The critical region in phase transitions of strong-coupling lattice QCD.

Barak Bringoltz

University of Oxford

Mostly Based on:
BB hep-lat/0511058
But also on:
BB and Michael Teper, PRD73:014517 (2006) hep-lat/0508021,

Large-N suppression of the critical region in phase transitions of strong-coupling lattice QCD at $T > 0$.

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General context - QCD phase diagram at finite T.

In particular : How can we learn from the 'simpler' large-N_c theory ?

Pressure deficit: $\frac{P_{\mathrm{QCD}}(T)}{P_{\mathrm{free}}(T)} \simeq 80\%$ even at $T \simeq 4T_c$ e.g. Boyd et al. '96

$\longrightarrow$ Happens at $SU(N_c \gg 1)$ as well BB and M. Teper '05.

$\longrightarrow$ connection with AdS/CFT Gusber et al. '98, Policastro et al. '01.

Relation between Hagedorn point T_H, and deconfinement : In QCD, $T_H =?$

$\longrightarrow$ signs for $\frac{T_H}{T_c} \simeq 10 - 15\%$ in $SU(N_c \gg 1)$ BB and M. Teper '05.

Critical region near T_c : How easily can you see nontrivial scaling in QCD ?

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Critical region near T_c : How easily can you see nontrivial scaling in QCD ?

Outline of the talk

I. **Motivation : does the critical region vanishes in QCD$_\infty$?**

II. **The effective Hamiltonian at $O(1/\lambda)$** .

III. **Large-N_c, fixed N_f, and fixed λ** : a classical magnet

IV. **Large-N_c, large-N_f, fixed $\lambda, \frac{N_f}{N_c}$** : a CP_N model.

V. **Large-N_f, fixed N_c, $\frac{N_f}{\lambda}$** : a Spin-Peierls transition.

VI. **Partial summary** .

VI. **Conclusions and possible (gauge) realizations in the continuum:** :

The Hagedorn transition in YM$_\infty$.

I. Motivation: How does planar QCD behave close to T_c ?

Studies of N-species (Yukawa, NJL,GN) models Rosenstein et al. '94,Kogut et al. '94-'98,Strouthos et al. '01-'05,Caracciolo '05.

Exactly solvable at $N = \infty$, can calculate:

- 2nd order at $T_c(N) = T_c(\infty) + O(1/N)$ where $Z_2 \to \emptyset$.
- A 'puzzle':
  - An exact solution : $\xi^{-1} \sim (T - T_c)^{1/2} \Leftrightarrow$ MF at $N = \infty$!
  - But from universality arguments: $\nu = \nu(\mathrm{Ising})$.
- Solution :
  - The range of T at which MF fails (= critical region) scales like

$$\frac{T - T_c}{T_c} \sim 1/N^p \quad ; \quad p \geq 0 \qquad \Leftrightarrow \qquad \lim_{N\to\infty} \lim_{T\to T_c} [\nu] \neq \lim_{T\to T_c} \lim_{N\to\infty} [\nu]$$

