Unexpected aspects of large amplitude nuclear collective motion

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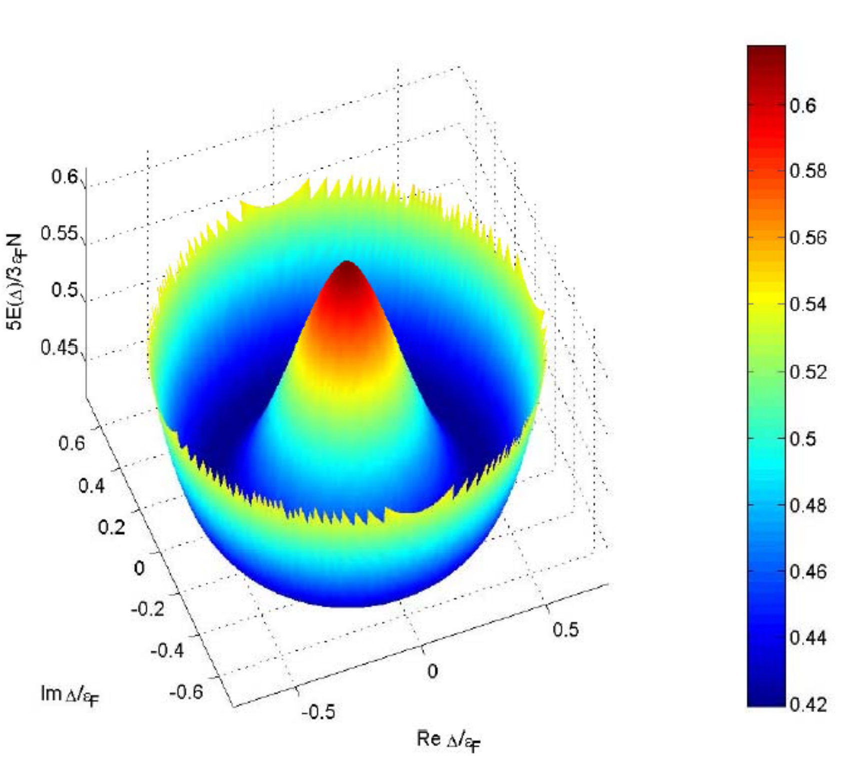
Bardeen, Cooper, and Schrieffer, 1957



$$\left|gs\right\rangle = \prod_{\vec{p}} \left(u_{\vec{p}} + v_{\vec{p}} a^{\dagger}_{\vec{p}\uparrow} a^{\dagger}_{\vec{p}\downarrow} \right) \left|vac\right\rangle$$

Global gauge invariance broken ⟹ gs is infinitely degenerate

$$\left|\varphi\right\rangle = \exp(i\varphi\hat{N})\left|gs\right\rangle = \prod_{\vec{p}} \left(u_{\vec{p}} + \exp(2i\varphi) v_{\vec{p}} a^{\dagger}_{\vec{p}\uparrow} a^{\dagger}_{\vec{p}\downarrow} \right) \left|vac\right\rangle$$

$$\hat{N} = \sum_{\vec{p}\sigma} a^{\dagger}_{\vec{p}\sigma} a_{\vec{p}\sigma}, \qquad \left\langle \varphi \middle| \hat{H} \middle| \varphi \right\rangle \equiv \left\langle gs \middle| \hat{H} \middle| gs \right\rangle$$

Goldstone theorem $\Longrightarrow$ existence of Goldstone bosons

The Bogoliubov-Anderson sound modes are the Goldstone bosons

Oscillations of the phase of the order parameter (pairing gap)

