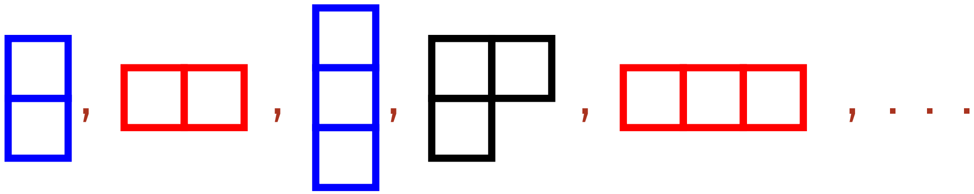
Strings in $SU(N)$ gauge theories in 2+1 dimensions:

Beyond the fundamental representation :



Barak Bringoltz

University of Oxford

(together with M. Teper)

Mostly based on:

BB and M. Teper, in progress.

But also on: BB and M. Teper, hep-th/0611286, PLB. A. Athenodorou, BB and M. Teper, in progress

Motivation

Set benchmarks & test analytic approaches, esp. the Karabali-Nair approach

Karabai-Kim-Nair '98

$$\boxed{\frac{\sqrt{\sigma_F}}{g^2 N} = \sqrt{\frac{1 - 1/N^2}{8\pi}} \quad ; \quad \sigma_{\mathcal{R}} \sim \text{Casimir}} \neq$$ lattice by a few %'s !!!.

Important to remove lattice systematics, esp. for $(\sigma_F)_{N=\infty}$ **[there diff is 1% !].**

$$E_{\text{string}} = \sigma l - \frac{\pi}{6l} \quad + \quad \text{higher order corrections}$$

Particularly interesting : higher order terms not universal :

$\rightarrow$ obtain valuable information on the effective string theory

Lattice calculation

Activity in last decade was rich : $3D, 4D$ with $Z_2, Z_4, U(1), SU(N \leq 6)$. For review Kuti '05, or Caselle and Co., Gliozzio and Co. Kuti and Co., Luscher and Weisz, Majumdar and Co., Teper and Co., Meyer.

Our approach : closed strings

Measure $\langle \boldsymbol{P_x P^*_{x+t}} \rangle$ of Polyakov loops with $\boldsymbol{p_\perp = 0}$ & Variational technique.

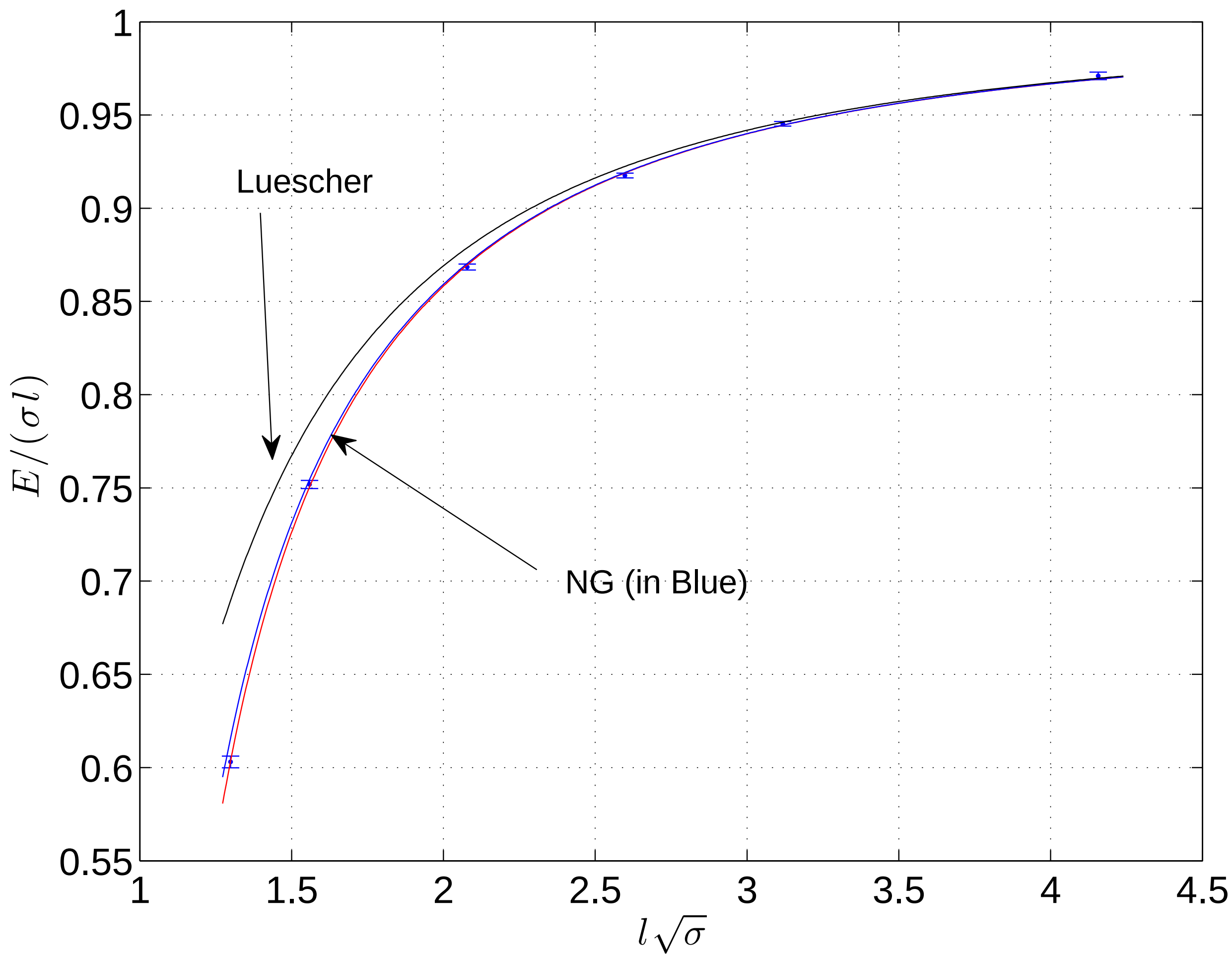
Strings that carry fluxes in any $\mathcal{R} \in \underbrace{\Box \times \ldots \times \Box}_{k}$ acquire z^k under Z_N

couple to all $\left(\mathrm{Tr}\, U^k, \mathrm{Tr}\, U^{k-1} \mathrm{Tr}\, U, \ldots, (\mathrm{Tr}\ (U))^k\right) \rightarrow$ transform the same.

MC of $SU(N)$ with $N = 2, 3, 4, 5, 6, 8$, S_W, $a \simeq 0.11, 0.08, 0.06$ fm, $l \simeq 0.68 - 2.3$ fm.

Fundamental representation

Ground state, $SU(5), a \simeq 0.13/\sqrt{\sigma} \simeq 0.06$ 'fm'



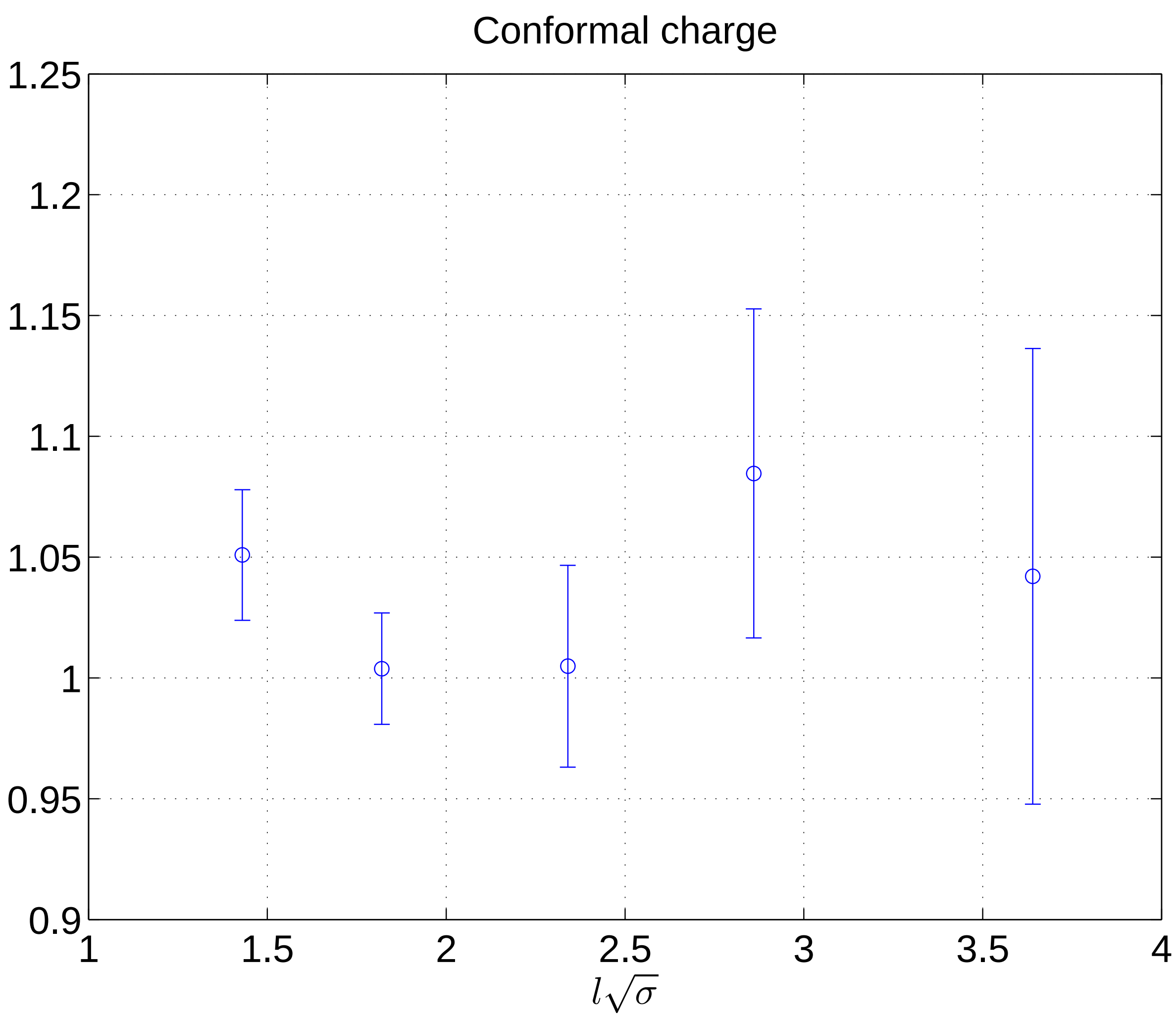
Fit : Luscher& Weisz '04, Drummond '04, Dass & Matlock '06

$$E_0^2 = \mathrm{NG} + \text{correction}$$

$$= (\sigma l)^2 + 8\pi\sigma\left(-\frac{1}{24}\right) - \frac{C(\simeq 0.1-0.2)}{\sqrt{\sigma} l^3}$$