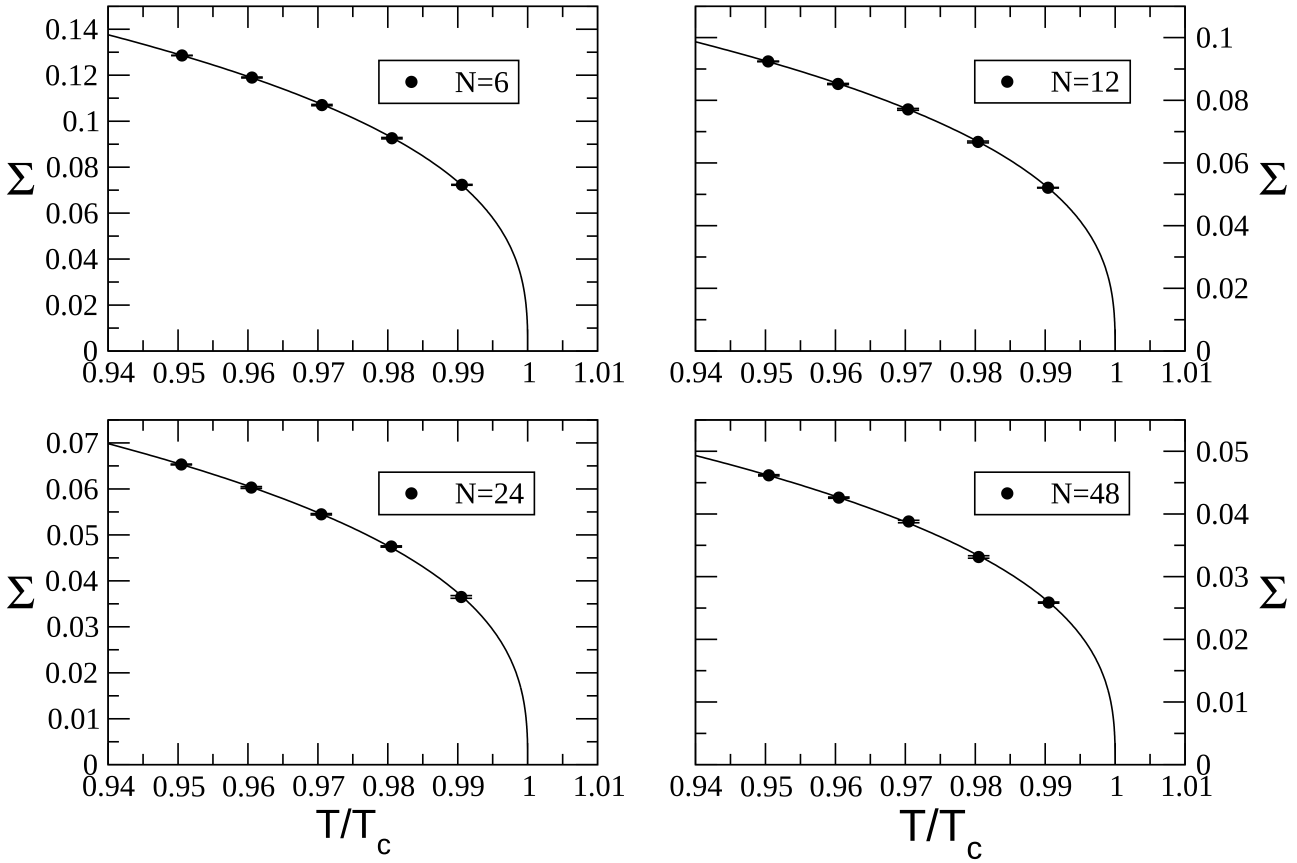
  - Kogut, Stephanov and Strothous '98: may happen in QCD where $3 \simeq \infty$
    $\to$ may be hard to observe nontrivial exponents in QCD.

Next Chandrasekharan and Strouthos '04 : MC's of strong-coupling QCD_∞, where Action $= \bar{\Psi} D(U) \Psi$

$$Z \simeq \int D\Psi D\bar{\Psi} \int DU \exp\left[\bar{\Psi} D(U) \Psi\right] = \int D\Psi D\bar{\Psi} \exp S_{\text{eff}}[(\bar{\Psi}\Psi)^2] = Z \begin{bmatrix} \text{monomers} \\ \text{\& dimers} \end{bmatrix}$$

Using advanced cluster algorithms enabled them to accurately see the chiral transition:

- The condensate $\Sigma = \langle \frac{\bar{\Psi}\Psi}{N_c} \rangle = 0$ at $T_c/N_c = 1.5525(3) + O(1/N_c)$.



– No supression of $\frac{T-T_c}{T_c}$.
– $\beta = \beta(3DXY) = 0.345.$

Leads to ask :

1. Can we understand this ?
2. How can we know *a priory* which large-N systems exhibit suppression ?
3. What happens in *continuum* planar QCD ?

An additional valid question is : How come $T_c \sim O(N_c)$ here ?

Turn to lattice Hamiltonian at strong-coupling. Why ?

- Different critical behaviors in different large-N 'corners' of $N_c - N_f - T$ space.

