The pressure, and a possible hidden Hagedorn transition at large-N

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Outline

A. $T \simeq T_c$: Hagedorn at large-N, In search of $T_H \gtrsim T_c$. In preparation

B. $T > T_c$: The pressure deficit of the deconfined phase at large-N. hep-lat/0506034

A. Hagedorn behaviour at large-N, background

Hagedorn '65 : predicted $\rho(E) \sim \exp(E/T_H) \longrightarrow$ ultimate 'Hagedorn' temperature T_H.

T_H in free-string models : $\longrightarrow \rho \sim e^{+cl/a}$ - wins $e^{-E/T} = e^{-\sigma l/T}$ above $T_H = a/c\sigma$
$\longrightarrow$ loops proliferate with $m(T = T_H) = 0$.

In practice have interactions $\Rightarrow$ deconfinement at T_c replaces Hagedorn:

$N \geq 3$: 1st order, $m_t(T_c) > 0 \longrightarrow T_H > T_c$.

Gocksch and Neri '84, Daamgard and Patkos '86, Billo et al. '94, Lucini, Teper and Wenger '03, Aharony et al. '05

But at large-N: interactions are $\mathcal{O}(1/N) \Rightarrow$ Hagedorn picture most attractive

We seek signs of $T_H > T_c$ at large-N.

A. Hagedorn behaviour at large-N, method

Strategy: calculating Polyakov lines' masses, $m_t(T)$ at $T > T_c$.

At $T > T_c$, Domain wall tension grows with N Lucini, Teper, and Wenger '05
$\longrightarrow$ easier to overheat at larger N, go deeper into ***metastable-confined*** phase.

As we over-heat :

Larger finite V effects
More tunnelings $\Rightarrow$ Cannot reach $T_H \Rightarrow$ extrapolate to $m_t(T = T_H) = 0$.

MC's info : Did $N = 8, 10, 12$, but focus on $N = 12$ on a $12^3 \times 5$.

Checked V dependence with $16^3 \times 5$ for $SU(12)$: OK within $\sim 1\sigma$.

Fix scale with σ: Extrapolate (Lucini, Teper and Wenger '05)'s $(a\sqrt{\sigma})_\beta$, and β_c, to $N = 12$.
Confirm at $\beta/2N^2 \simeq 0.344$.

A. Hagedorn behaviour at large-N, results for $SU(12)$

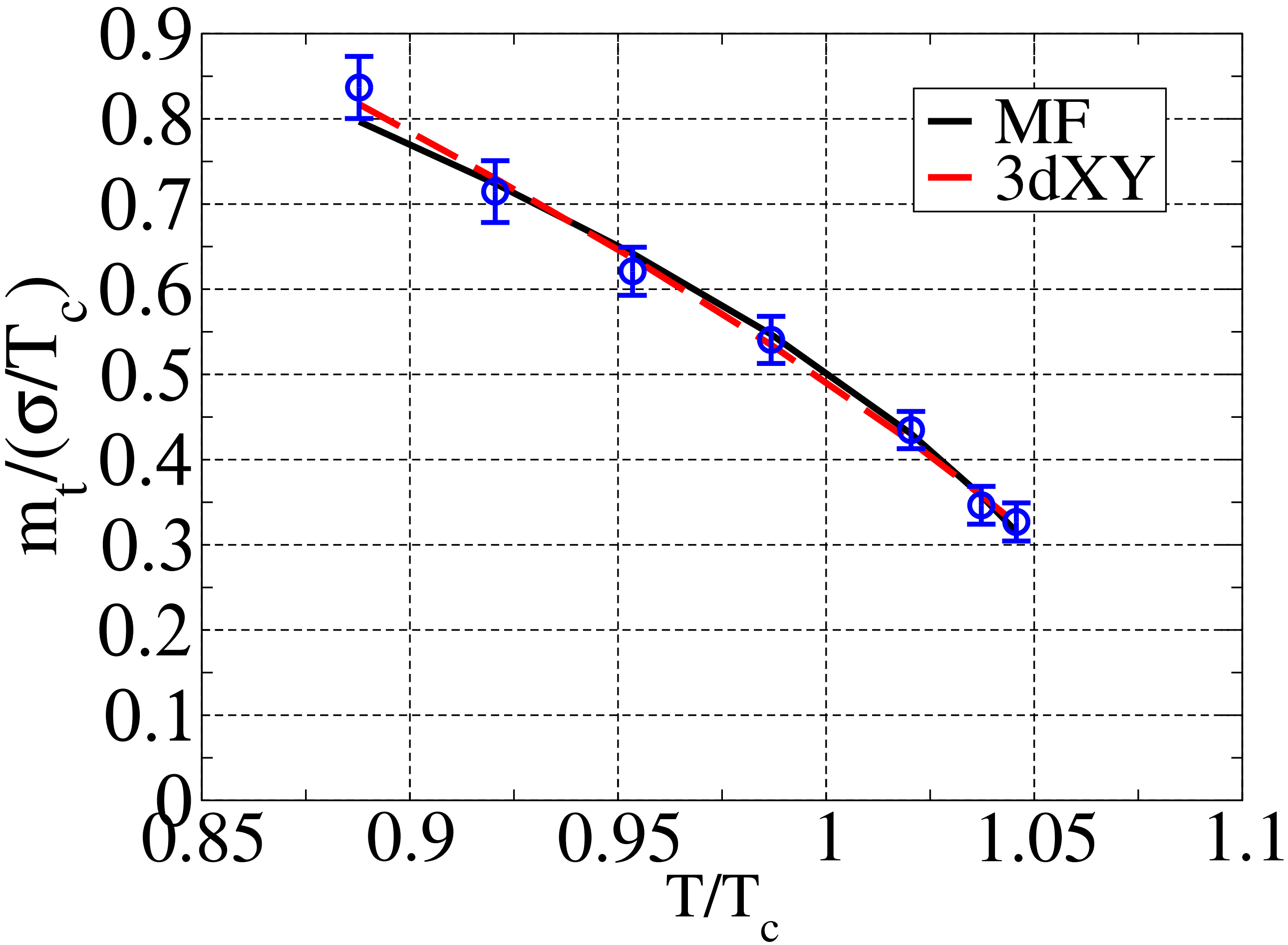
Fit: $m_t(T)/(\sigma/T_c) \equiv$ $\boxed{\sigma_{\text{eff}}/\sigma \times T/T_c}$ with $A(T_H/T_c - T/T_c)^\nu$, $\nu =$MF, 3dXY.