Fundamental representation

Ground state, $SU(5), a \simeq 0.13/\sqrt{\sigma} \simeq 0.06$ 'fm'



In $SU(2), \sim 50M$:

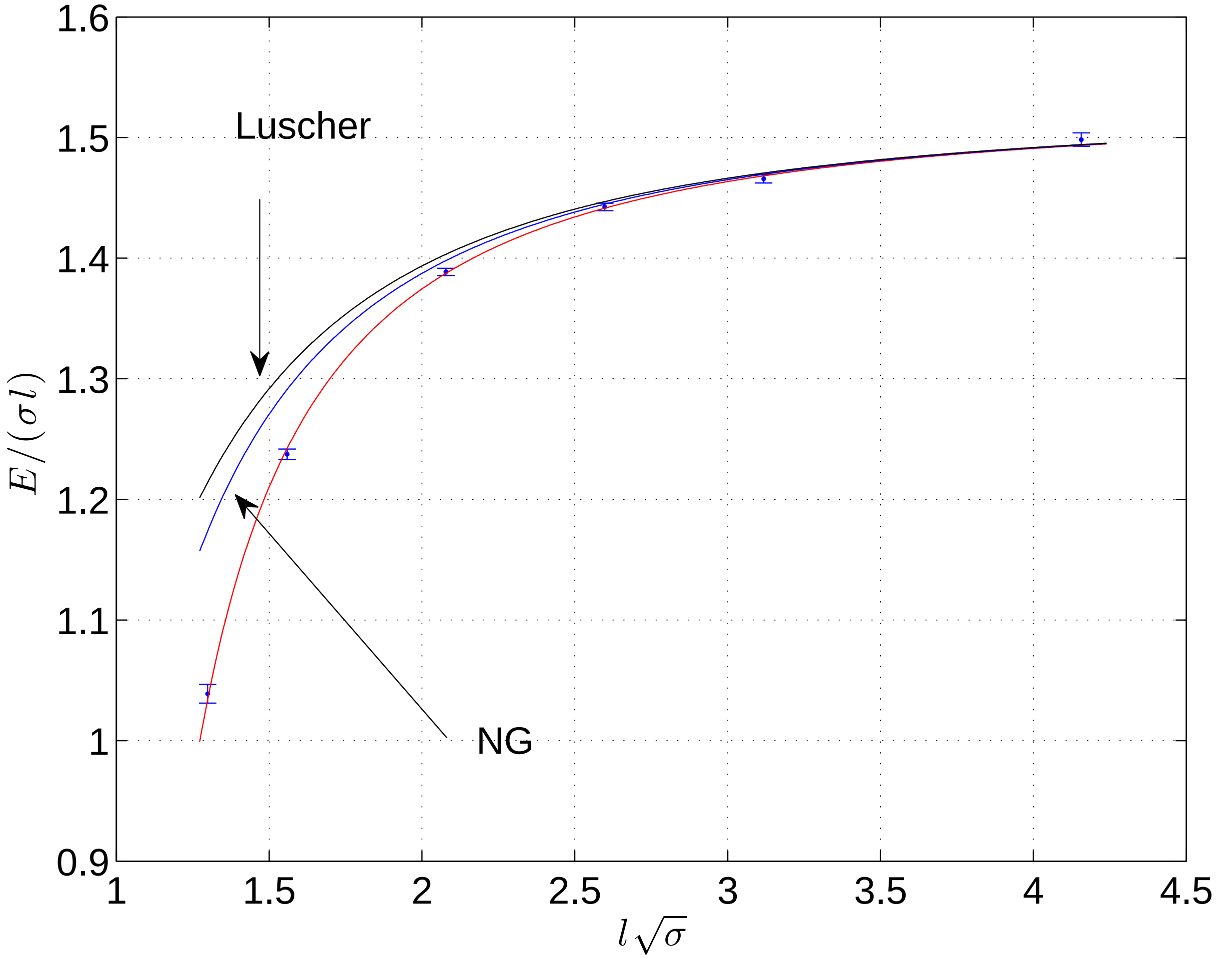
Conformal anomaly $= 1.0004(66)$

When $l \simeq 1.5/\sqrt{\sigma} \simeq 0.68$ fm.

Using NG+correction : $(\sigma_F)_{\mathrm{KKN}} - (\sigma_F)_{\mathrm{Lat.}} \simeq 1\%$ for $SU(\infty)$ is $6-8$ sigma.

Fundamental representation

Ground state, $k = 2$, $SU(5), a \simeq 0.13/\sqrt{\sigma} \simeq 0.06$ 'fm'



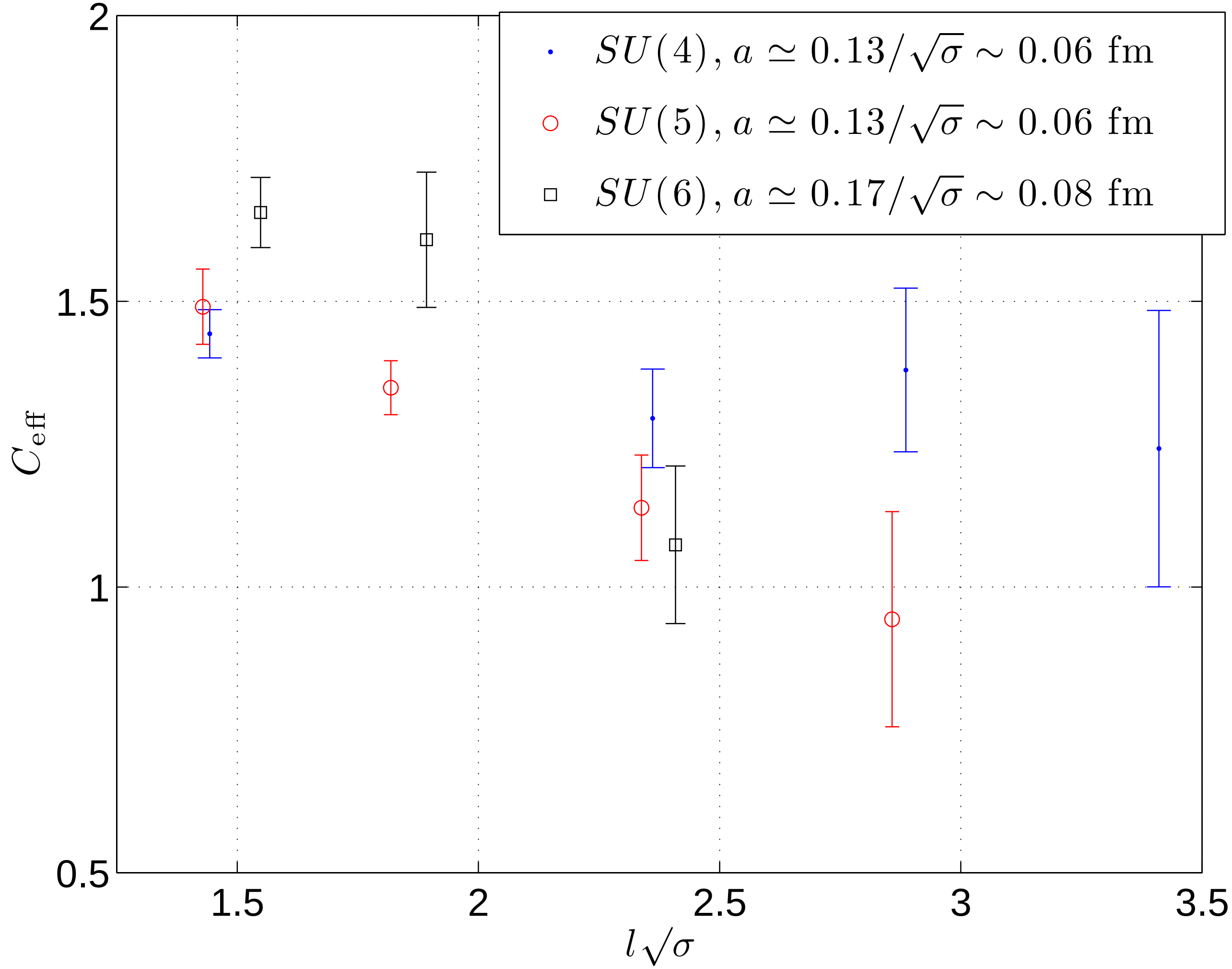
Fit : Luscher& Weisz '04, Drummond '04, Dass & Matlock '06

$$E_0^2 = \mathrm{NG} + \text{correction}$$

$$= (\sigma l)^2 + 8\pi\sigma\left(-\frac{1}{24}\right) - \frac{C(\simeq 1-3)}{\sqrt{\sigma} l^3}$$

Fundamental representation

Ground state, $k = 2$, $SU(5), a \simeq 0.13/\sqrt{\sigma} \simeq 0.06$ 'fm'