III. Hamiltonian QCD at $O(1/\lambda)$ Smit '80 (For simplicity take N_f-naive fermions.)

$$H_{\rm E} = \underbrace{\frac{\lambda}{2N_c}\sum_{\mathbf{n}\mu} E^2_{\mathbf{n}\mu}}_{H_0} - \underbrace{\left[i\sum_{\mathbf{n}\mu} \psi^\dagger_{\mathbf{n}}\,\alpha_\mu\, U_{\mathbf{n}+\hat\mu,\mu}\psi_{\mathbf{n}+\mu} + h.c.\right]}_{H_F} + \frac{N_c}{2\lambda}\sum_{\rm P} {\rm Tr}\left[1 - U_{\rm P} - U^\dagger_{\rm P}\right].$$

At $\lambda = \infty$, $[N_c$ fixed]: $\quad H = H_0 \to |\Omega\rangle = |E=0\rangle \quad$ with huge degeneracy

(can put $m = \psi^\dagger\psi, B = \psi_1\psi_2\cdots\psi_{N_c}$ on any site)

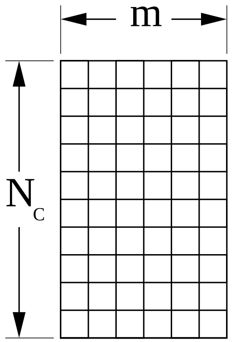
Split deg. with $H_F \quad \Rightarrow \quad H^{(2)}_{\rm eff} = H_F \frac{|n\rangle\langle n|}{E^{(0)}_0 - E^{(0)}_n \,_{(\sim\lambda)}} H_F \sim \frac{1}{\lambda N_c}(\psi^\dagger\psi)_{\mathbf{n}}(\psi^\dagger\psi)_{\mathbf{n}+\mu}$

$$H^{(2)}_{\rm eff} \equiv \frac{J}{N_c}\sum_{\mathbf{n}\mu} \vec S_{\mathbf{n}} \cdot \vec S_{\mathbf{n}+\hat\mu} \quad ; \quad J \sim 1/\lambda$$

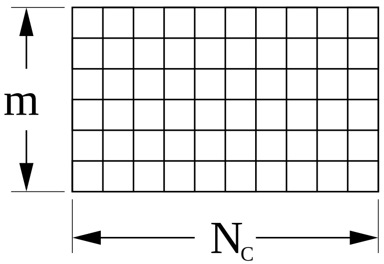
$S^\eta_{\mathbf{n}} = \sum_{a=1}^{N_c}\sum_{\alpha,\beta=1}^{4N_f} \Psi^{\dagger\alpha a}_{\mathbf{n}} M^\eta_{\alpha\beta}\Psi_{\beta a n} \quad ; \quad M^\eta = \Gamma^A_{\rm Dirac}\otimes\tau^a_{\rm flavor} \to$ generate a global $U(4N_f)$.

So $H_{\rm eff}^{(2)} = \dfrac{J}{N_c}\sum_{\mathbf{n}\mu} \vec{S}_{\mathbf{n}} \cdot \vec{S}_{\mathbf{n}+\hat{\mu}}$ with $S^{\eta}_{\mathbf{n}} = \sum_{a=1}^{N_c} \sum_{\alpha,\beta=1}^{4N_f} \Psi^{\dagger\alpha a}_{\mathbf{n}} M^{\eta}_{\alpha\beta} \Psi_{\beta a \mathbf{n}}$

Gauss law : $a, b \in$



Fermions $\Rightarrow$ $\alpha, \beta \in$



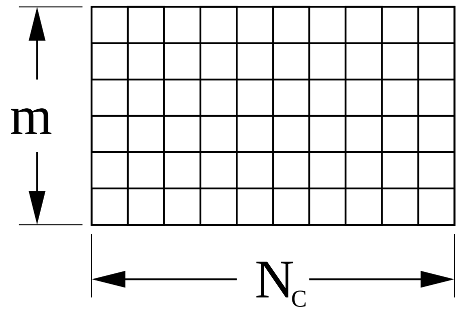
In different corners of the $N_f - N_c - T$ **diagrams get** :

Different ground states and Different critical behaviors.