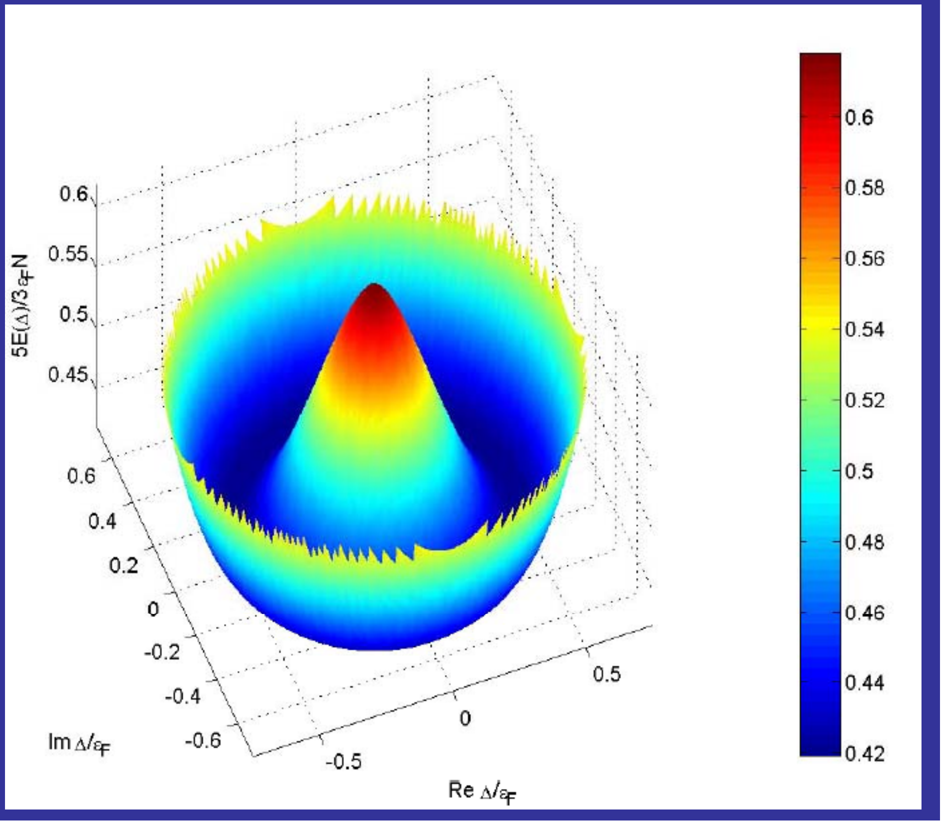
- **T=0, no collisions**
- **Momentum distribution is spherical**

$$\varphi \Rightarrow \varphi(\vec{r},t) = \vec{k}\cdot\vec{r} - \omega t$$

$$\exp\left[i(\vec{k}\cdot\vec{r} - \omega t)\hat{N} \right] | gs \rangle$$

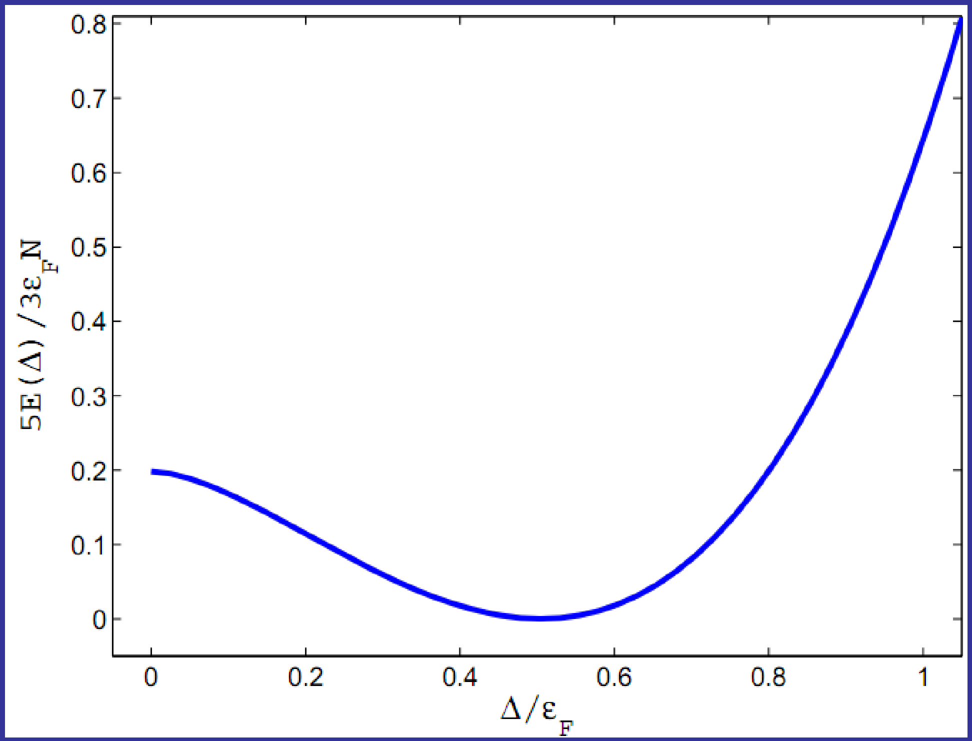
$$\omega = ck = \frac{v_F}{\sqrt{3}} k \text{ (BCS limit)}$$

Somewhat surprising, it has the same speed as regular (first/collisional) sound!