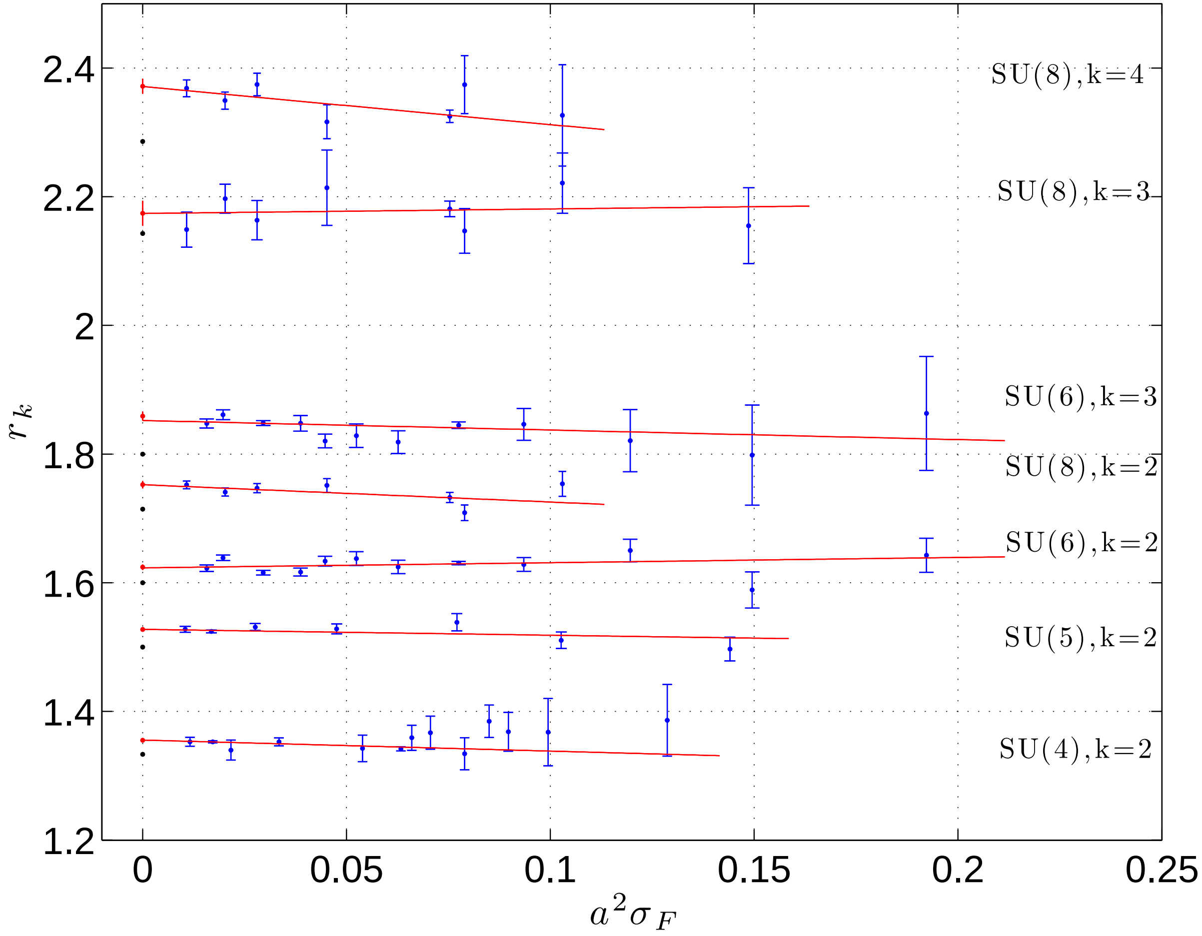


Conformal anomaly decreases to 1

Disagrees with Gliozzi an co. '07

Using NG+correction : tensions for $a \simeq 0.06 - 0.2$ fm with $l \gtrsim 3/\sqrt{\sigma} \simeq 1.35$ fm

Continuum and comparison with Casimir : $r_k = \sigma_k/\sigma_F$

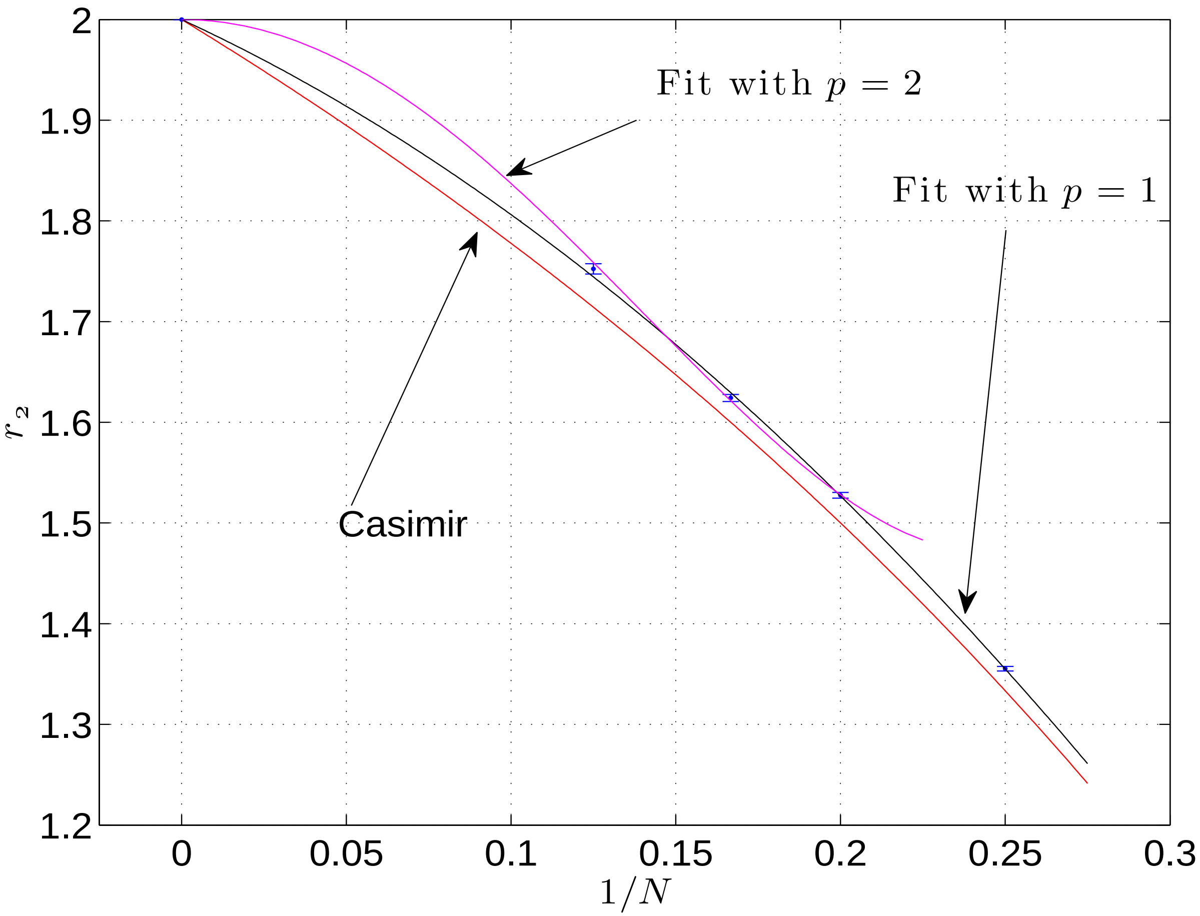


N	k	r_k(lattice)	Casimir
4	2	1.3553(23)	1.3334 . . .
5	2	1.5275(26)	1.5
6	2	1.6242(35)	1.6
8	2	1.7524(51)	1.7142 . . .
6	3	1.8590(63)	1.8
8	3	2.1742(187)	2.1429. . .
8	4	2.3725(111)	2.2857 . . .

Lattice-Casimir$\simeq 1.4 - 3.7\%$, looks many sigma away, but still has systematics

Ready for $1/N$ extrapolations . . .

Large-N extrapolation : $k = 2$



Fit : $\boldsymbol{r_2(N) = 2 - \frac{a}{N^p} - \frac{b}{N^{2p}}}$

p	a	b	χ^2/dof
1	1.51(46)	4.3(2)	$2.2 \to 1$
2	17.8(3)	$-149(9)$	2.14, no $SU(4)$

$p = 1$ seems to fit our data more naturally

Conclusions

String behavior : $\boldsymbol{E^2 - E^2_{\mathrm{NG}} \simeq +\frac{1}{2}\%}$ for $\boldsymbol{l \gtrsim 2.8/\sqrt{\sigma} \simeq 1.25}$ fm.

Conformal anomaly :

Clearly approaches 1 for $k = 2$, going down for $k = 3, 4$ as well.

Disagrees with Gliozzi et al. '07

Strings' tensions :

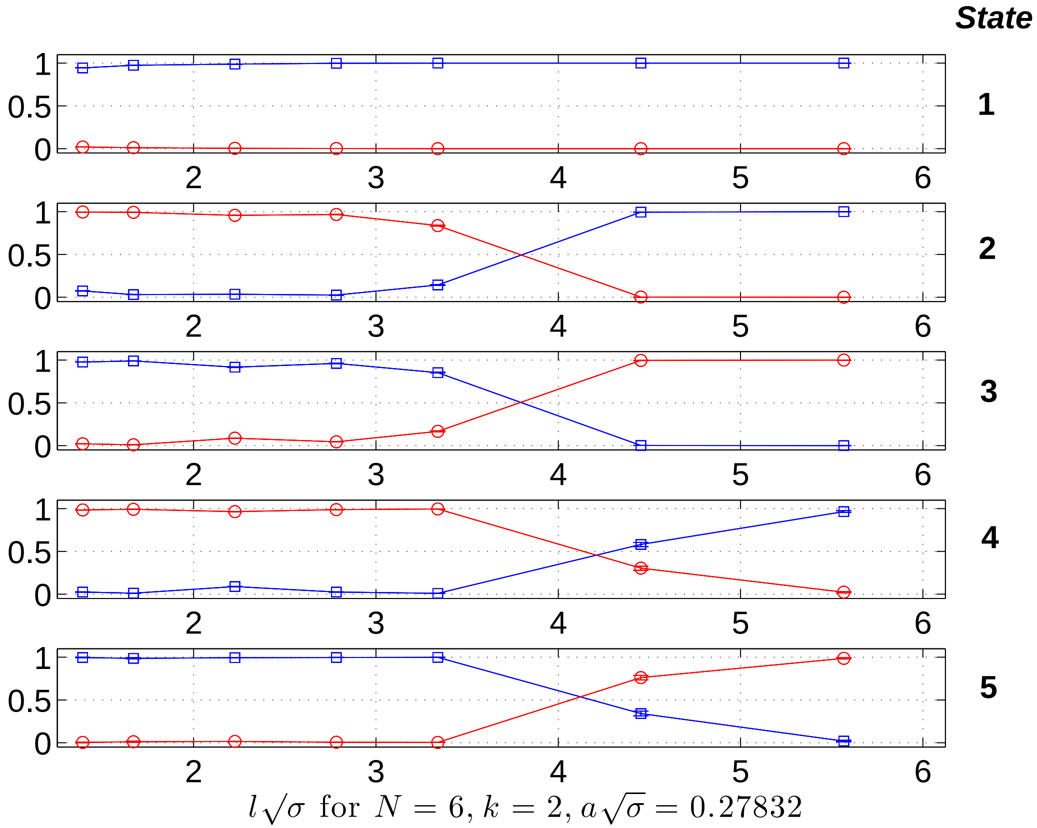
$$\boldsymbol{\left(\frac{\sigma_k}{\sigma_F}\right)_{\mathrm{Lattice}} - \left(\frac{\sigma_k}{\sigma_F}\right)_{\mathrm{Casimir}} = +(1.4 - 3.7)\%}$$