IV. Large-N_c, fixed N_f, λ ($\longrightarrow$ fixed J) Read & Sachdev '89

Study the $U(4N_f)$ **Hamiltonian** :

$$H = \frac{J}{N_c} \sum_{\mathbf{n}\mu} \vec{S}_{\mathbf{n}} \cdot \vec{S}_{\mathbf{n}+\mu} \quad ; \quad \vec{S}_{\mathbf{n}} = \sum_{a=1}^{N_c} \psi^\dagger_{\mathbf{n}} \vec{M} \, \psi_{\mathbf{n}}$$

Simplicity : replace $U(4N_f)$ with $SU(2)$. ($\alpha, \beta \in$ 

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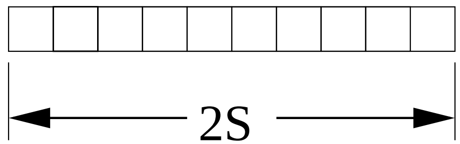
Simplicity : replace $U(4N_f)$ with $SU(2)$. ($\alpha, \beta \in$ 

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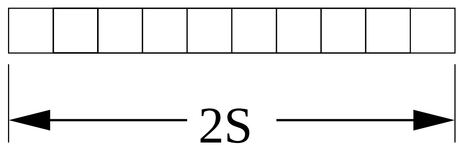
Simplicity : replace $U(4N_f)$ with $SU(2)$. $(\alpha, \beta \in$ 

).

Go to coherent states' basis , $|\vec{\Omega}\rangle = e^{i\vec{S}\vec{\Omega}}|S_z = \frac{N_c}{2}\rangle$

$$Z = \mathrm{Tr}\, e^{-\frac{\beta}{N_t}H} \cdots e^{-\frac{\beta}{N_t}H} = \int_0^{4\pi} D\Omega \, \exp \frac{N_c}{2} \int_0^\beta dt \left[i\, A_{\mathrm{kin}}(\partial_t) + J \sum_{\mathbf{n}\mu} \vec{\Omega}_{\mathbf{n}}(t) \cdot \vec{\Omega}_{\mathbf{n}+\mu}(t) \right]$$

$$\langle \Omega | \vec{S} | \Omega \rangle = \frac{N_c}{2} \vec{\Omega}$$

For any β can take $N_c \gg 1$ and get $A_{\mathrm{kin}} = 0 \rightarrow \partial_t \Omega = 0$.

$$Z = \int D\Omega \exp \left[\frac{JN_c}{T} \sum_{\mathbf{n},\mu} \vec{\Omega}_{\mathbf{n}} \cdot \vec{\Omega}_{\mathbf{n}+\mu} \right] \equiv \text{a classical spin system.}$$

So at large-$N_c(= 2S)$ get classical spins $Z = \int D\Omega \exp\left[\frac{JN_c}{T}\sum_{\mathbf{n},\mu}\vec{\Omega}_{\mathbf{n}}\cdot\vec{\Omega}_{\mathbf{n}+\mu}\right]$.

broken $U(4N_f)$ at low-$T \rightarrow$ broken χ-symmetry.

Also immediatly see that $T_c \sim O(N_c J)$. $Z = \int D\Omega \exp\left[\#\frac{T_c}{T}\sum_{\mathbf{n},\mu}\vec{\Omega}_{\mathbf{n}}\cdot\vec{\Omega}_{\mathbf{n}+\mu}\right]$.

Finally about critical exponents :

- No reason to expect MF exponents at $N_c = \infty$ here !
- In terms of T/T_c no suppression of critical region.

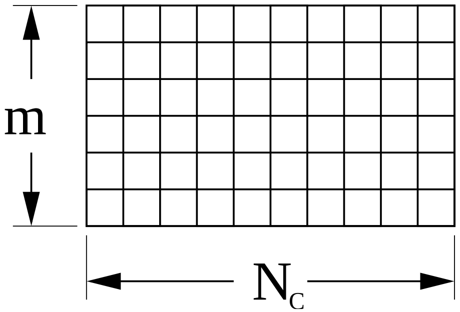
Also note that $T_c \sim N_c$: like Damgaard et al. '84.
Only nonzero fluxes to get it realistic Gocksh & Ogilvie '84.