A different type of collective excitation: <u>Higgs mode</u>

Small amplitude oscillations of the modulus of the order parameter (pairing gap)



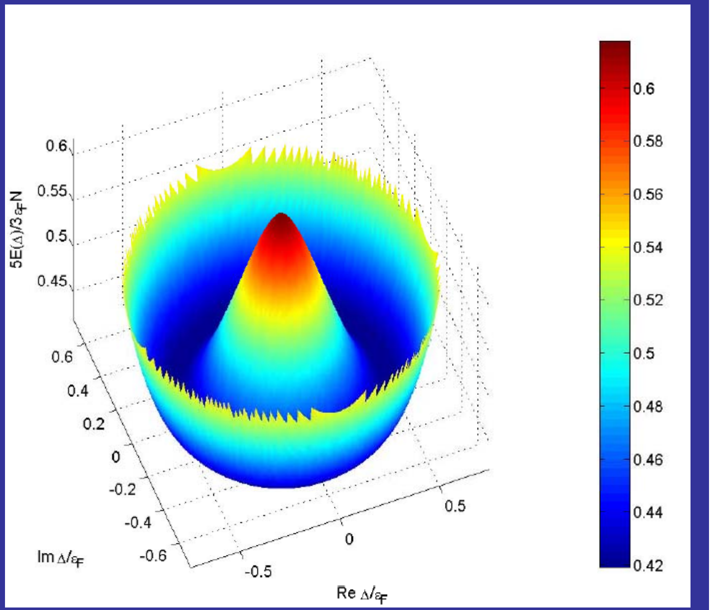
$$\hbar\Omega_H = 2\Delta_0$$

This mode has a bit more complex character
***cf.* Volkov and Kogan 1972 (a bit later about it)**

The Bogoliubov-Anderson sound modes are routinely described within Quantum Hydrodynamics approximation (Landau, here at T=0) Their properties are well confirmed experimentally

$$\dot{n} + \vec{\nabla} \cdot [\vec{v}n] = 0$$

$$m\dot{\vec{v}} + \vec{\nabla}\left\{ \frac{m\vec{v}^2}{2} + \mu[n] + V_{ext} \right\} = 0$$



Landau-Ginzburg-like /effective action approaches are often advocated for the dynamics of the order parameter

$$i\hbar\dot{\Psi}(\vec{r},t) = -\frac{\hbar^2\Delta}{4m}\Psi(\vec{r},t) + U\left(\left|\Psi(\vec{r},t)\right|^2\right)\Psi(\vec{r},t) + V_{ext}(\vec{r},t)\Psi(\vec{r},t)$$

Very brief/skewed summary of DFT

Kohn-Sham theorem

$$H = \sum_{i}^{N} T(i) + \sum_{i<j}^{N} U(ij) + \sum_{i<j<k}^{N} U(ijk) + \ldots + \sum_{i}^{N} V_{ext}(i)$$

$$H\Psi_0(1,2,\ldots N) = E_0\Psi_0(1,2,\ldots N)$$

$$n(\vec{r}) = \left\langle \Psi_0 \middle| \sum_{i}^{N} \delta(\vec{r} - \vec{r}_i) \middle| \Psi_0 \right\rangle$$

Injective map (one-to-one)

$$\Psi_0(1,2,\ldots N) \Leftrightarrow V_{ext}(\vec{r}) \Leftrightarrow n(\vec{r})$$

$$E_0 = \min_{n(\vec{r})} \int d^3r \left\{ \frac{\hbar^2}{2m^*(\vec{r})}\tau(\vec{r}) + \varepsilon\left[n(\vec{r})\right] + V_{ext}(\vec{r})n(\vec{r}) \right\}$$

$$n(\vec{r}) = \sum_{i}^{N} \left|\varphi_i(\vec{r})\right|^2, \qquad \tau(\vec{r}) = \sum_{i}^{N} \left|\vec{\nabla}\varphi_i(\vec{r})\right|^2$$

Universal functional of particle density alone

Independent of external potential

Normal Fermi systems only!