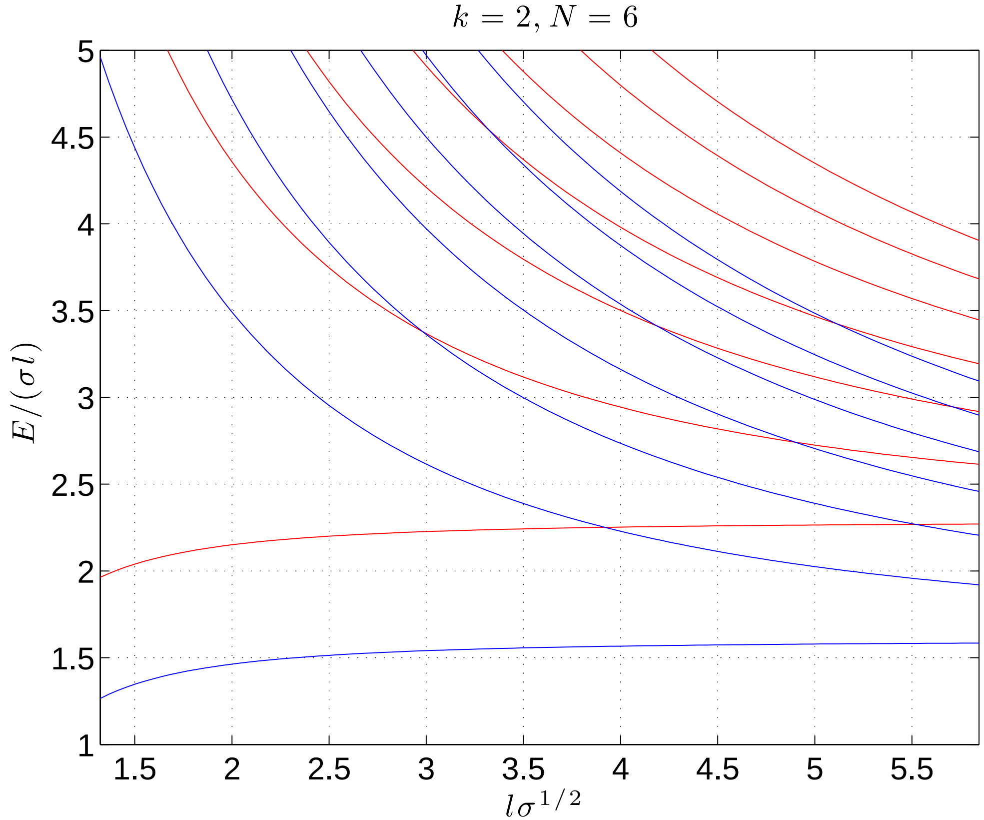
$\boldsymbol{1/N^p}$ **corrections** : to $\boldsymbol{\frac{\sigma_2}{\sigma_F}}$ and $\boldsymbol{\frac{\sigma_{N/2}}{\sigma_F}}$: $\boldsymbol{p = 1}$ seems to fit data more naturally.

Unstable strings : evidence for $\boldsymbol{k}$—NG towers $\longrightarrow$ explains degeneracy of real states.

Puzzles in the $k-$string spectrum

Model : $k-$noninteracting NG 'towers' : $E_k^2 = (\sigma_k l)^2 + 8\pi\sigma_k \left(n_k - \frac{1}{24}\right)$

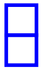


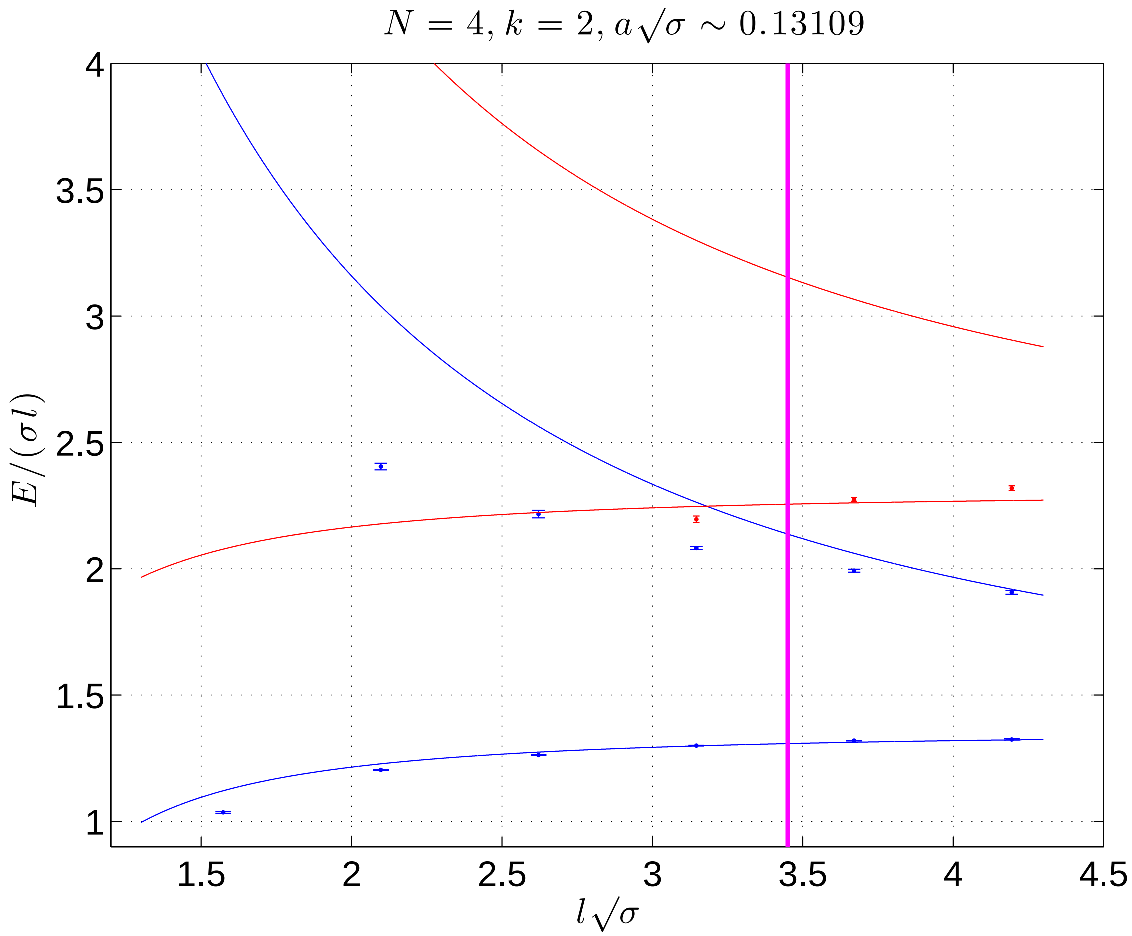


$\sum_i \left|u_{\mathcal{R}}^{(i)}\right|^2$, $\mathcal{R} = \begin{smallmatrix}\square\\\square\end{smallmatrix}, \square\square$

$NG(\sigma_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}, l), NG(\sigma_{\square\square}, l)$ with $\sigma_{\mathcal{R}} \sim C_{\mathcal{R}}$

Puzzles in the $k-$string spectrum

Step 2: Energies of states - separated according to $\mathcal{R} =$ , 

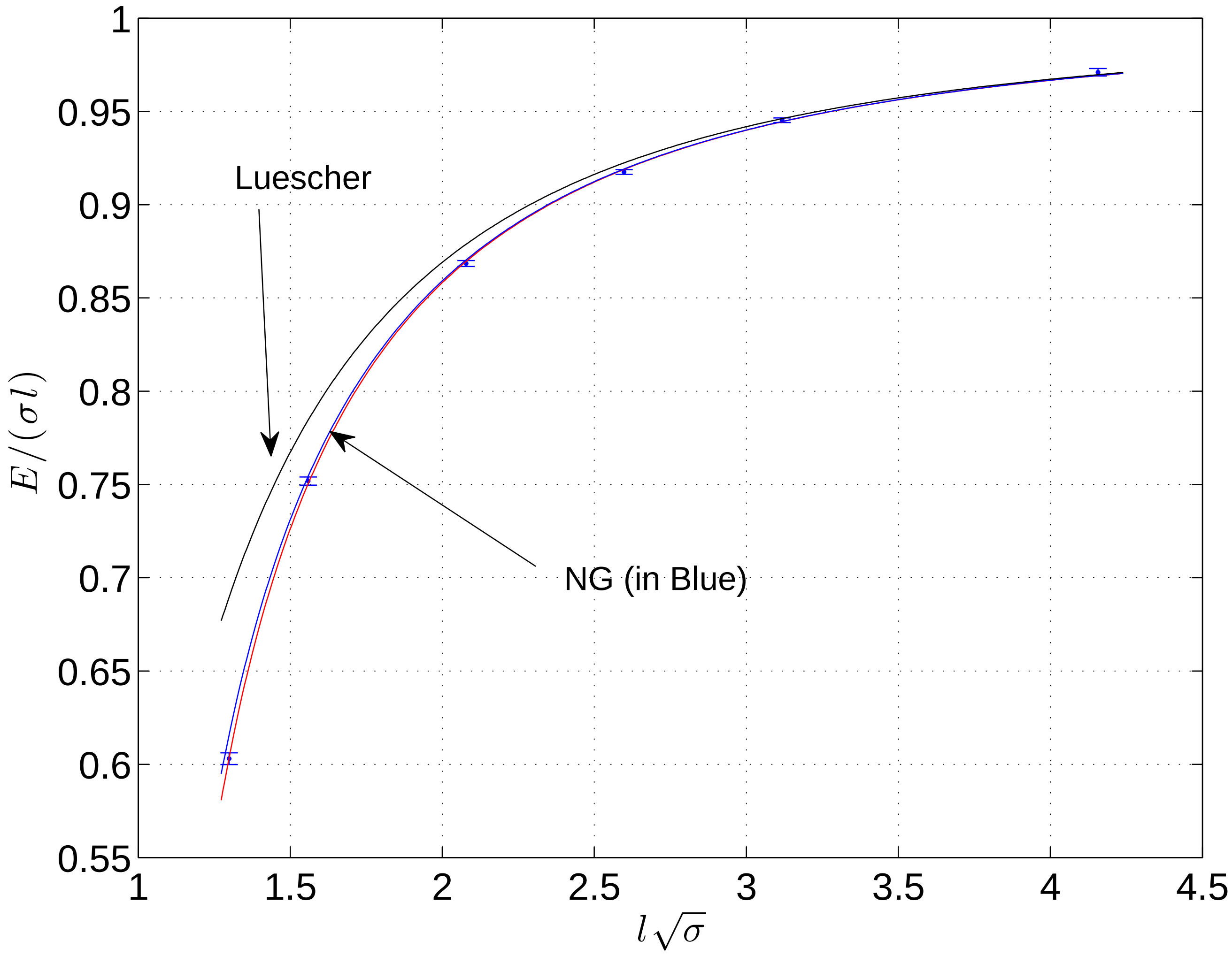


Puzzling degeneracies look like degeneracies between the different towers.