Here large-N_c is *not* soluble $\forall\, T$.

What happens when it does ? Do you get MF exponents then ?

IV. Large-N_c, Large N_f, fixed $\lambda, N_f/N_c$ ($\longrightarrow$ fixed J) Arovas & Auerbach '88

Study : $H = \frac{J}{N_c} \sum_{\mathbf{n}\mu} \vec{S}_{\mathbf{n}} \cdot \vec{S}_{\mathbf{n}+\mu} \quad ; \quad \vec{S}_{\mathbf{n}} = \sum_{a=1}^{N_c} \psi^\dagger_{\mathbf{n}} \vec{M} \, \psi_{\mathbf{n}} \qquad \in$



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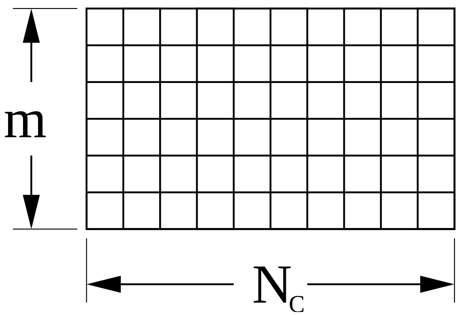

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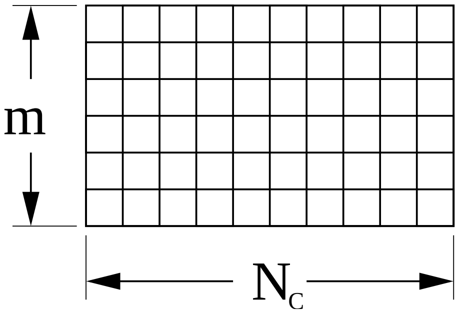

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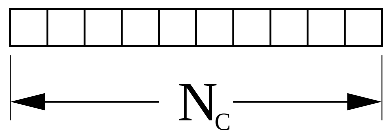

Simplicity : choose $m_{\rm even} = 1$ and its conjugate on odd, and replace

$$\vec{S}_{\mathbf{n}} \rightarrow \left[\begin{array}{ll} +b^\dagger_{\mathbf{n}} \vec{M} b_{\mathbf{n}} & \text{even} \\ -b^\dagger_{\mathbf{n}} (\vec{M})^* b_{\mathbf{n}} & \text{odd} \end{array} \right. \quad ; \quad \sum_{\alpha=1}^{4N_f} b^{\dagger\alpha}_{\mathbf{n}} b^{\alpha}_{\mathbf{n}} = N_c \quad \Rightarrow \quad H \sim (b^\dagger_{\mathbf{n}} b_{\mathbf{n}})(b^\dagger_{\mathbf{n}+\mu} b_{\mathbf{n}+\mu})$$

Next $Z = \mathrm{Tr}\left[e^{-\beta H} \times \delta(b^\dagger b - N_c)\right] = \displaystyle\int D\lambda \, \mathrm{Tr}\exp\left[-\beta H - i \sum_{\mathbf{n}} \lambda_{\mathbf{n}} (b^\dagger_{\mathbf{n}} b_{\mathbf{n}} - N_c)\right]$

Path integral ,

$$A = -\int_0^{1/T} dt \left[\sum_{\mathbf{n}\alpha} b^*_{\alpha\mathbf{n}} \frac{\partial}{\partial_t} b_{\alpha\mathbf{n}} + \frac{J}{N_f} \sum_{\substack{\mathbf{n}\mu \\ \alpha\beta}} b^*_{\alpha\mathbf{n}} b^*_{\alpha\mathbf{n}+\mu} b_{\beta\mathbf{n}} b_{\beta\mathbf{n}+\mu} + i \sum_{\mathbf{n}} \lambda_{\mathbf{n}} (b^\dagger_{\mathbf{n}\alpha} b_{\mathbf{n}\alpha} - N_c) \right]$$