However, not everyone is normal!

Superconductivity and superfluidity in Fermi systems

✓ Dilute atomic Fermi gases	$T_c \approx 10^{-12} – 10^{-9}$ eV
✓ Liquid ^{3}He	$T_c \approx 10^{-7}$ eV
✓ Metals, composite materials	$T_c \approx 10^{-3} – 10^{-2}$ eV
✓ Nuclei, neutron stars	$T_c \approx 10^{5} – 10^{6}$ eV
• QCD color superconductivity	$T_c \approx 10^{7} – 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

Extension of Kohn-Sham to superfluid fermionic systems:

Superfluid Local Density Approximation (SLDA)
The case of a unitary Fermi gas

Why would one want to study this system?

One (very good) reason:

(for the nerds, I mean the hard-core theorists, not the phenomenologists)

Bertsch's Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

The unitary gas is really a pretty good model for dilute neutron matter

Dilute fermion matter

The ground state energy is given by a function:

$$E_{gs} = f(N, V, \hbar, m, a, r_0)$$

Taking the scattering length to infinity and the range of the interaction to zero, we are left with:

$$E_{gs} = F(N, V, \hbar, m) = \frac{3}{5}\varepsilon_F N \times \xi$$

$$\frac{N}{V} = \frac{k_F^3}{3\pi^2}, \qquad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

Pure number
Bertsch's parameter

The renormalized SLDA energy density functional

$$\varepsilon(\vec{r}) = \left[\alpha \frac{\tau_c(\vec{r})}{2} - \Delta(\vec{r})\nu_c(\vec{r})\right] + \beta \frac{3(3\pi^2)^{2/3} n^{5/3}(\vec{r})}{5}$$

$$n(\vec{r}) = 2\sum_k \left|\mathrm{v}_k(\vec{r})\right|^2, \quad \tau_c(\vec{r}) = 2\sum_{E<E_c} \left|\vec{\nabla}\mathrm{v}_k(\vec{r})\right|^2, \quad \nu_c(\vec{r}) = \sum_{E<E_c} \mathrm{u}_k(\vec{r})\mathrm{v}_k^*(\vec{r})$$

$$\frac{1}{g_{eff}(\vec{r})} = \frac{n^{1/3}(\vec{r})}{\gamma} - \frac{k_c(\vec{r})}{2\pi^2\alpha}\left[1 - \frac{k_0(\vec{r})}{2k_c(\vec{r})}\ln\frac{k_c(\vec{r}) + k_0(\vec{r})}{k_c(\vec{r}) - k_0(\vec{r})}\right]$$

$$E_c + \mu = \alpha\frac{k_c^2(\vec{r})}{2} + U(\vec{r}), \qquad \mu = \alpha\frac{k_0^2(\vec{r})}{2} + U(\vec{r})$$

$$U(\vec{r}) = \beta\frac{(3\pi^2)^{2/3} n^{2/3}(\vec{r})}{2} - \frac{\left|\Delta(\vec{r})\right|^2}{3\gamma n^{2/3}(\vec{r})} + V_{ext}(\vec{r})$$

$$\Delta(\vec{r}) = -g_{eff}(\vec{r})\nu_c(\vec{r})$$

No free/fitting parameters, EDF is fully determined by *ab initio* calculations

Bulgac, Phys. Rev. A <u>76</u>, 040502(R) (2007)

Time Dependent Phenomena and Formalism

The time-dependent density functional theory is viewed in general as a reformulation of the exact quantum mechanical time evolution of a many-body system when only single-particle properties are considered.