$\longrightarrow$ Picture for $k = 3$ is similar.

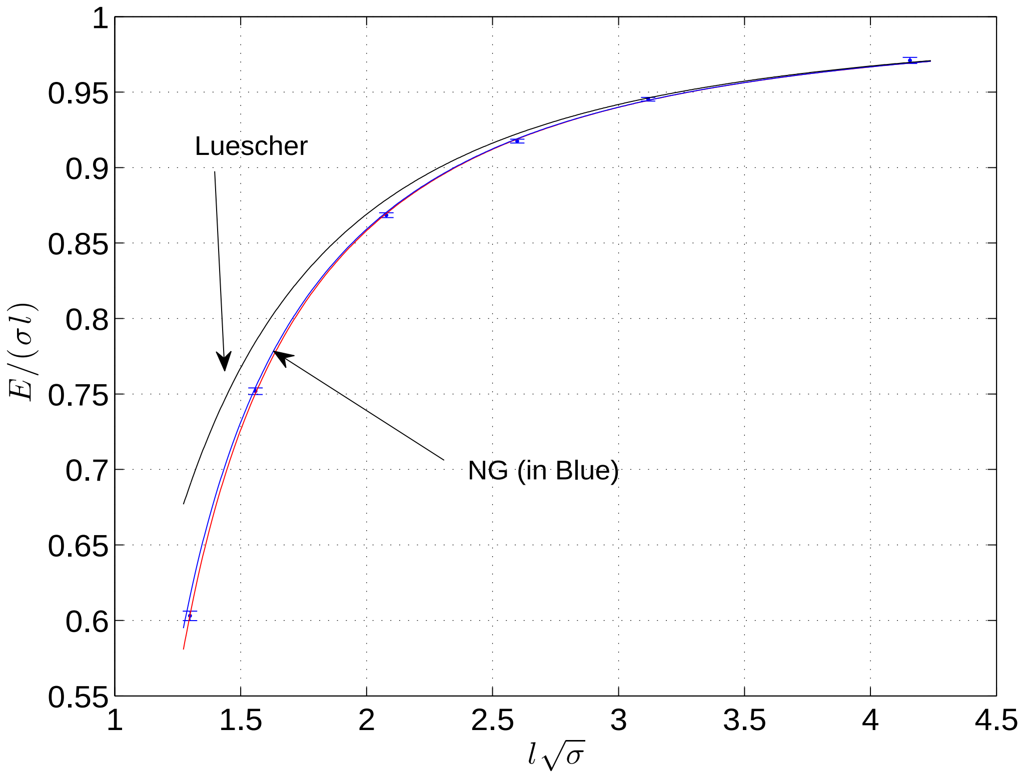
Appendix A : Fundamental representation

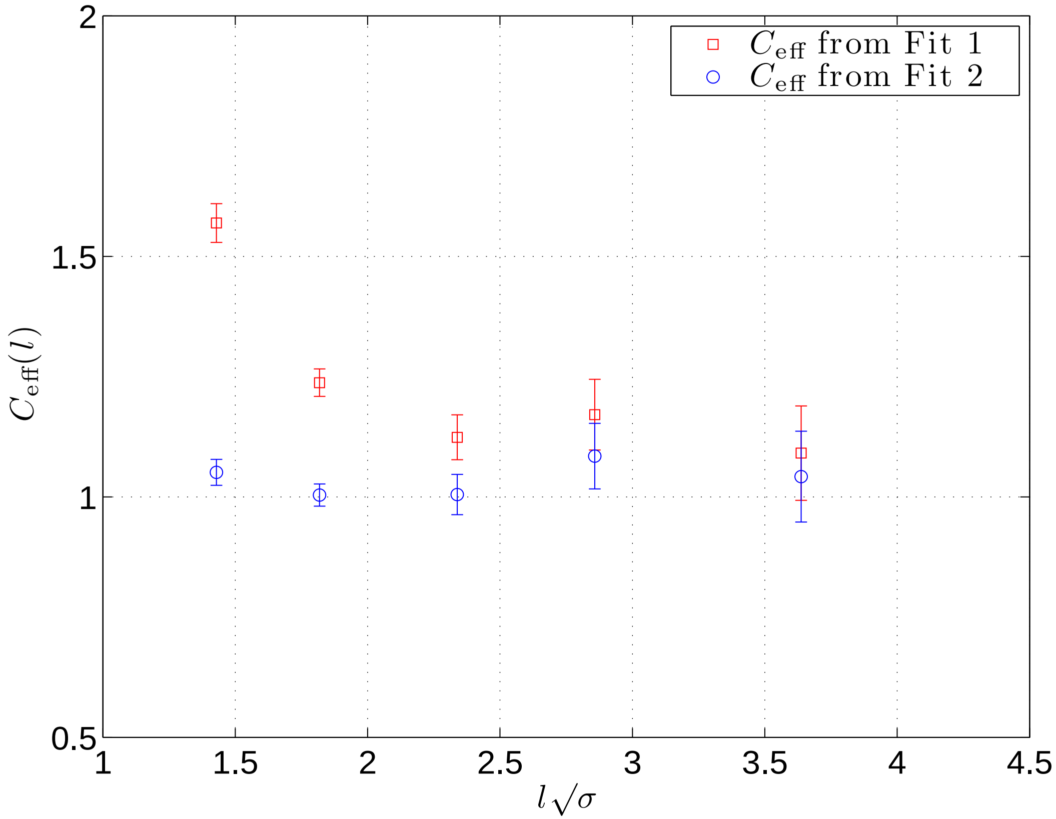
Ground state, $SU(5), a \simeq 0.13/\sqrt{\sigma} \simeq 0.06$ 'fm', Fit : $E_0^2 = \underbrace{(\sigma l)^2 - \frac{\pi\sigma}{3}}_{\text{NG}} - \frac{C}{\sqrt{\sigma} l^3}$



Appendix A : Fundamental representation

Ground state, $SU(5), a \simeq 0.13/\sqrt{\sigma} \simeq 0.06$ 'fm', Fit : $E_0^2 = \underbrace{(\sigma l)^2 - \frac{\pi\sigma}{3}}_{\text{NG}} - \frac{C}{\sqrt{\sigma} l^3}$





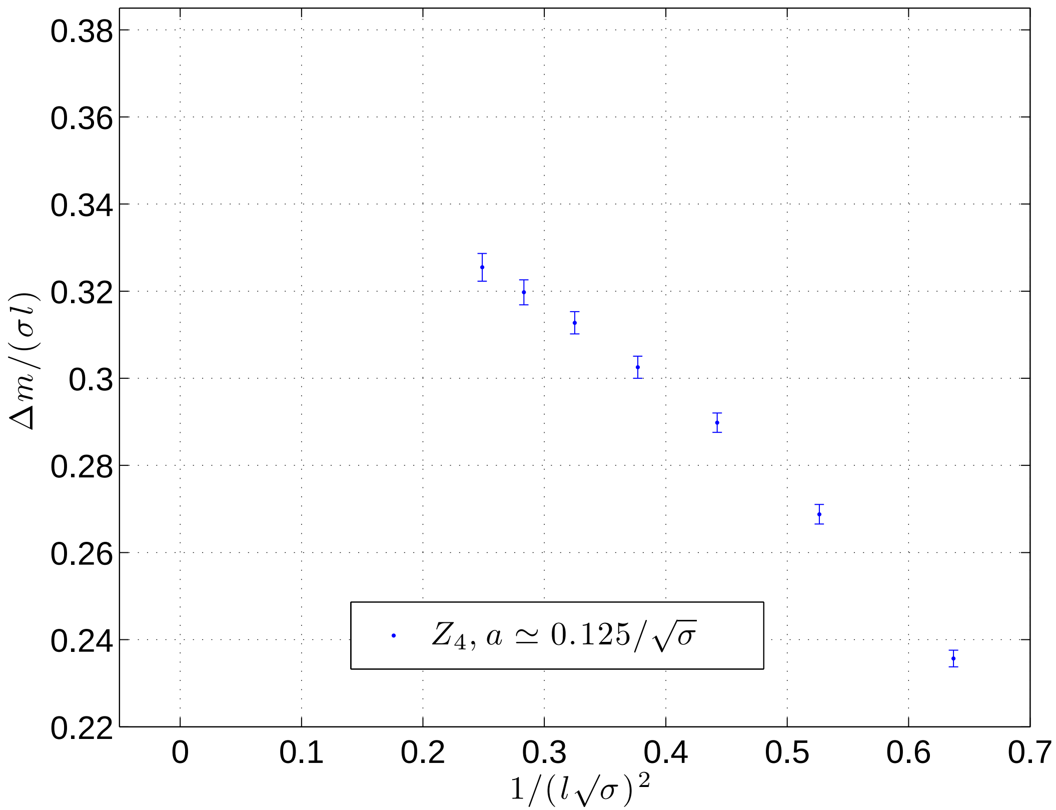
- Fit 1. $E = \sigma l - \frac{\pi}{6l} C_{\rm eff}$ $\Leftrightarrow$ comparing $c_{\rm eff}(r) \sim \frac{\partial^2 V(r)}{\partial r^2}$ with $-\pi/24$
- Fit 2. $E^2 = (\sigma l)^2 - \frac{\pi\sigma}{3} C_{\rm eff}$ $\Leftrightarrow$ analog to $c_{\rm eff}(r) \sim \frac{\partial^2 V(r)}{\partial r^2}$ with NG(open)

Appendix B : Comparison with Z_4.

