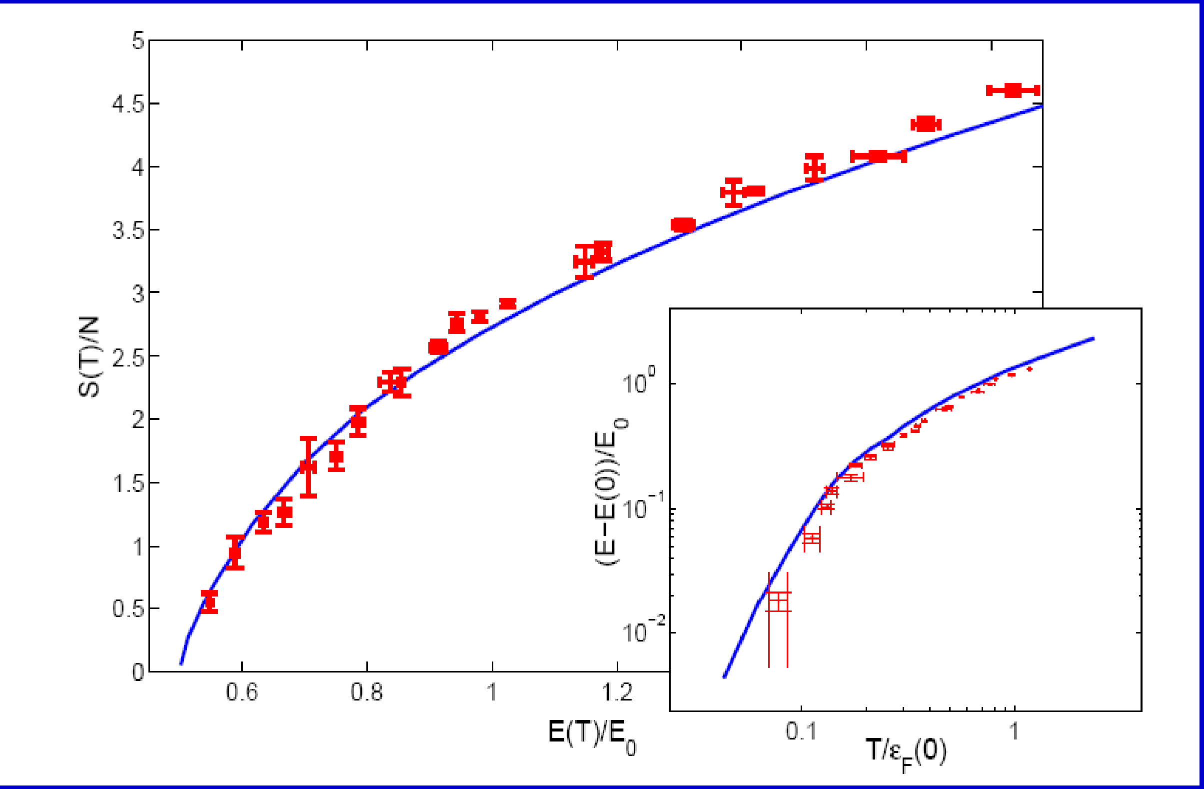
$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}} \exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

Experiment (about 100,000 atoms in a trap):