A.K. Rajagopal and J. Callaway, Phys. Rev. B 7, 1912 (1973)
V. Peuckert, J. Phys. C 11, 4945 (1978)
E. Runge and E.K.U. Gross, Phys. Rev. Lett. 52, 997 (1984)

http://www.tddft.org

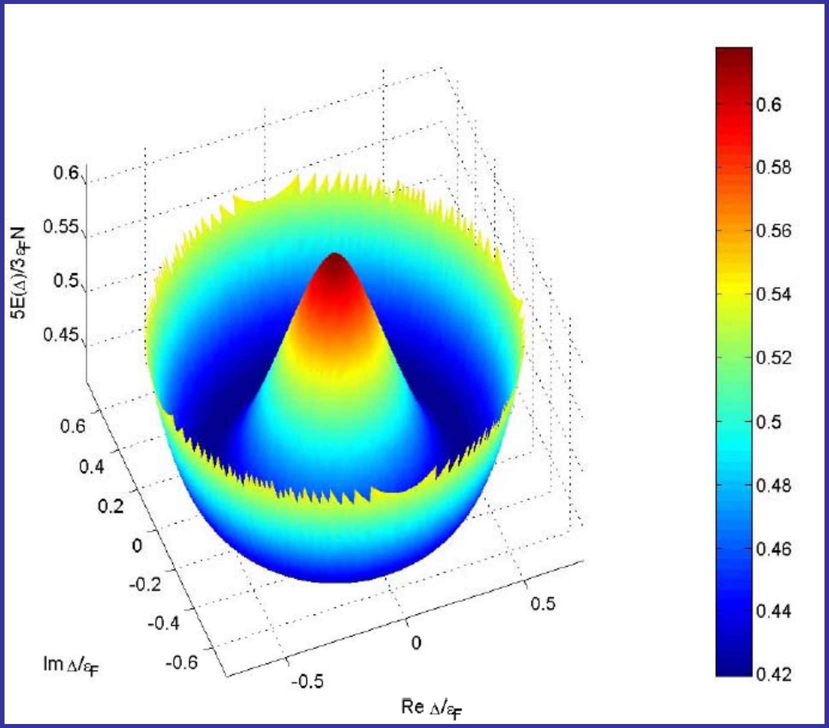
$$\begin{cases} [h(\vec{r},t)+V_{ext}(\vec{r},t)-\mu]\mathrm{u}_i(\vec{r},t)+[\Delta(\vec{r},t)+\Delta_{ext}(\vec{r},t)]\mathrm{v}_i(\vec{r},t)=i\hbar\dfrac{\partial \mathrm{u}_i(\vec{r},t)}{\partial t} \\ [\Delta^*(\vec{r},t)+\Delta^*_{ext}(\vec{r},t)]\mathrm{u}_i(\vec{r},t)-[h(\vec{r},t)+V_{ext}(\vec{r},t)-\mu]\mathrm{v}_i(\vec{r},t)=i\hbar\dfrac{\partial \mathrm{v}_i(\vec{r},t)}{\partial t} \end{cases}$$

Full 3D implementation of TD-SLDA is a petaflop problem and is almost complete for both nuclear systems and cold dilute atomic gases

Bulgac and Roche, J. Phys. Conf. Series 125, 012064 (2008)

Lots of contributions due to Yu, Yoon, Luo, Magierski, and Stetcu

Energy of a (unitary) Fermi system as a function of the pairing gap



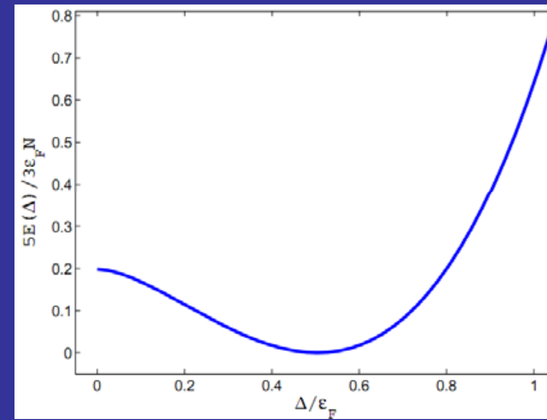
$$\dot{n} + \vec{\nabla} \cdot \left[\vec{v} n \right] = 0$$