- $\frac{\sigma_{\text{eff}}(T_c)}{\sigma} \simeq 0.5$

- **MF:** $\chi^2/dof \simeq 0.5$
  $A = 1.842$
  $\frac{T_H}{T_c} = 1.075(7)$
  $\frac{T_H}{\sqrt{\sigma}} = 0.617(4)$

- **3dXY:** $\chi^2/dof \simeq 0.3$
  $\frac{T_H}{T_c} = 1.099(10)$
  $\frac{T_H}{\sqrt{\sigma}} = 0.630(9)$
  $A = 2.321$

- $SU(2)$: $\frac{T_c}{\sqrt{\sigma}} \simeq 0.7$



B. The pressure deficit in the deconfined phase, background

Recent years : simple QGP is unsuitable for $T_c \leq T \leq 4T_c$. Here we focus on:

$10 - 20\%$ pressure deficit from free gas Boyd et al. '96

Approached by many, a few: perturbation+IR div. Kajantie et al. '02, ressumations Blaizot et al. '03 (review), quasi-particles Peshier et al. '96, Levai and Heinz '97, loosely bound states Shuryak and Zahed '04, loop models Pisarski '00, AdS/CFT Gusber et al. '98, small volume Aharony et al. '03.

At $N = \infty$ these simplify (and sometimes become soluble). .

- **Q: What happens to the bulk thermodynamics at large-N ?**
- **A: Can constrain models, point to important ingredients.**

B. The pressure deficit in the deconfined phase, the method

Larger N $\longrightarrow$ smaller L_t $\rightarrow$ larger a $\rightarrow$ "integral" method Boyd et al. '96:

- If $V = \infty$, then $F = V f(T)$, and

$$p(T) = \frac{T}{V}\log Z = \frac{1}{a^4(\beta)L_s^3 L_t}\int_{\beta_0}^{\beta} d\beta' \underbrace{\frac{\partial \log Z}{\partial \beta'}}_{6L_t L_s^3 u_p(\beta)} = \frac{6}{a^4(\beta)}\int_{\beta_0}^{\beta} d\beta'\, u_p(\beta')$$

- Regularize: $p(T;\beta) \longrightarrow p(T;\beta) - p(0;\beta)$

$$u_p(\beta) \longrightarrow \delta u_p(\beta) \equiv u_p(L_s^3 L_t) - u_p(L_s^4) \longrightarrow \text{need 2 sets of MC's.}$$

Useful to define the $\Delta/T^4 = \frac{\partial(p/T^4)}{\partial \log T} = 6L_t^4\,\delta u_p \times \frac{\partial \beta}{\partial \log a^{-1}}$,

$\longrightarrow \epsilon = \Delta + 3p, s\,T = \Delta + 4p.$

Lattice volumes : $16^3 5$ for $SU(4)$, $8^3 5$ for $SU(8)$, $20^3 5$ for $SU(3)$

B. The pressure deficit in the deconfined phase, finite V check

On $L_s^3 \times 5$: Need $L_s/\xi \gg 1$

$\rightarrow \xi = 12.5$ for $SU(3)$, and $5.2, 2.4$ for $SU(4,8)$ Lucini, Teper, and Wenger '05