Giudice, Gliozzi and Lottini '07 study $\boldsymbol{Z_4@3D}$ and Suggest $C_{\text{eff}}^{(k=2)} = \frac{\sigma_k}{\sigma_F} > 1$ and so

$$\frac{m_{k=2} - m_{k=1}}{\sigma l} = (r_2 - 1)\left(1 - \frac{\pi}{6\sigma l^2}\right) + O(1/l^4)$$

In contrast if $C_{\text{eff}}^{(k=2)} = 1$ then $\frac{m_{k=2} - m_{k=1}}{\sigma l} = (r_2 - 1) + O(1/l^4)$

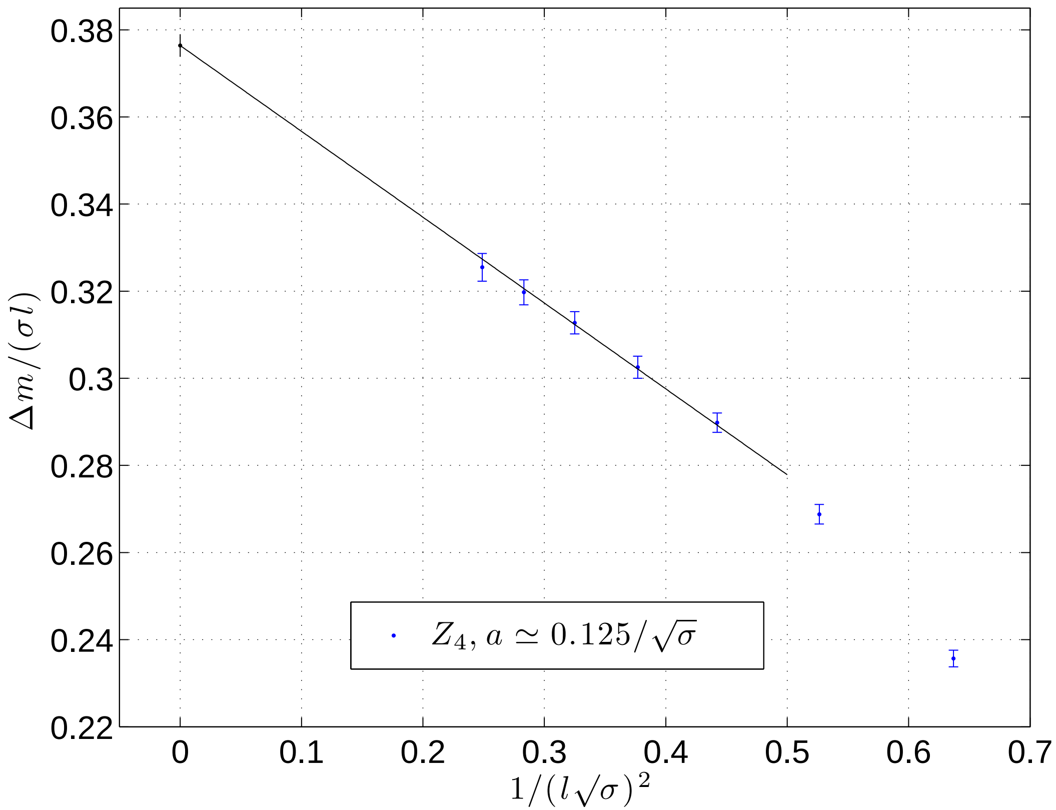


Appendix B : Comparison with Z_4.

Giudice, Gliozzi and Lottini '07 study $\boldsymbol{Z_4@3D}$ and Suggest $C_{\text{eff}}^{(k=2)} = \frac{\sigma_k}{\sigma_F} > 1$ and so

$$\frac{m_{k=2} - m_{k=1}}{\sigma l} = (r_2 - 1)\left(1 - \frac{\pi}{6\sigma l^2}\right) + O(1/l^4)$$

In contrast if $C_{\text{eff}}^{(k=2)} = 1$ then $\frac{m_{k=2} - m_{k=1}}{\sigma l} = (r_2 - 1) + O(1/l^4)$

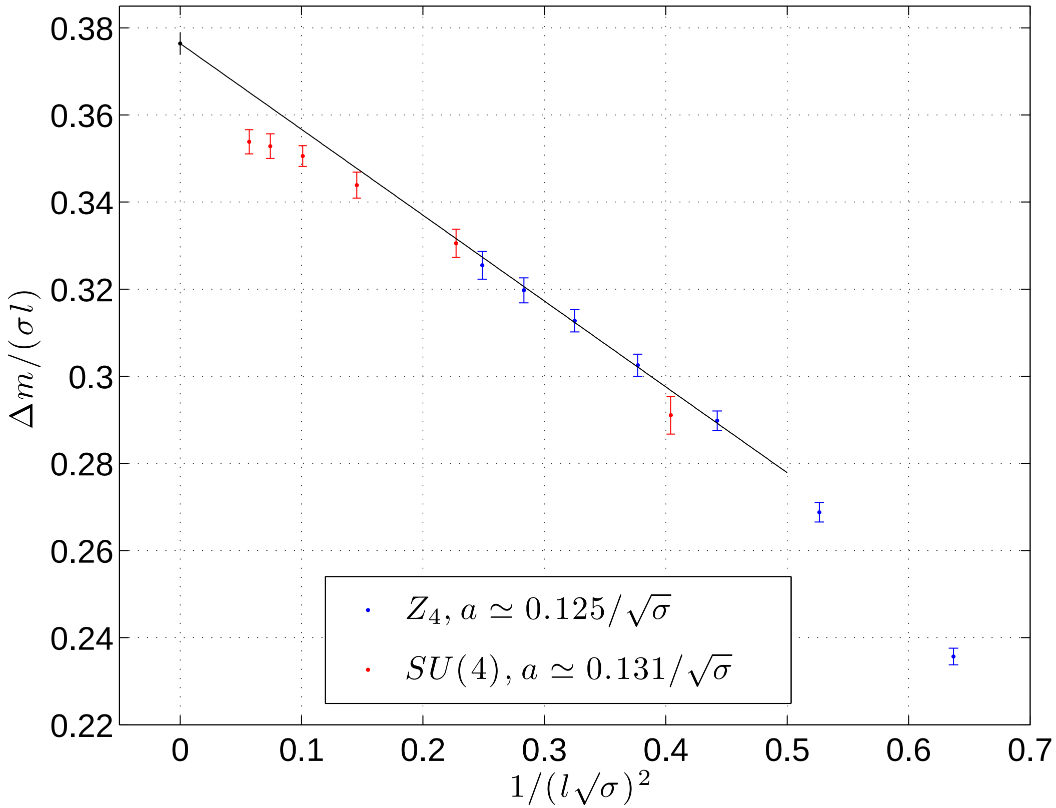


Appendix B : Comparison with Z_4.

Giudice, Gliozzi and Lottini '07 study $\boldsymbol{Z_4@3D}$ and Suggest $C_{\text{eff}}^{(k=2)} = \frac{\sigma_k}{\sigma_F} > 1$ and so

$$\frac{m_{k=2}-m_{k=1}}{\sigma l} = (r_2 - 1)\left(1 - \frac{\pi}{6\sigma l^2}\right) + O(1/l^4)$$

In contrast if $C_{\text{eff}}^{(k=2)} = 1$ then $\frac{m_{k=2}-m_{k=1}}{\sigma l} = (r_2 - 1) + O(1/l^4)$



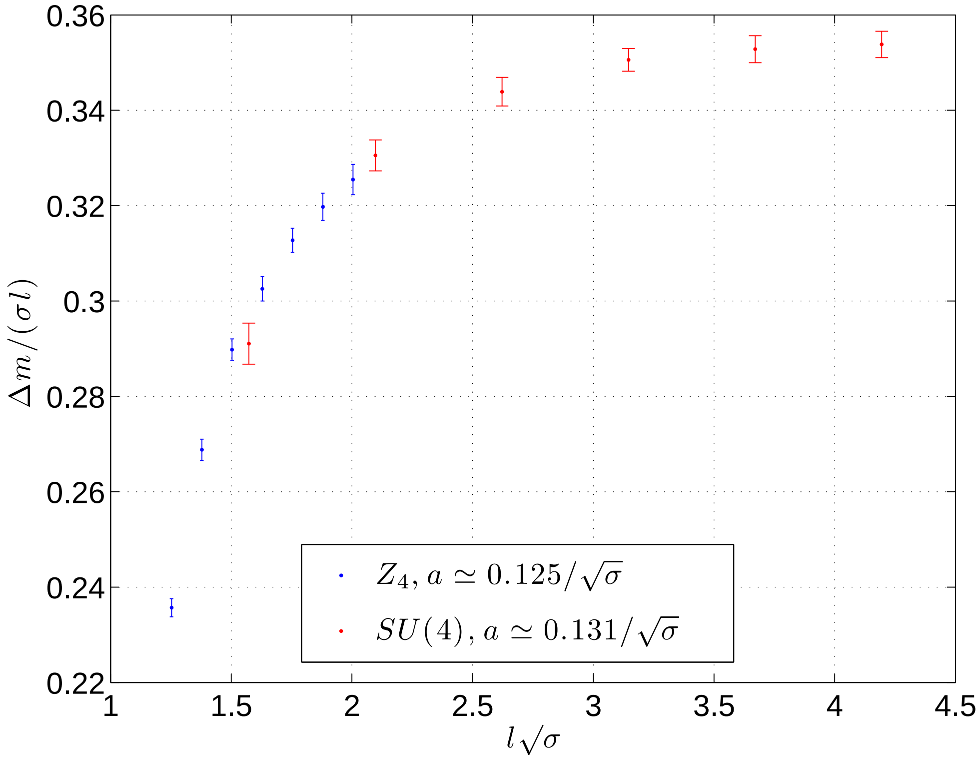
Appendix B : Comparison with Z_4.