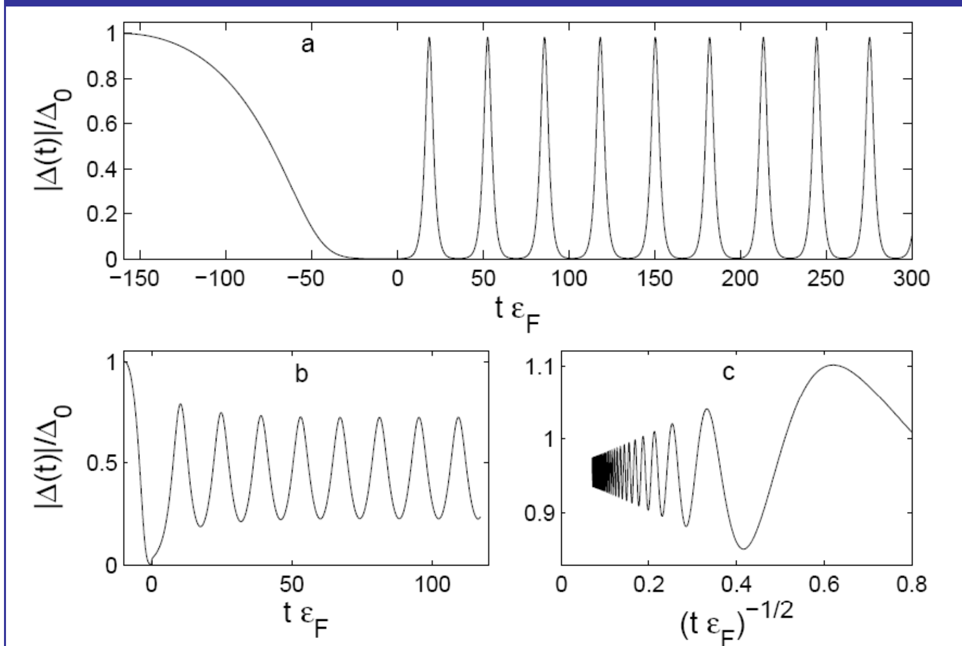
$$m\dot{\vec{v}} + \vec{\nabla} \left\{ \frac{m\vec{v}^2}{2} + \mu\left[n\right] \right\} = 0$$

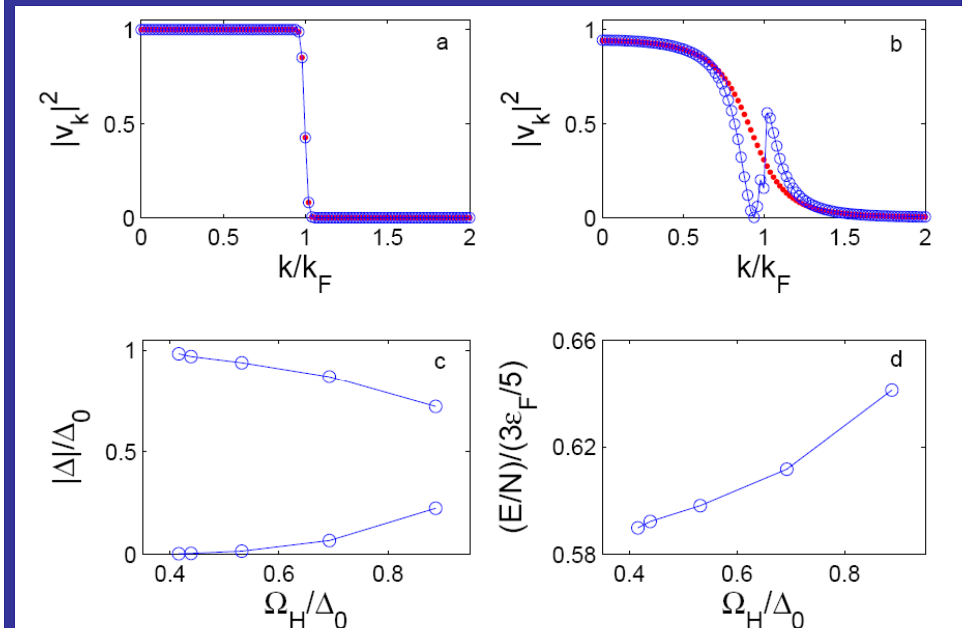
$$i\hbar\dot{\Psi}(\vec{r},t) = -\frac{\hbar^2 \Delta}{4m}\Psi(\vec{r},t) + U\left(\left|\Psi(\vec{r},t)\right|^2 \right)\Psi(\vec{r},t)$$