$\rightarrow SU(8)$: At most 2σ between $L_s = 8, 14$. ✓ (mostly 1σ)

On L_s^4: Need $V^{1/3}T_c \gg 1$

$\rightarrow$ But for $N = 8$ get $V^{1/3}T_c = 1$ at $T = 1.6T_c$

$\rightarrow SU(8)$: a huge 16σ between $L_s = 8, 16$. ✗

We use (Lucini and Teper '01, Lucini, Teper and Wenger '04)'s u_p from lattices up to $L_s = 16$, and fit $u_p(\beta)$ with

$$u_p(\beta) = \underbrace{u_p^{PT}(\beta)}_{\text{Alles et al. '98}} + \frac{\pi^2}{12}\frac{G_2}{N}a^4(\beta) + c_4 g^8 + c_5 g^{10}$$

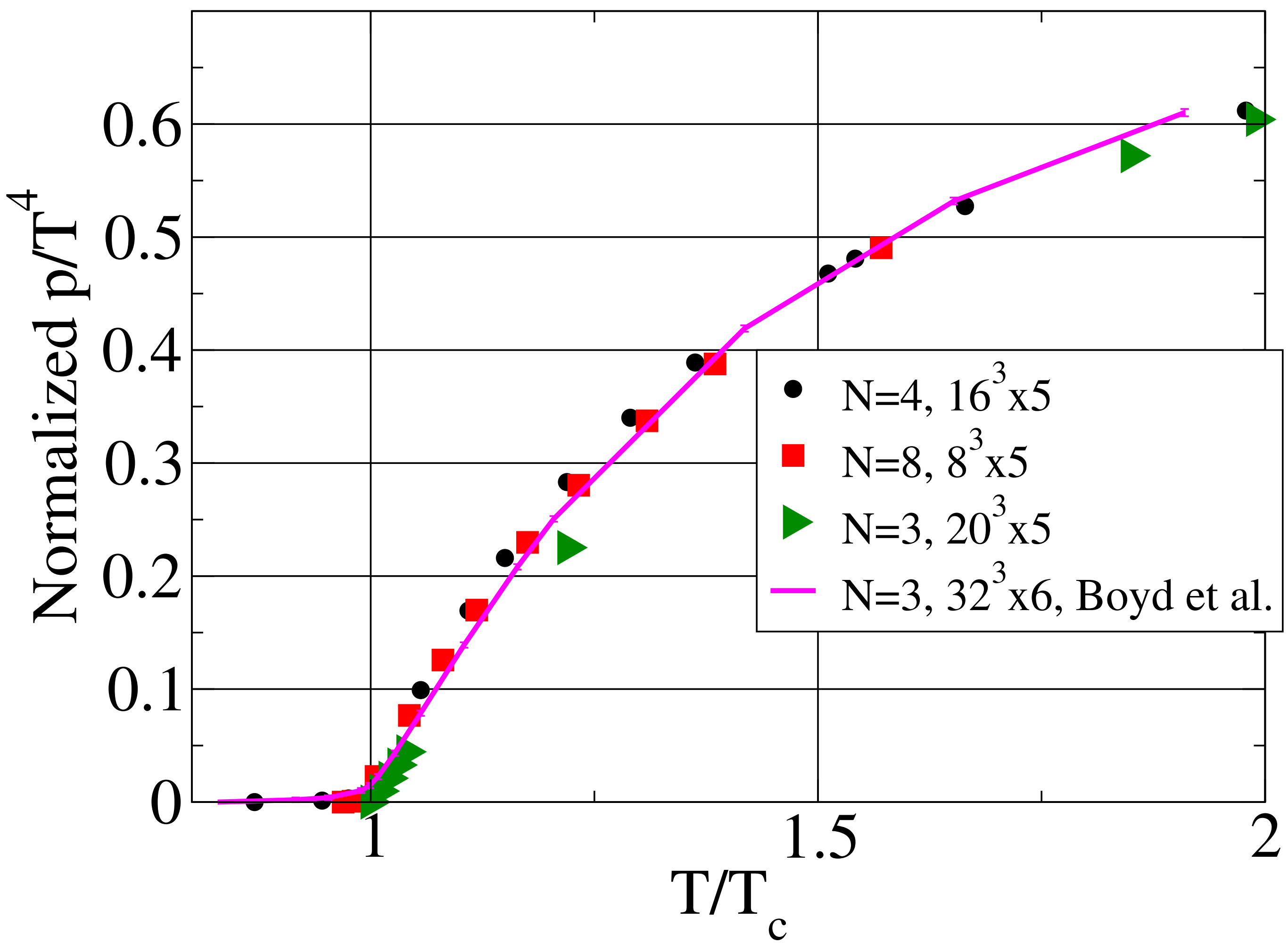
Finally for $N = 4, 8$: we separate phases according to $\beta_c(V = \infty)$ Lucini, Teper and Wenger '05.