Giudice, Gliozzi and Lottini '07 study $\boldsymbol{Z_4@3D}$ and Suggest $C_{\rm eff}^{(k=2)} = \frac{\sigma_k}{\sigma_F} > 1$ and so

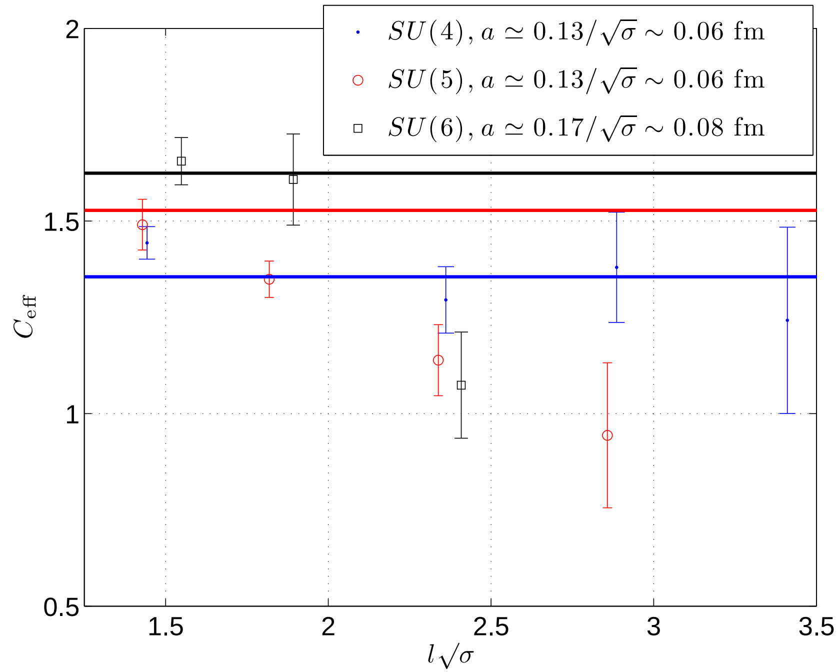
$$\frac{m_{k=2} - m_{k=1}}{\sigma l} = (r_2 - 1)\left(1 - \frac{\pi}{6\sigma l^2}\right) + O(1/l^4)$$

In contrast if $C_{\rm eff}^{(k=2)} = 1$ then $\frac{m_{k=2} - m_{k=1}}{\sigma l} = (r_2 - 1) + O(1/l^4)$

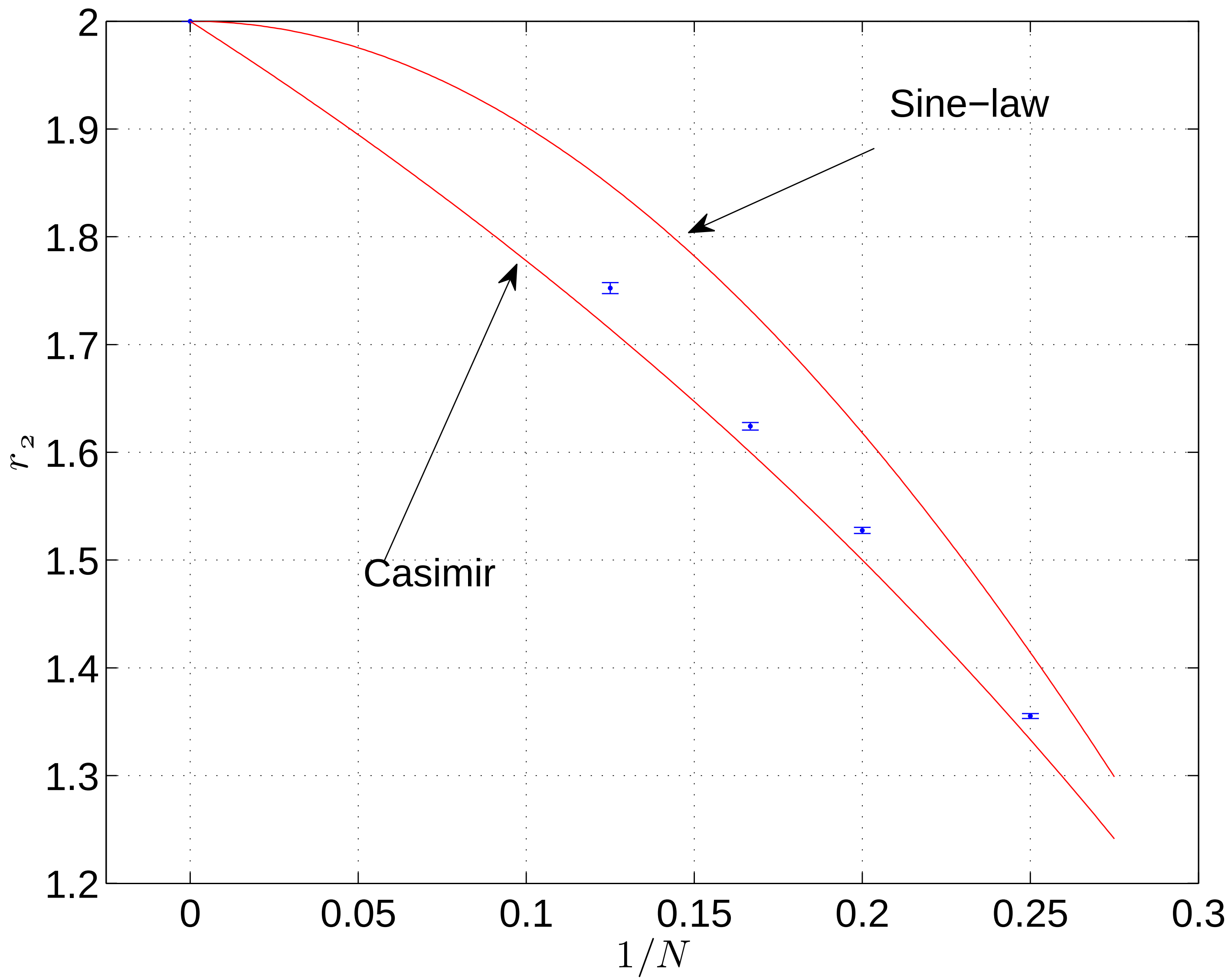
$l\sqrt{\sigma} \simeq 2$ is far from asymptotic ...



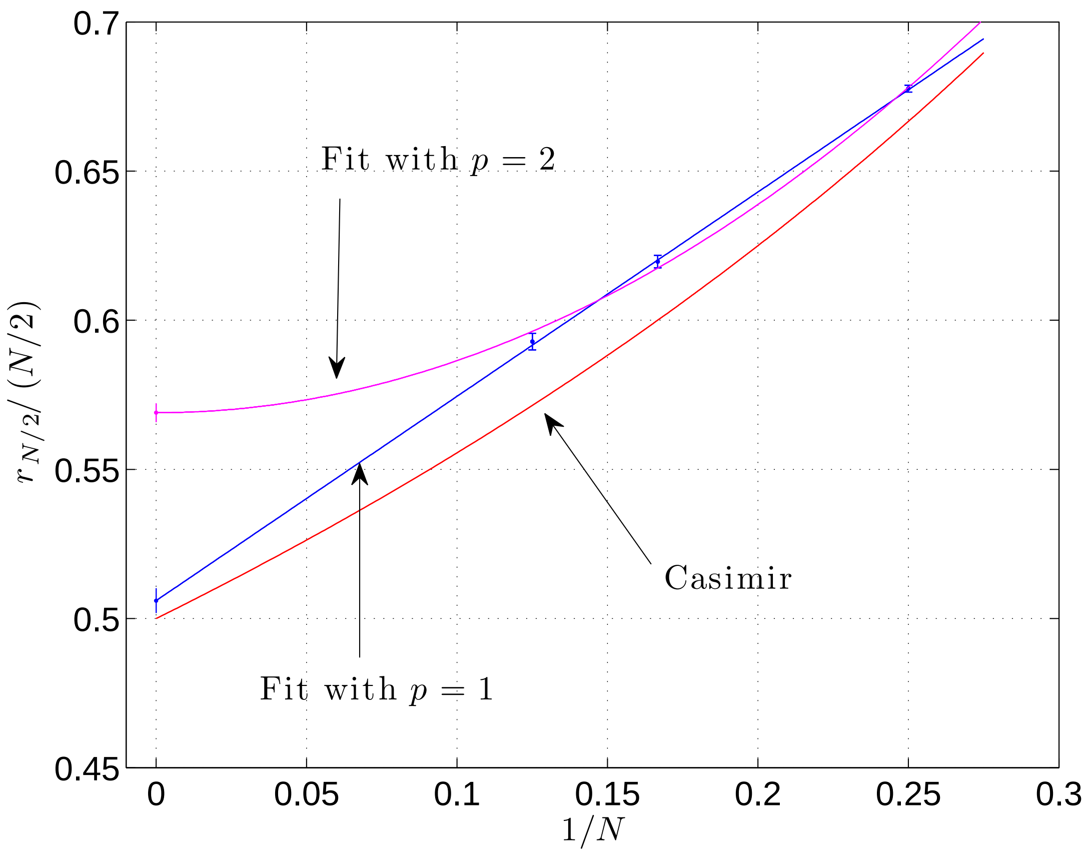
Indeed :



Appendix C : Large-N extrapolation : $k = 2$



Appendix C : Large-N extrapolation : $k = N/2$



Fit : $r_{N/2}(N) = \frac{N}{2}\left(a + \frac{b}{N^p}\right)$

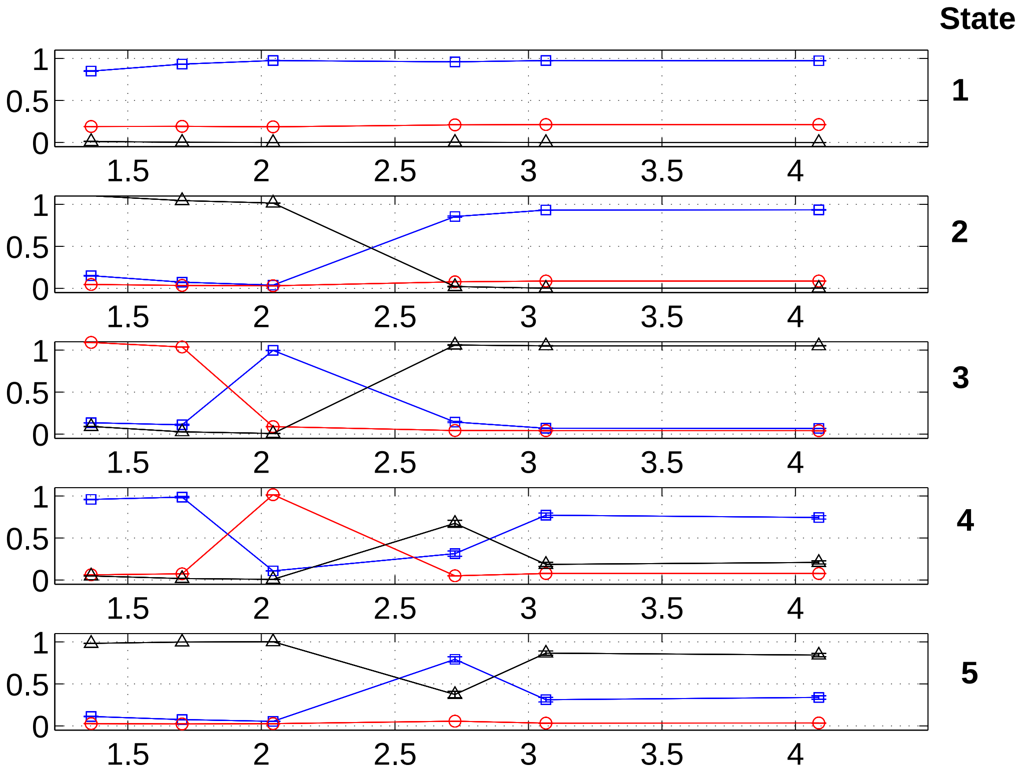
p	a	b	Con. level
1	0.506(4)	0.685(20)	92%
2	0.569(3)	1.745(54)	26%