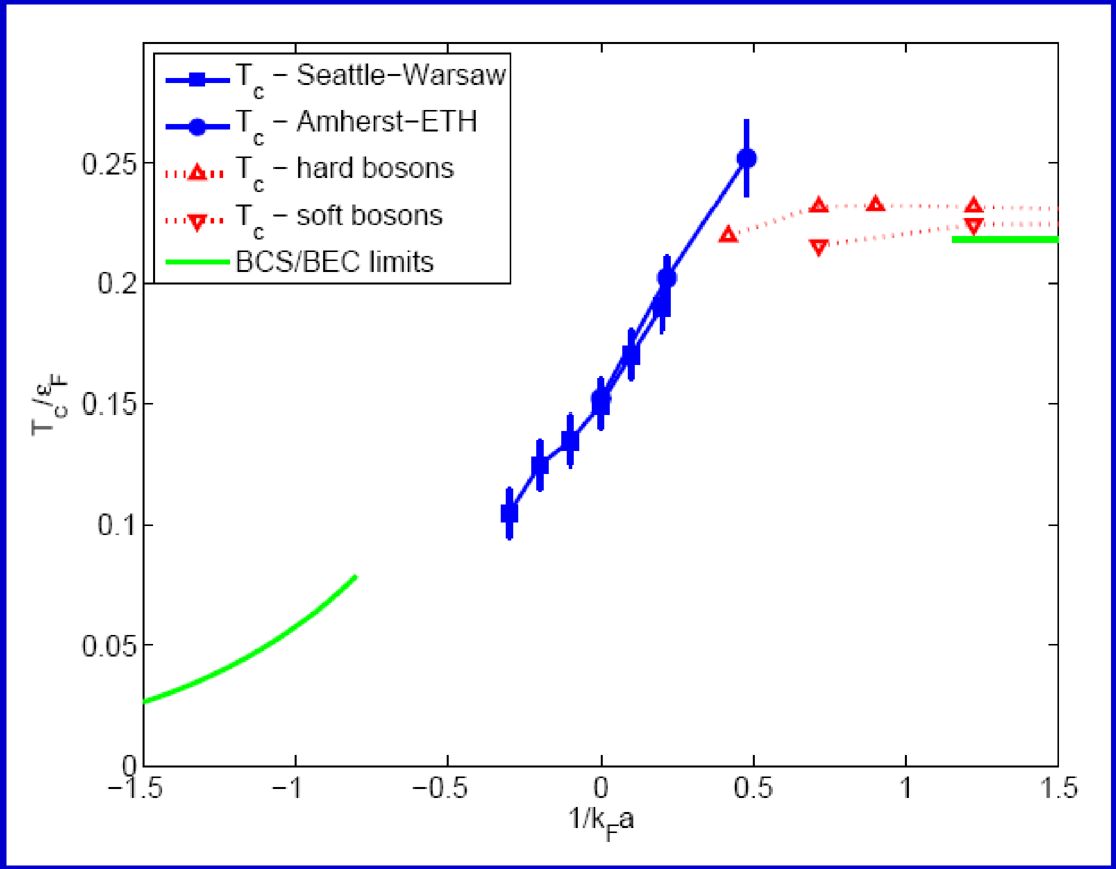
Measurement of the Entropy and Critical Temperature of a Strongly Interacting Fermi Gas

Luo, Clancy, Joseph, Kinast, and Thomas, Phys. Rev. Lett. 98, 080402 (2007)



Ab initio **theory (no free parameters)**
Bulgac, Drut, and Magierski, Phys. Rev. Lett. 99, 120401 (2007)

Critical temperature for superfluid to normal transition



Bulgac, Drut, and Magierski, Phys. Rev. A 78, 023625 (2008)

Amherst-ETH: Burovski et al. Phys. Rev. Lett. 101, 090402 (2008)

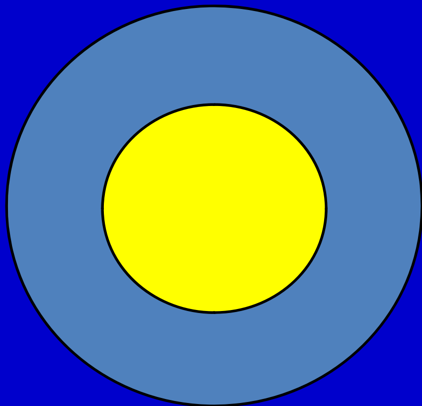
Hard and soft bosons: Pilati et al. Phys. Rev. Lett. 100, 140405 (2008)

Until now we kept the numbers of spin-up and spin-down equal.

What happens when there are not enough partners for everyone to pair with?

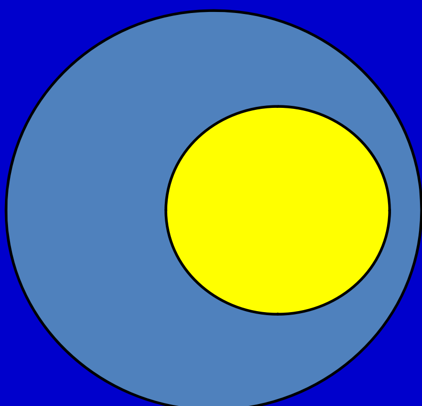
(In particular this is what one expects to happen in color superconductivity, due to the heavier strange quark)

What theory tells us?

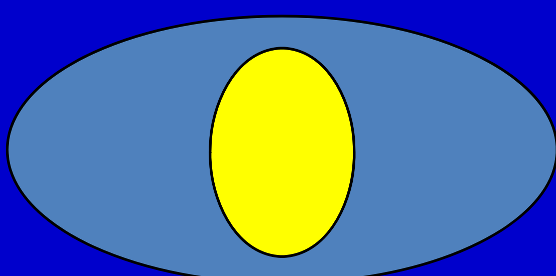


Gray – Fermi sphere of spin-up fermions
Yellow – Fermi sphere of spin-down fermions

If $\left|\mu_\uparrow - \mu_\downarrow\right| < \frac{\Delta}{\sqrt{2}}$ the same solution as for $\mu_\uparrow = \mu_\downarrow$



LOFF/FFLO solution (1964)
Pairing gap becomes a spatially varying function
Translational invariance broken