B. The pressure deficit in the deconfined phase, results for pressure

$\frac{p/T^4}{\text{free}}$, free$=(N^2-1)\frac{\pi^2}{45}\left[1+\frac{30}{63}\left(\frac{\pi}{L_t}\right)^2+\ldots\right]$ Boyd et al. '96, Heller and Karsch '84.

$\downarrow$
$\mathbf{1}$

Pressure plots lie almost on top of each other.

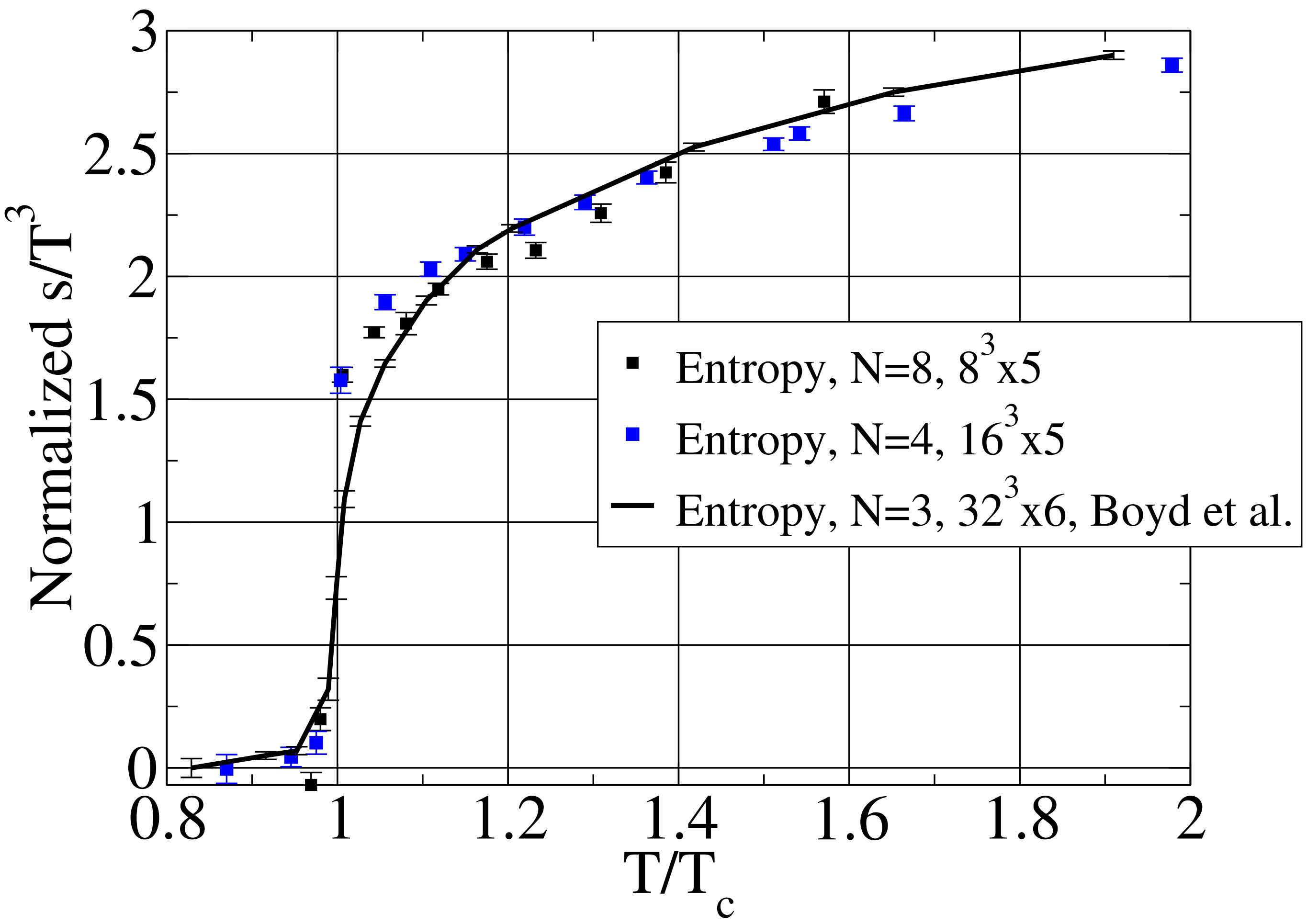


B. The pressure deficit in the deconfined phase, results and entropy

$\frac{s/T^3}{\text{free}}$, free$=(N^2-1)\frac{\pi^2}{45}\left[1+\frac{30}{63}\left(\frac{\pi}{L_t}\right)^2+\ldots\right]$ Boyd et al. '96, Heller and Karsch '84.