$p = 1$ seems to fit our data more naturaly

Appendix D: Puzzles in the $k-$string spectrum

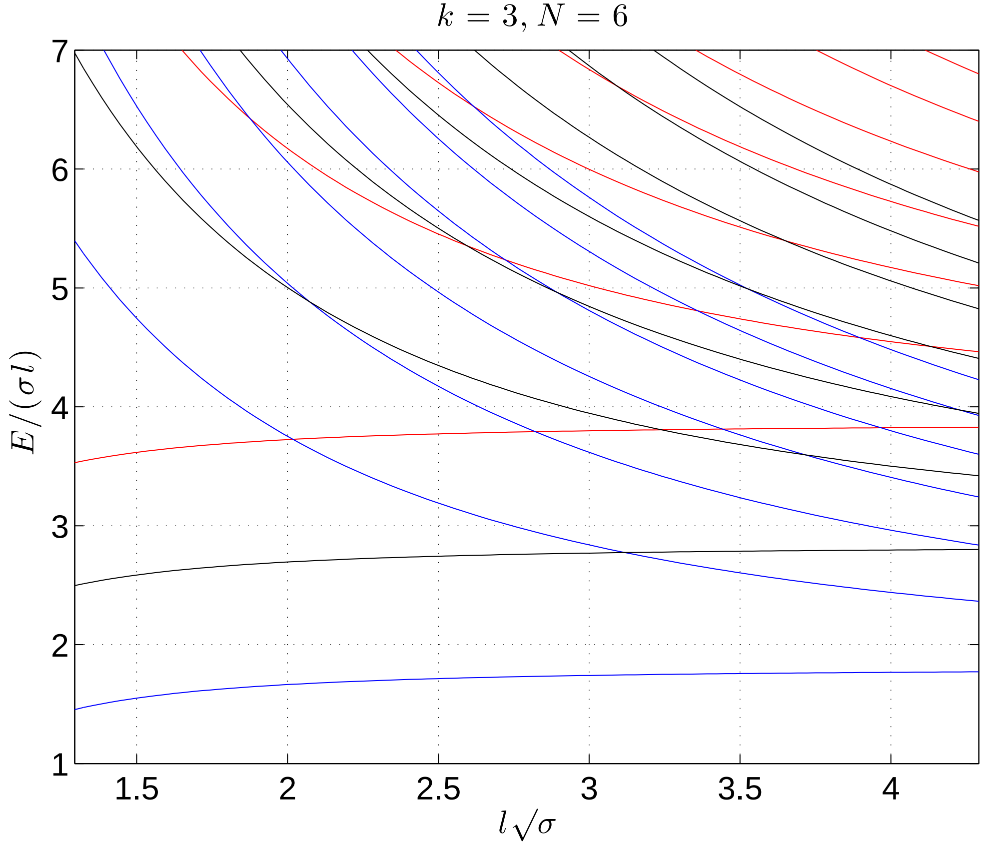
Step 1: $\mathcal{R}-$content of states. $$\left|\Psi^{(k)}_{\text{state}}(x)\right\rangle = \sum_{\substack{\mathcal{R}\\ \text{Nality}(\mathcal{R})=k}} \sum_i \left(u_{\mathcal{R}}\right)^{(i)} \left|\phi_i^{\mathcal{R}}(x)\right\rangle.$$

Model : $k-$noninteracting NG 'towers' : $$E_k = \sqrt{\left(\sigma_k l\right)^2 + 8\pi\left(n_k - \frac{1}{24}\right)}$$



$l\sqrt{\sigma}$ for $N=6, k=3, a\sqrt{\sigma}=0.17029$

$\mathcal{R} =$ [Young diagrams: column of three boxes, mixed (2,1) diagram, row of three boxes]

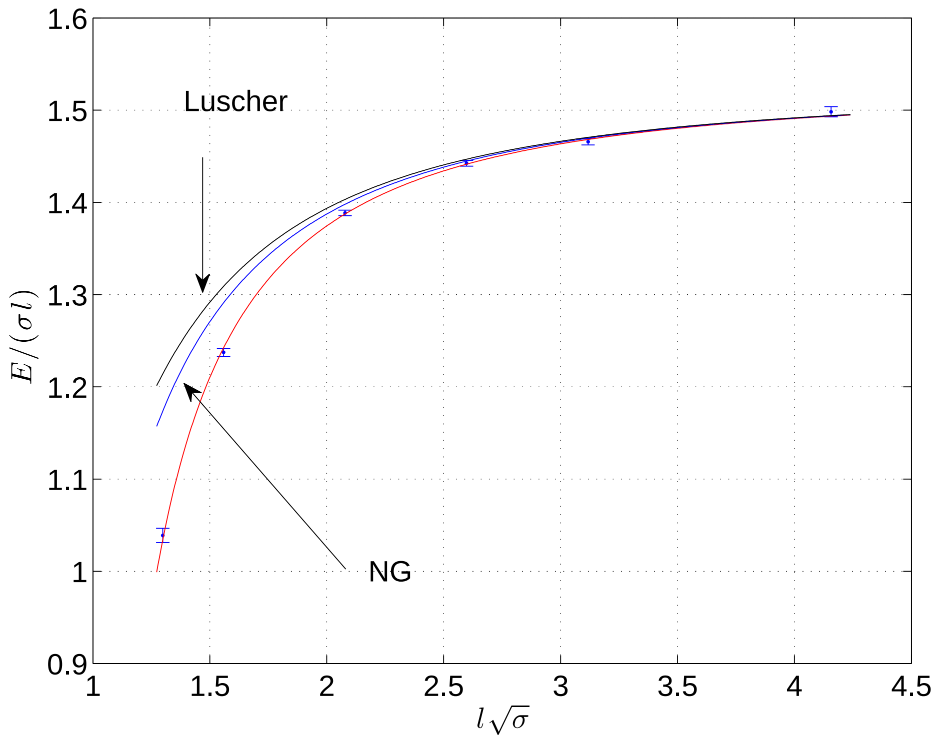


$NG(\sigma_{\mathcal{R}_1}, l)$, $NG(\sigma_{\mathcal{R}_2}, l)$, $NG(\sigma_{\mathcal{R}_3}, l)$ with $\sigma_{\mathcal{R}} \sim C_{\mathcal{R}}$

Appendix E : comparison of $k = 1$ and $k = 2$

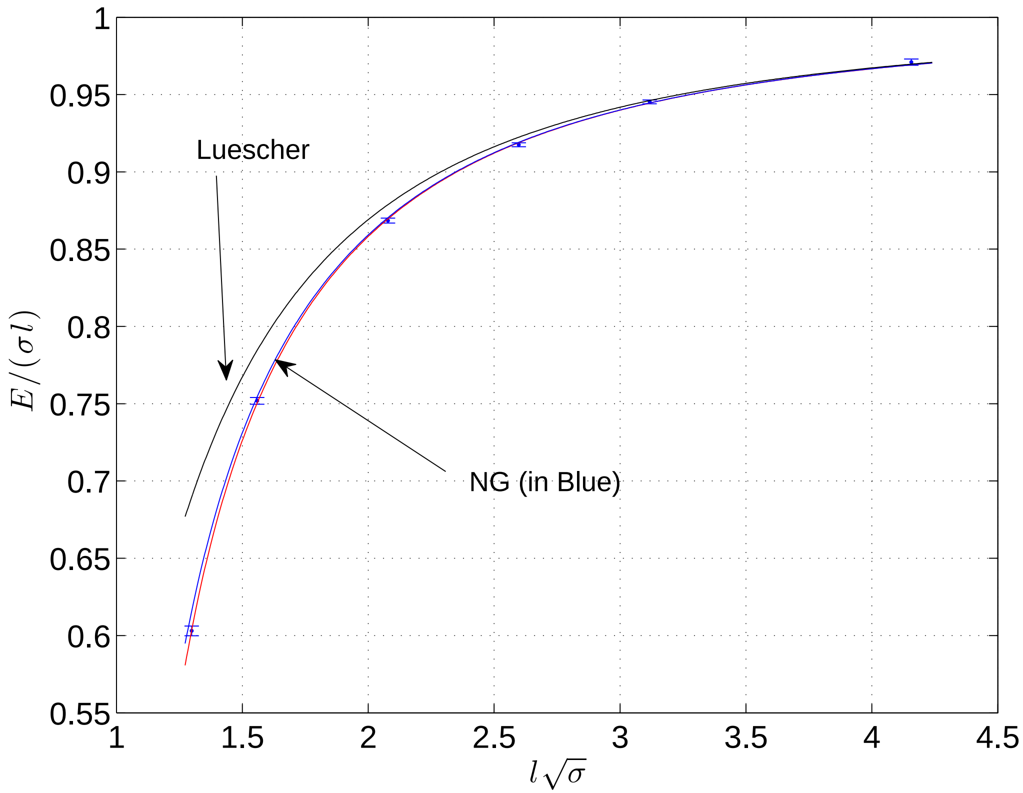
$k = 2$, $SU(5), a \simeq 0.13/\sqrt{\sigma} \simeq 0.06$ 'fm', Fit :$E_0 = \sqrt{\underbrace{(\sigma_k l)^2 - \frac{\pi\sigma_k}{3}}_{\text{NG}} - \frac{C}{\sqrt{\sigma_k} l^3}}$

$k = 2$



$C = 1.407(73)$

$k = 1$



$C = 0.055(14).$

Appendix F : Other systematic errors to controlling for $\boldsymbol{k > 1}$

Bayesian fitting of excited states' contamination.

$\boldsymbol{k}$**−string/**$\boldsymbol{(N - k)}$**−string** mixings in $SU(4)$.

More operators for k**−strings' excited states** .