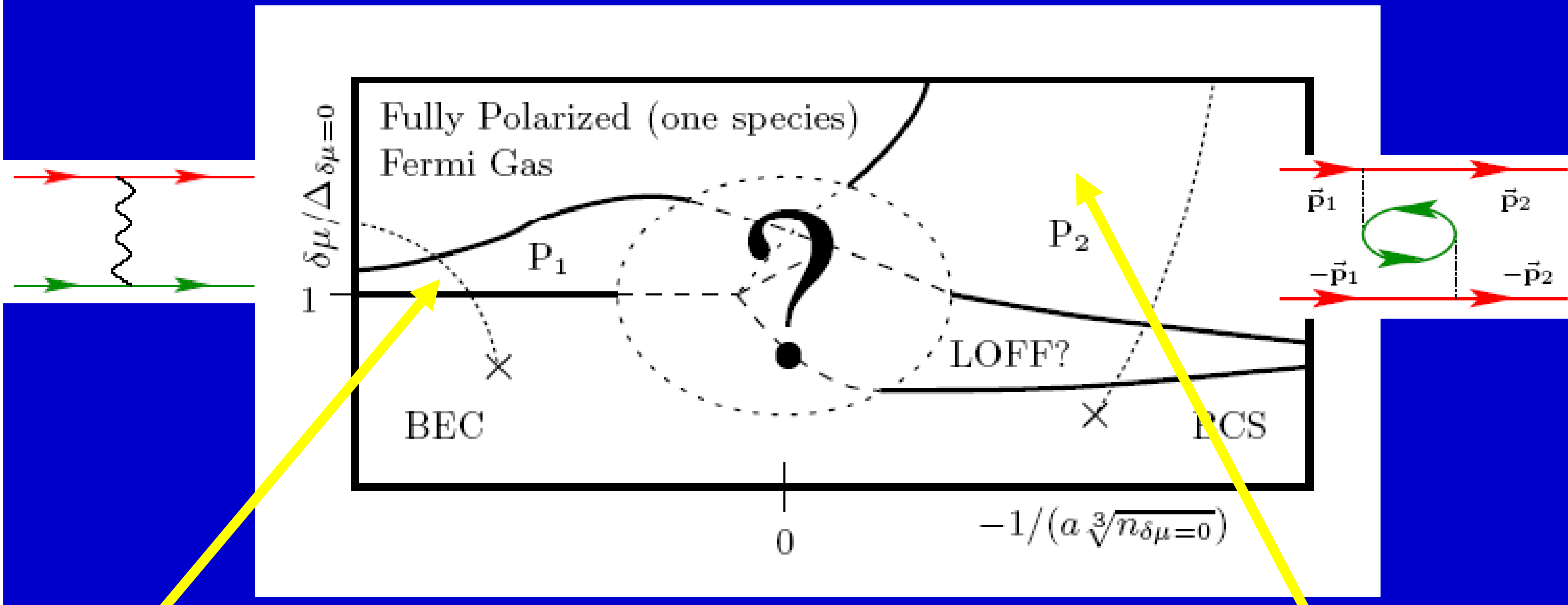


Muether and Sedrakian (2002)
Translational invariant solution
Rotational invariance broken

What we think is happening in spin imbalanced systems?