Response of a unitary Fermi system to changing the scattering length with time

Tool: TD-SLDA





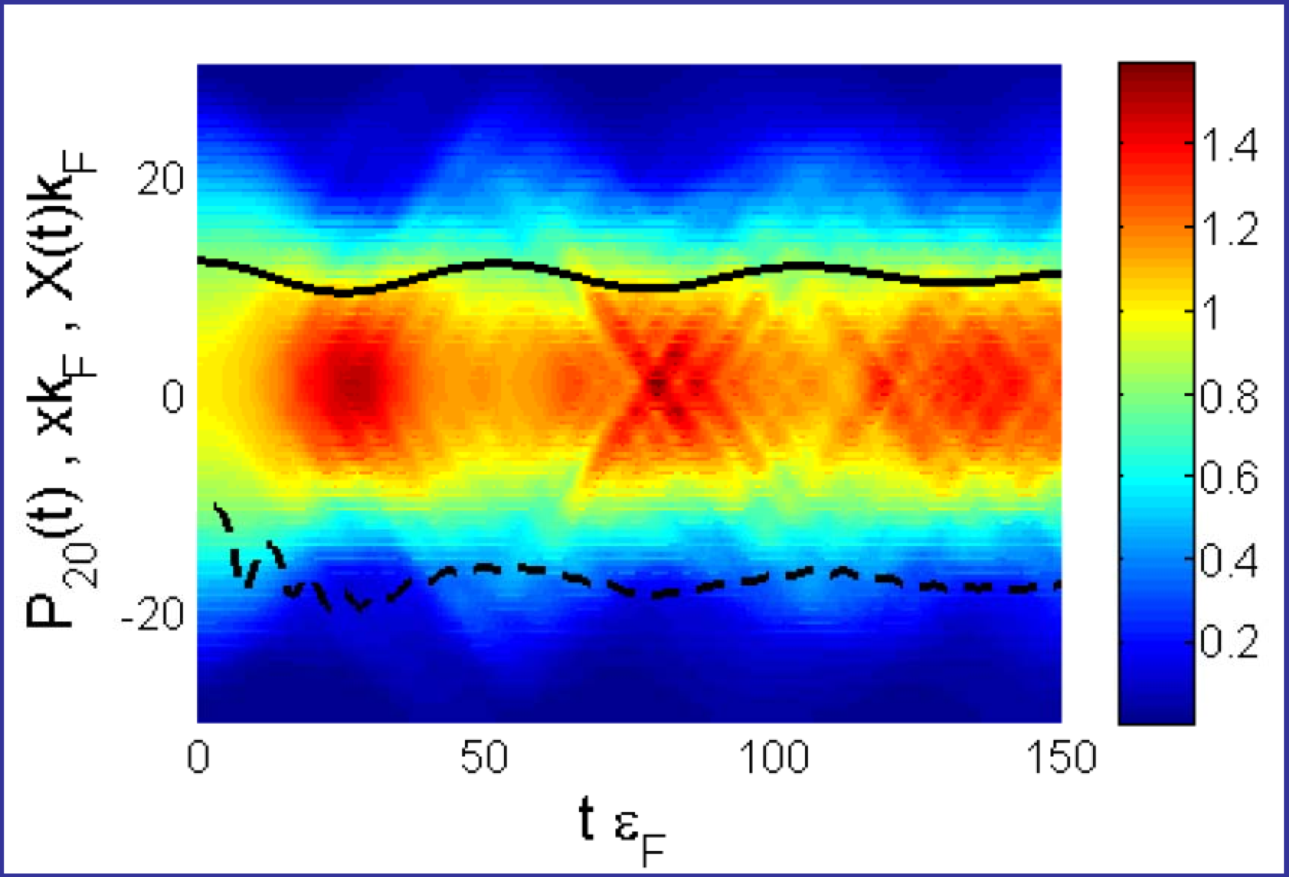


- All these modes have a very low frequency below the pairing gap and a very large amplitude and excitation energy as well
- None of these modes can be described either within Quantum Hydrodynamics or Landau-Ginzburg like approaches

Bulgac and Yoon, Phys. Rev. Lett. 102, 085302 (2009)

3D unitary Fermi gas confined to a 1D HO potential well (pancake)

New qualitative excitation mode of a superfluid Fermi system (non-spherical Fermi momentum distribution)



Black solid line – Time dependence of the cloud radius
Black dashed line – Time dependence of the quadrupole moment of momentum distribution

Bulgac and Yoon, Phys. Rev. Lett. 102, 085302 (2009)