$\downarrow$
$\mathbf{4}$

(1) s/T^3 have modest $O(1/N)$ corrections.
(2) Have a sharper jump for $N=4,8$ than for $N=3$.
(3) s/T^3 is not that far from $3/4$.



B. Summary

For $T \gtrsim T_c$ examine decrease of $m_t(T)$ at large-N in metastable confined phase, extrapolate to zero $\rightarrow$ estimate of T_H, and

We find signs of Hagedorn behaviour:

- $T_H/T_c = 1.075(7) - 1.099(10)$ for $SU(12)$.
- $\sigma_{\rm eff}/\sigma \simeq 0.5$, also for $N = 8, 10$.

$\Rightarrow$ Valuable for effective potential in loop models, central charge in string models.

For $T > T_c$ we calculate bulk termodynamics with the integral method,

We find that the pressure deficit is present at large-N:

- $\frac{p/T^4}{\rm free}$ - weak N-dependence.
- $\frac{\epsilon/T^4, \Delta/T^4, s/T^3}{\rm free}$ - weak N-dependence

(sharper jump at T_c for $N = 4, 8$).

$\Rightarrow$

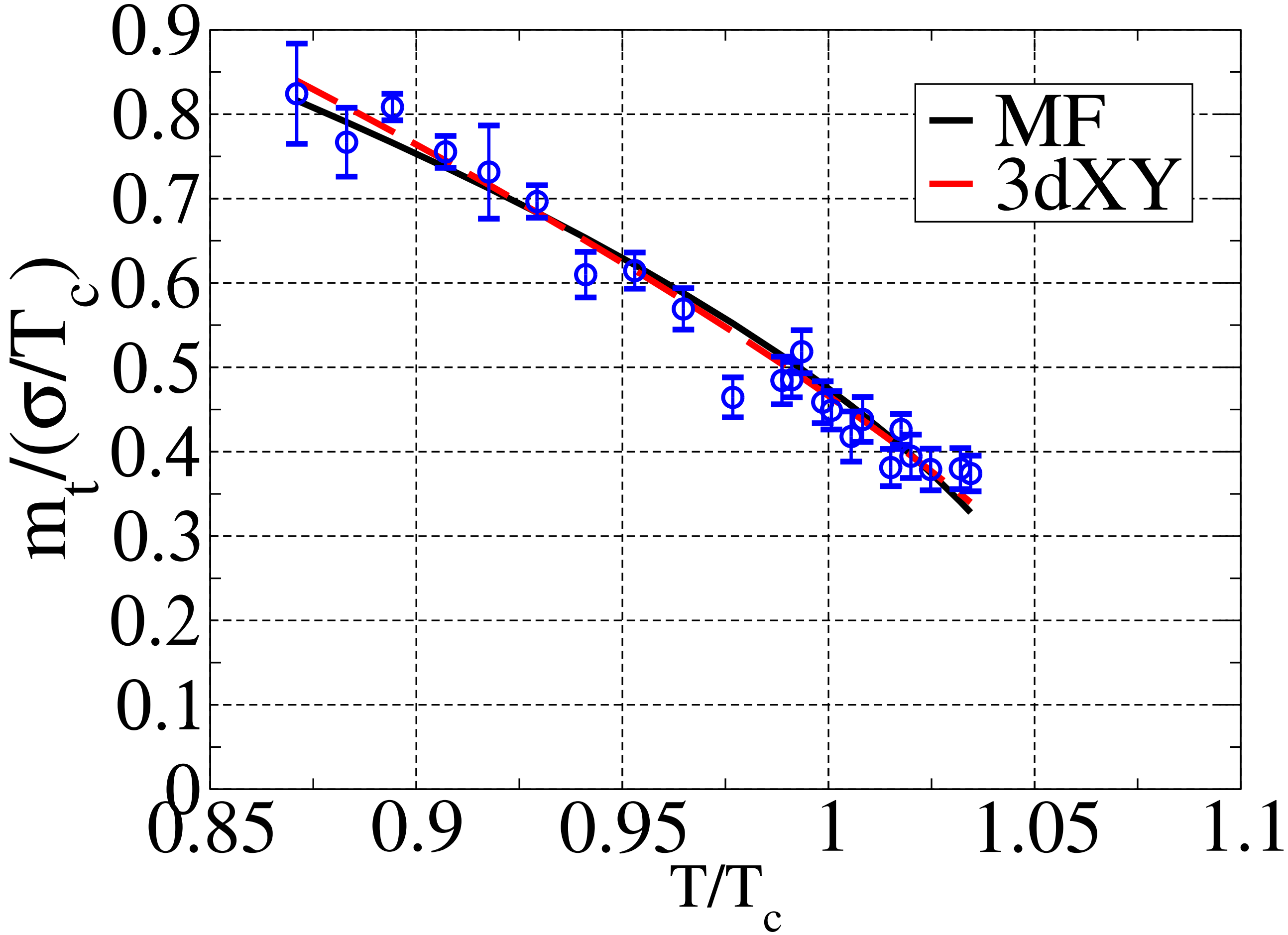
- Descriptions (diagramatic, bound states, loops, quasi-gluons) of the pressure deficit should survive the $N \rightarrow \infty$ limit.
- No role for topology Lucini, Teper and Wenger '04
- Bridge to AdS/CFT predictions Del Debbio et al '04

Questions and coffee . . .