Induced P-wave superfluidity

Two new superfluid phases where before they were not expected

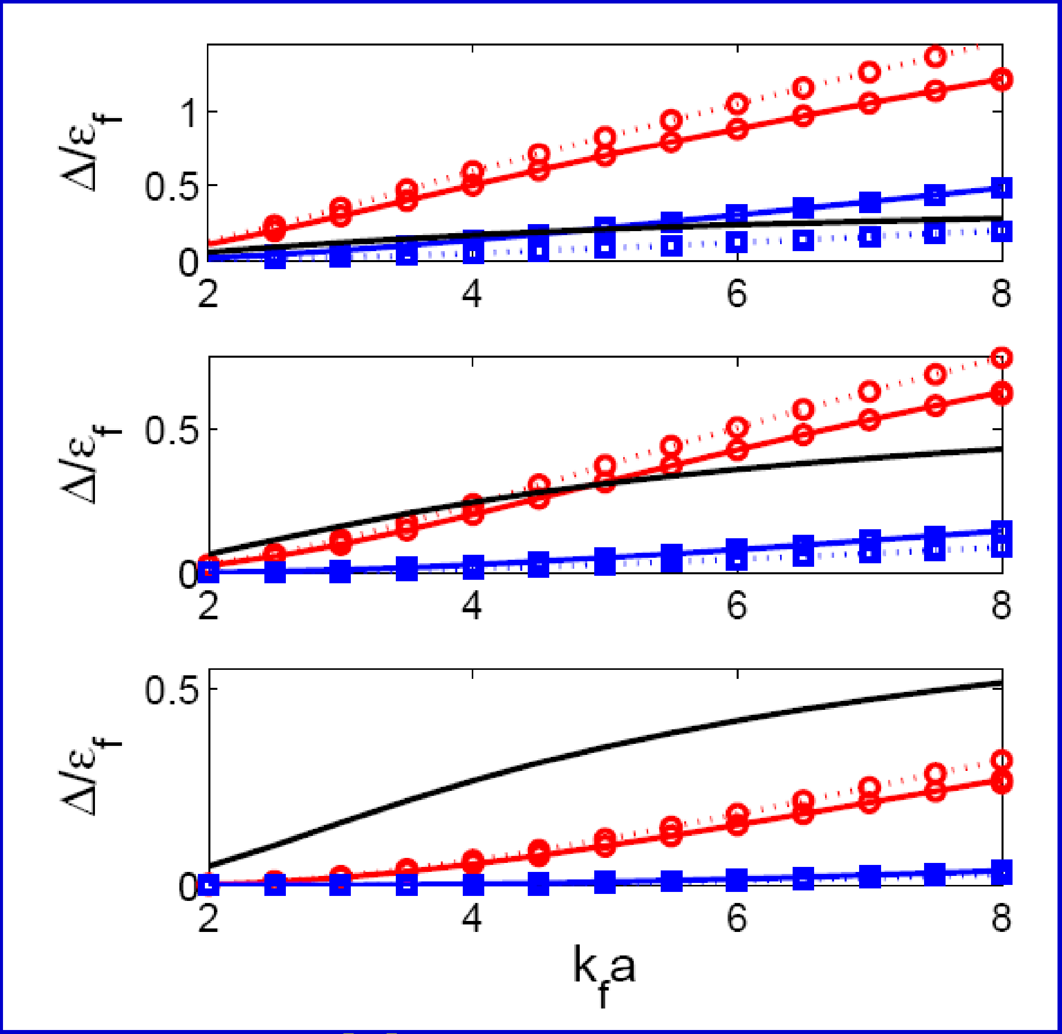


One Bose superfluid coexisting with one P-wave Fermi superfluid

Two coexisting P-wave Fermi superfluids

Bulgac, Forbes, Schwenk, Phys. Rev. Lett. 97, 020402 (2006)

Beyond the naïve BCS approximation
(*The interaction is mediated by Bogoliubov-Anderson-phonons and both finite range and retardation effects are important*)



BCS approx. (black)

Eliashberg approximation frequency dependence only (blue)

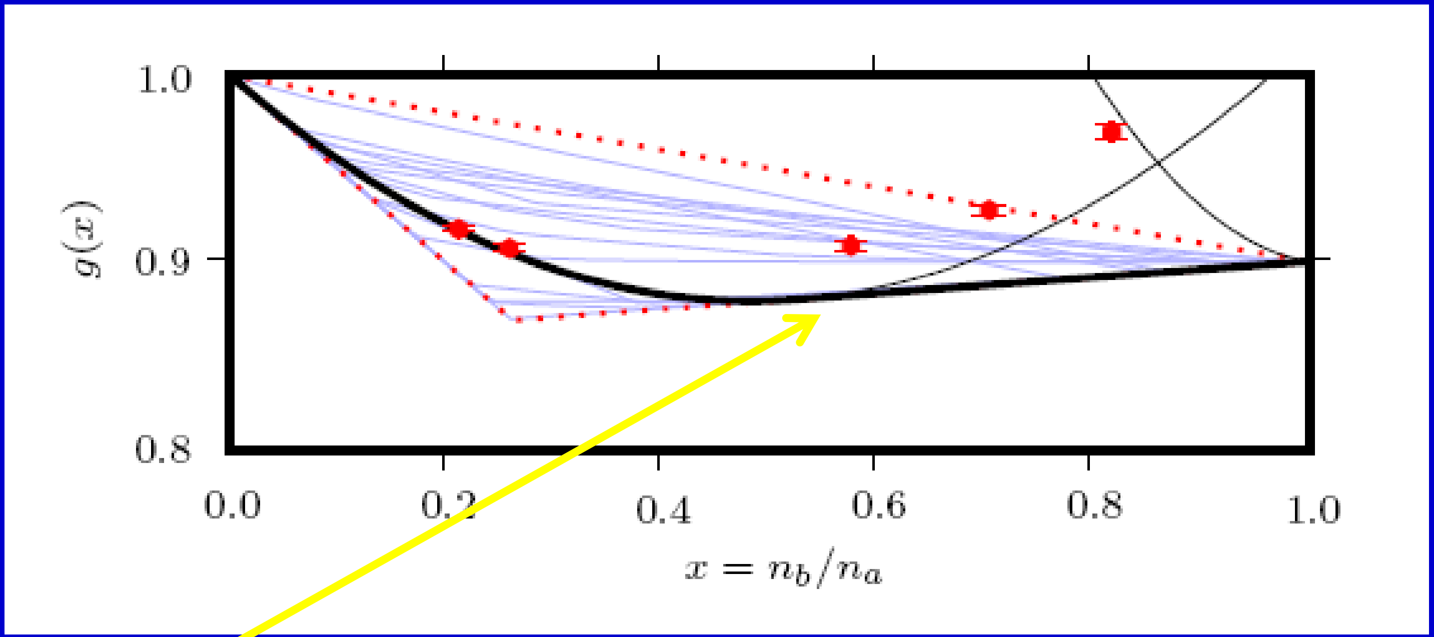
Full momentum and frequency dependence(blue)

From top panel down
# of dimers/# of fermions = 2.0, 1.0 and 0.5

Bulgac and Yoon, arXiv:0901.0348

What happens at unitarity?