Vortex generation and dynamics

See movies at

http://www.phys.washington.edu/groups/qmbnt/vortices_movies.html

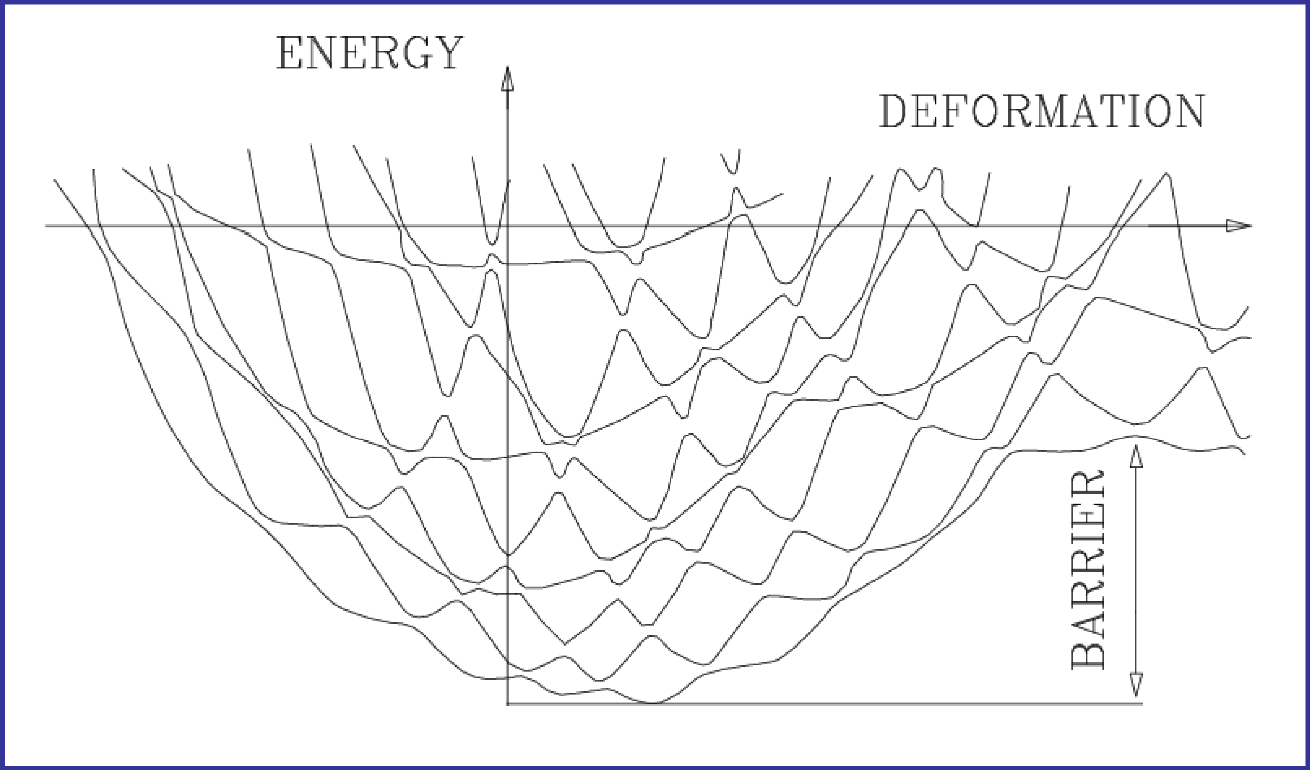
Time-Dependent Superfluid Local Density Approximation

This is a general many-body problem with direct applications, which will provide the time dependent response of superfluid fermionic systems to a large variety of external probes for both cases of small and large amplitude collective motion.

- **Nuclear physics: fission, heavy-ion collision, nuclear reactions, response electromagnetic fields, beta-decay, …**
- **Neutron star crust, dynamics of vortices, vortex pinning mechanism**
- **Cold atom physics, optical lattices, …**
- **Condensed matter physics**

- **Next frontier: Stochastic TDSLDA**

Generic adiabatic large amplitude potential energy SURFACES



- In LACM adiabaticity is not a guaranteed
- The most efficient mechanism for transitions at level crossing is due to pairing
- Level crossings are a great source of :

  entropy production (dissipation)
  dynamical symmetry breaking
  non-abelian gauge fields