A. Hagedorn behaviour at large-N, results for $SU(10)$

Fit: $m_t(T)/(\sigma/T_c) \equiv \boxed{\sigma_{\rm eff}/\sigma \times T/T_c}$ with $A(T_H/T_c - T/T_c)^{\nu}$, ν =MF, 3dXY.

- $\frac{\sigma_{\rm eff}(T_c)}{\sigma} \simeq 0.5$
- **MF:** $\chi^2/dof \simeq 2$
  $A = 1.849$
  $\frac{T_H}{T_c} = 1.066$
  $\frac{T_H}{\sqrt{\sigma}} = 0.614$
- **3dXY:** $\chi^2/dof \simeq 1.4$
  $A = 2.314$
  $\frac{T_H}{T_c} = 1.092$
  $\frac{T_H}{\sqrt{\sigma}} = 0.629$



A.appendix. Hagedorn behaviour at large-N, comparison with other groups.

$\sigma_{\text{eff}} = m_t/T$

$N = 2$

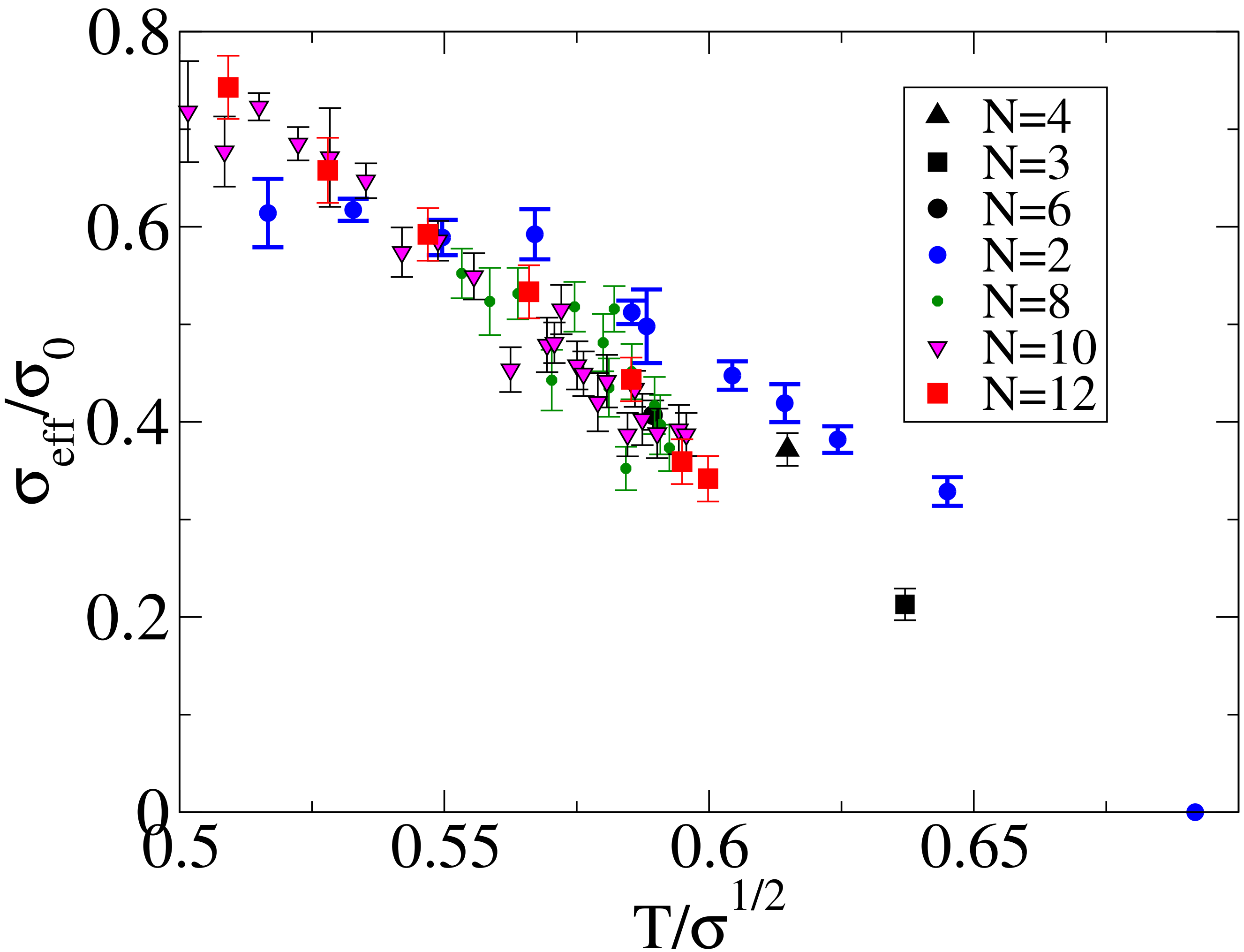
$N = 3, 4, 6$ Lucini, Teper

$N = 8, 10, 12$ and Wenger '05

(1) $SU(2)$ seems to be different.

(2) For $N \geq 3$ $\sigma_{\text{eff}}/\sigma_0 \stackrel{N\to\infty}{\longrightarrow} \sim 0.5$

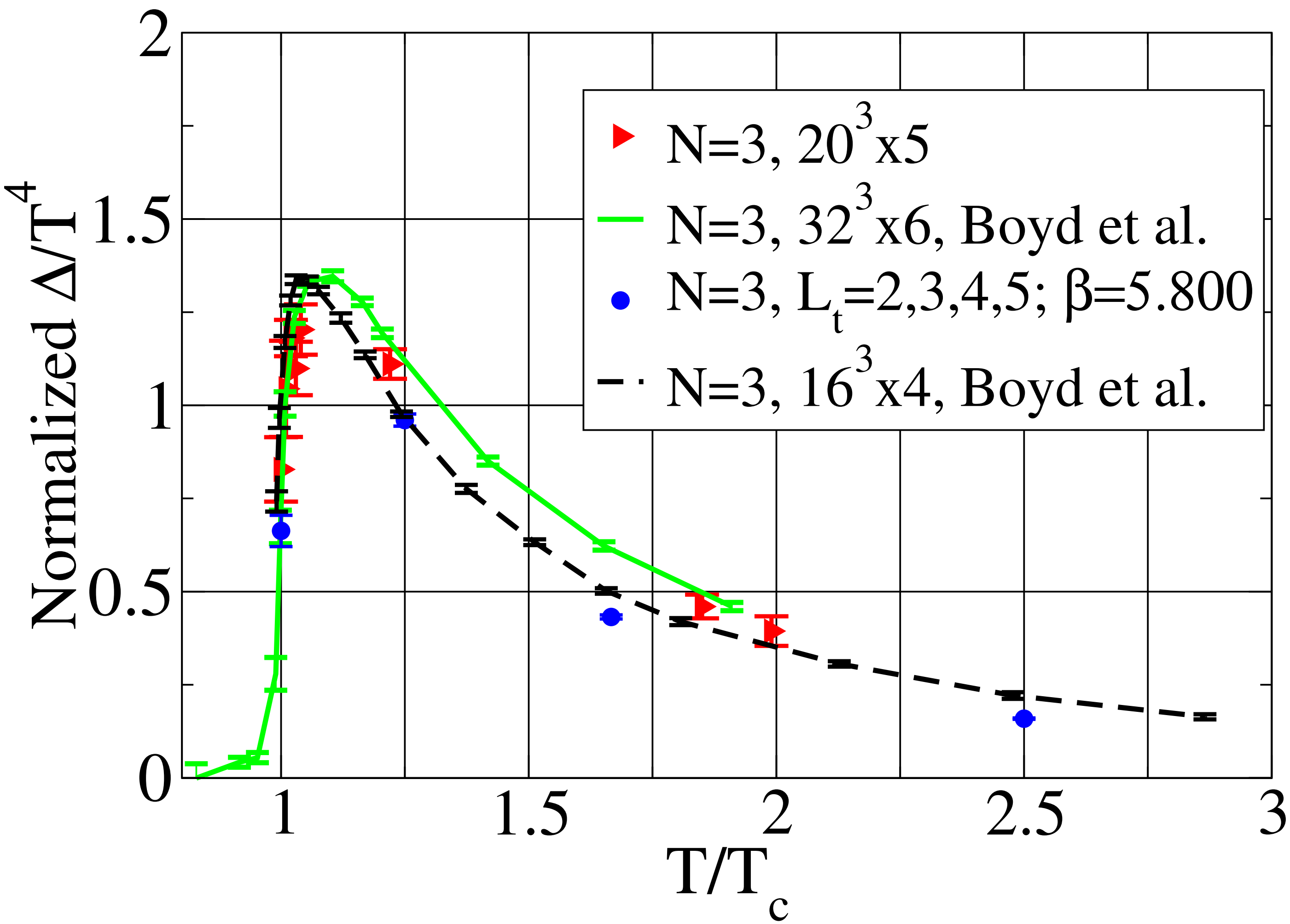
(3) Seems that $T_H/\sqrt{\sigma} \searrow$ with N.