Bulgac and Forbes, PRA 75, 031605(R) (2007)

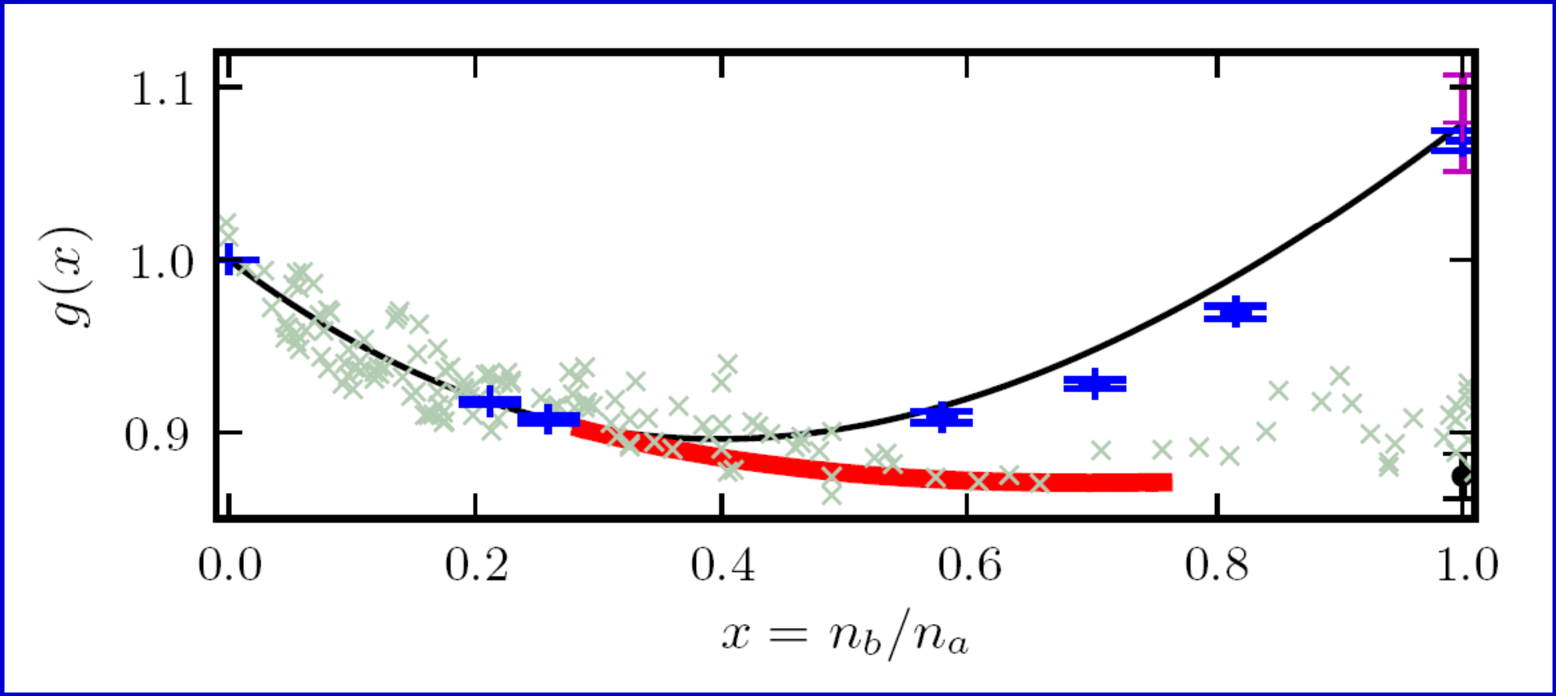


Predicted quantum first phase order transition, subsequently observed in MIT experiment , Shin *et al*. Nature, 451, 689 (2008)

Red points with error bars – subsequent DMC calculations for normal state due to Lobo et al, PRL 97, 200403 (2006)

$$\frac{E(n_a,n_b)}{V} = \frac{3}{5}\frac{(6\pi^2)^{2/3}\hbar^2}{2m}\left[n_a g\left(\frac{n_b}{n_a}\right)\right]^{5/3}, \qquad n_a \geq n_b$$

A refined EOS for spin unbalanced systems



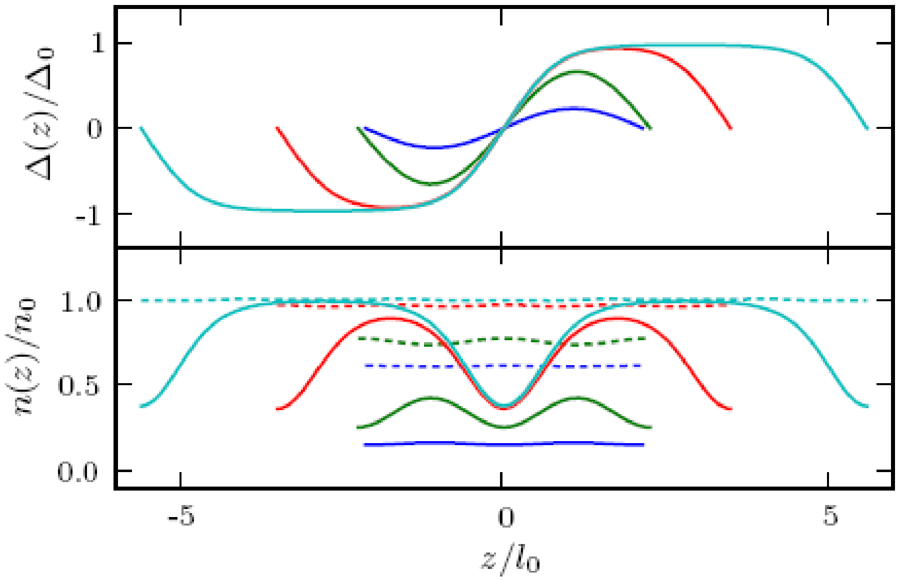
Red line: Larkin-Ovchinnikov phase

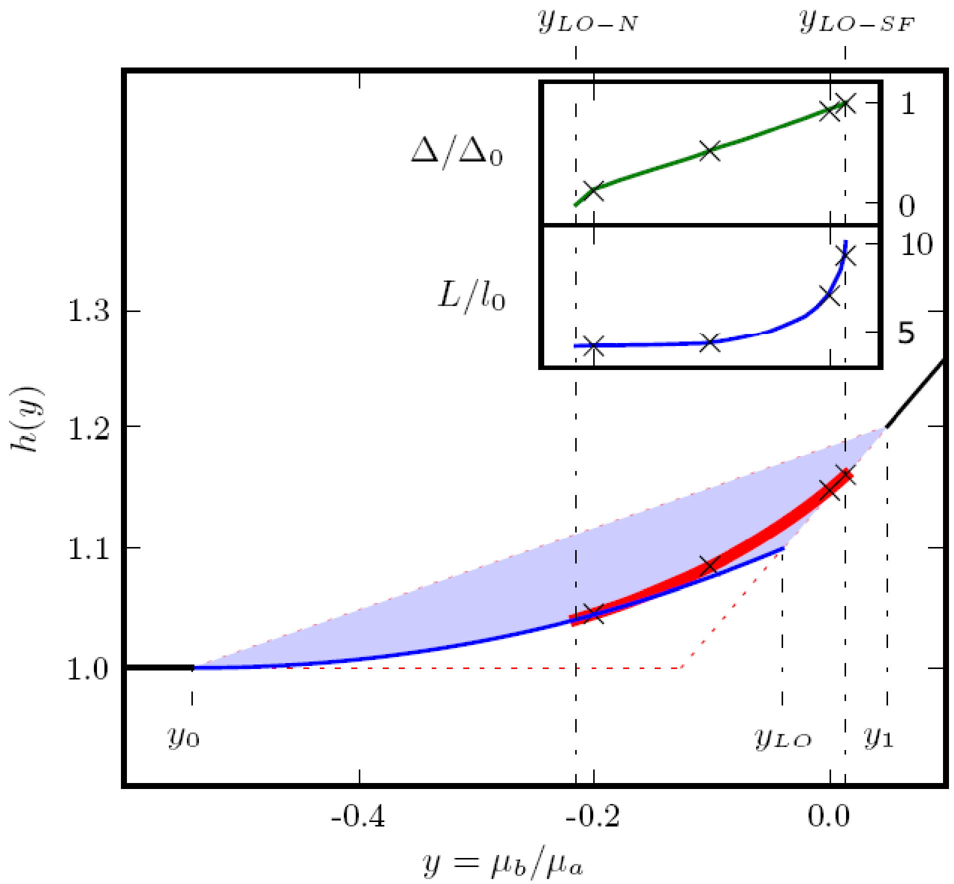
Black line: normal part of the energy density
Blue points: DMC calculations for normal state, Lobo et al, PRL 97, 200403 (2006)
Gray crosses: MIT experimental EoS, Shin, Phys. Rev. A 77, 041603(R) (2008)

$$\frac{E(n_a,n_b)}{V} = \frac{3}{5}\frac{(6\pi^2)^{2/3}\hbar^2}{2m}\left[n_a g\left(\frac{n_b}{n_a}\right)\right]^{5/3}$$