B.appendix. The pressure deficit in the deconfined phase, results for $\boldsymbol{\Delta}$ of $SU(3)$.

$L_t = 4, 5, 6.$

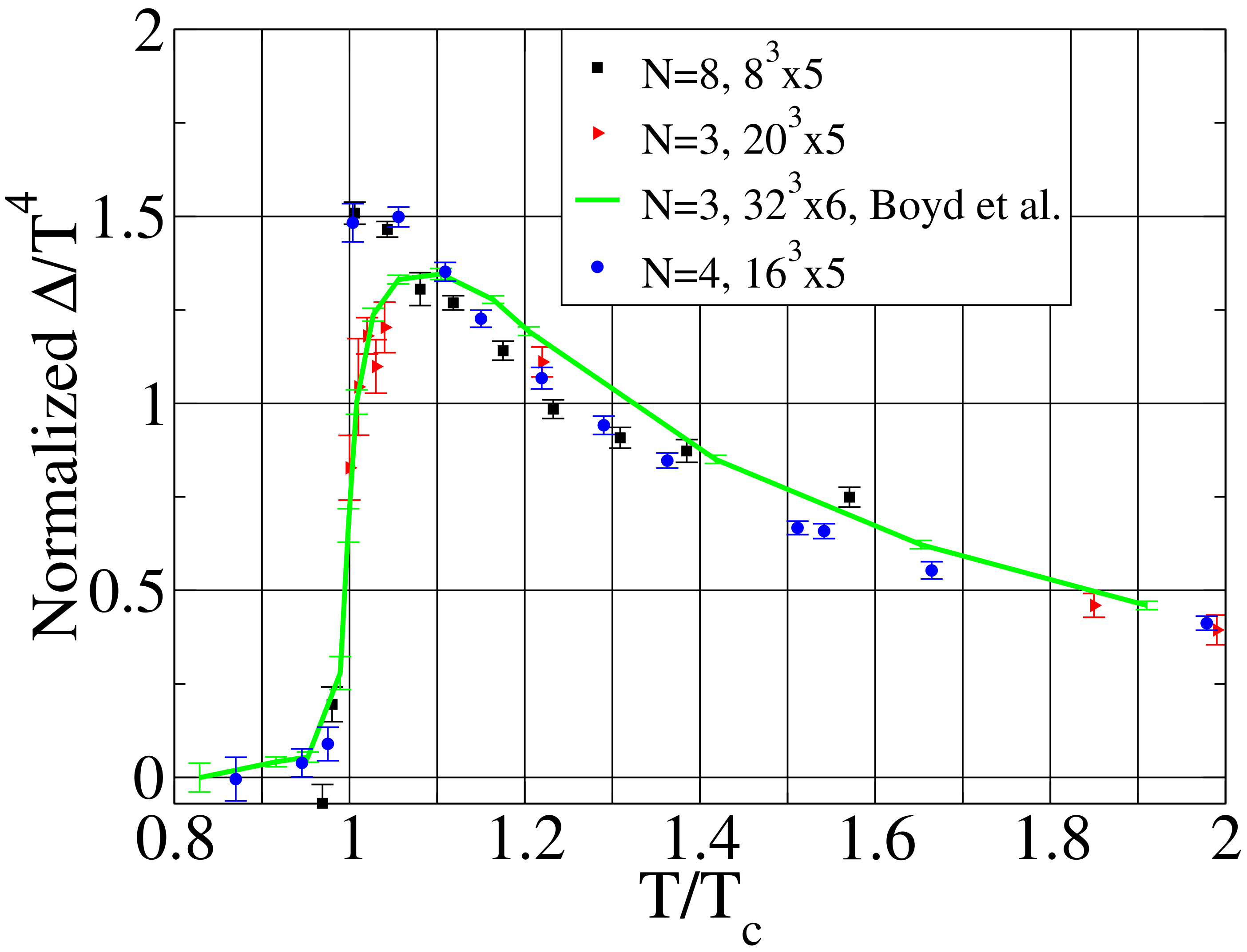
Δ has systematic change with L_t $\rightarrow$ we are not "missing" anything.



B.appendix. The pressure deficit in the deconfined phase, results for $\boldsymbol{\Delta}$.

$L_t = 5$.

(1) Δ has modest $O(1/N)$ corrections.
(2) At $T = T_c$ Δ is different, possibly due to phase separation.
(3) See systematic L_t dep.



B.appendix. The pressure deficit in the deconfined phase, results for $\boldsymbol{\Delta}$.

$L_t = 2, 3, 4, 5$

Δ has modest $O(1/N)$ corrections up to $T = 2.5T_c$.