Bulgac and Forbes,
Phys. Rev. Lett. 101, 215301 (2008)

A Unitary Fermi Supersolid: the Larkin-Ovchinnikov phase



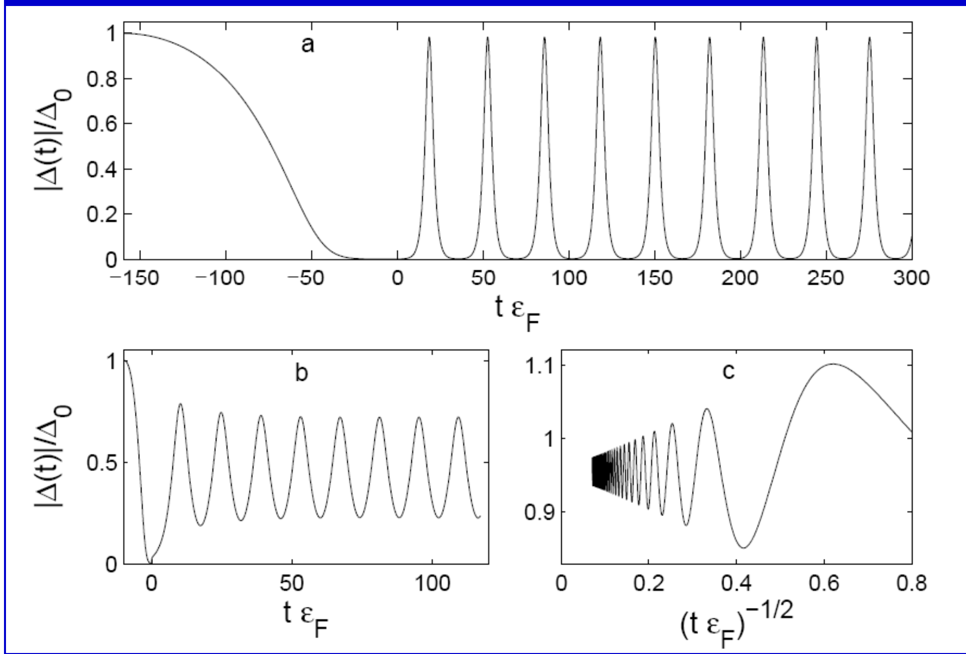


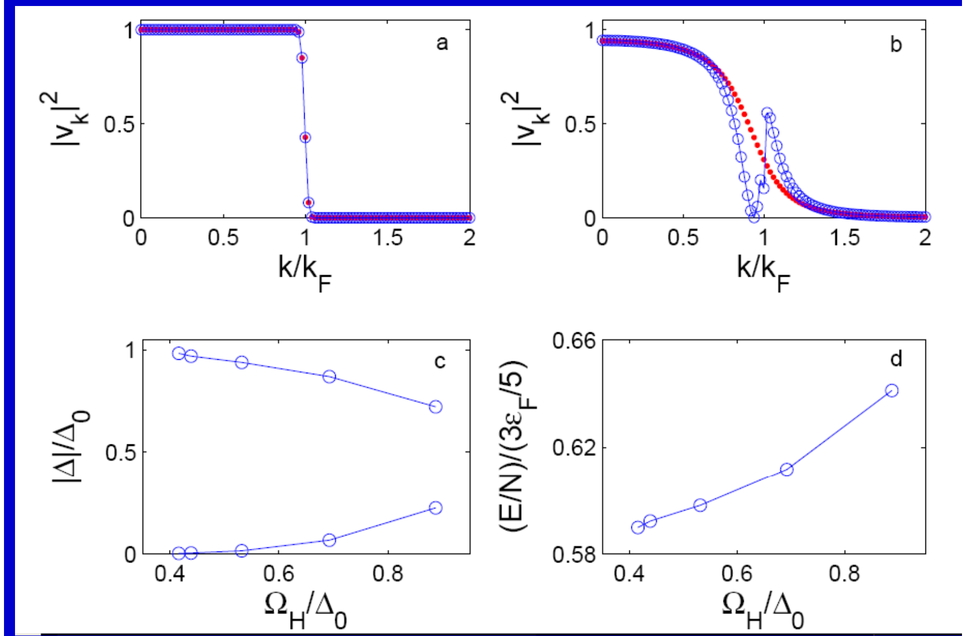
$$P[\mu_a, \mu_b] = \frac{2}{30\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

Bulgac and Forbes,
Phys. Rev. Lett. 101, 235301 (2008)

Response of a unitary Fermi system to changing the scattering length with time

Tool: TD DFT extension to superfluid systems (TD-SLDA)





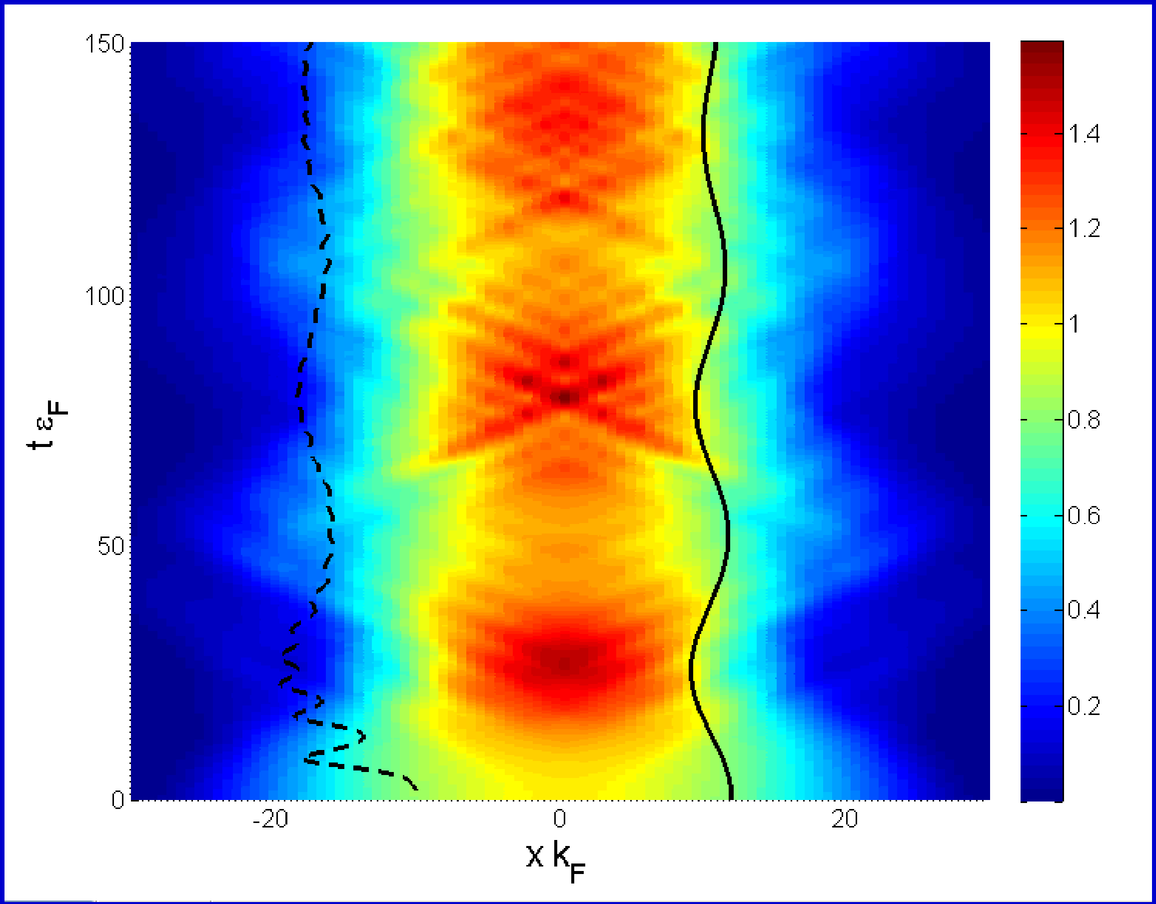
- All these modes have a very low frequency below the pairing gap and a very large amplitude and excitation energy as well

- None of these modes can be described either within Quantum Hydrodynamics or Landau-Ginzburg like approaches

Bulgac and Yoon, arXiv:0812:3643

3D unitary Fermi gas confined to a 1D ho potential well (pancake)

New qualitative excitation mode of a superfluid Fermi system (non-spherical Fermi momentum distribution)



Black solid line – Time dependence of the cloud radius
Black dashed line – Time dependence of the quadrupole moment of momentum distribution

Bulgac and Yoon, arXiv:0812:3643

<u>Some</u> of the lessons learned so far:

• We have (finally) control over the calculation of the pairing gap in dilute fermion/neutron matter and the thermodynamics of this system

• At moderate spin imbalance the system turns into a supersolid with pairing of the LOFF/FFLO type and the system displays first and second order quantum phase transitions to and from a Unitary Fermi Supersolid

• At large spin imbalance two *<u>simbiotic</u>* superfluids appear, one bosonic the other fermionic in character (induced P-wave superfluidity)

• There exist a new excitation mode of superfluid systems, a Higgs-like excitation mode with a non–spherical Fermi distribution. Only the Landau zero sound for normal system is the only other system with somewhat similar features.