Next : Hubbard-Stratonovich transformation : $-\frac{J}{N_f}(b^\dagger b)^2 \rightarrow \frac{N_f}{J} Q^2 + 2Q b^\dagger b$

$$A = \int dt \left[\sum_{\mathbf{n}\mu} \frac{N_f}{J} |Q_{\mathbf{n}\mu}|^2 - 2\mathrm{Re} \sum_{\mathbf{n}\mu} Q^*_{\mathbf{n}\mu} b_{\alpha\mathbf{n}} b_{\alpha\mathbf{n}+\mu} + i \sum_{\mathbf{n}} \left(N_c - b^\dagger_{\mathbf{n}} b_{\mathbf{n}} \right) \lambda_{\mathbf{n}} + \sum_{\mathbf{n}\alpha} b^\dagger_{\alpha\mathbf{n}} \partial_\tau b_{\mathbf{n}\alpha} \right]$$

Re-arrange :

$$A = \int dt \left[\sum_{\mathbf{n}\mu} \frac{N_f}{J} |Q_{\mathbf{n}\mu}|^2 - i N_c \sum_{\mathbf{n}} \lambda_{\mathbf{n}} + \sum_{\mathbf{n}\alpha} b^\dagger_{\alpha\mathbf{n}} \left[\partial_\tau + i\lambda_{\mathbf{n}} \right] b_{\alpha\mathbf{n}} - 2\mathrm{Re} \sum_{\mathbf{n}\mu} Q^*_{\mathbf{n}\mu} b_{\alpha\mathbf{n}} b_{\alpha\mathbf{n}+\mu} \right].$$

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Integration over b $\longrightarrow$ $4N_f \mathrm{tr}\,\log G^{-1}(Q,\lambda)$

Integrate over b :

$$A_{eff} = 4N_f \left\{ \int dt \left[\sum_{\mathbf{n}\mu} \frac{1}{4J} |Q_{\mathbf{n}\mu}|^2 + i \sum_{\mathbf{n}} \frac{N_c}{4N_f} \lambda_{\mathbf{n}} \right] + \text{tr} \log G^{-1} \right\},$$

For any β can take $N_f, N_c \gg 1$, fixed $\kappa = \frac{N_c}{4N_f}$, J and try $Q_{\mathbf{n}\mu}(t)$, $i\lambda_{\mathbf{n}}(t)$=const,real.

$$\frac{A_{\text{MF}}}{\text{Volume}} = \frac{4N_f}{T} \left[\frac{dQ^2}{4J} - \lambda \left(\kappa + \frac{1}{2} \right) + T \int \left(\frac{dk}{2\pi} \right)^d \log 2 \sinh \left(\frac{\lambda \, \omega_{\mathbf{k}}}{2T} \right) \right].$$

At $\mathbf{k} \ll 1$: $\boxed{\omega_{\mathbf{k}}^2 = \Delta_{Q,\lambda}^2 + c_{Q,\lambda}^2 \, \mathbf{k}^2}$

$\Rightarrow$ So at $\Delta = 0$, bosons condense $\langle b \rangle \neq 0 \rightarrow$ breaks $U(4N_f)$ and χ-symmetry.

$\boxed{U(4N_f) \rightarrow U(4N_f - 1) \times U(1)}$ $\Rightarrow$ Expect $\boxed{CP_{4N_f-1} \text{ criticality}}$

$\dfrac{\delta}{\delta Q}, \dfrac{\delta}{\delta \lambda} \rightarrow$**"MF" equations** :

$$\frac{4\lambda}{dJ} = \int \left(\frac{dk}{2\pi}\right)^d \frac{\gamma_{\mathbf{k}}^2}{\omega_{\mathbf{k}}} \coth \frac{\omega_{\mathbf{k}}}{2T}\,, \qquad \kappa + \frac{1}{2} = \int \left(\frac{dk}{2\pi}\right)^d \frac{1}{\omega_{\mathbf{k}}} \coth \frac{\lambda\omega_{\mathbf{k}}}{2T}.$$

Whose solution gives exact phase diagram at $N = \infty$:

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- $\Delta = 0$ at $T_c/J = f(\kappa) < \infty$ AA '88, Sarker et al. '89.

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$$\kappa + \frac{1}{2} = \int \left(\frac{dk}{2\pi}\right)^d \frac{1}{\sqrt{\Delta^2 + c^2 \mathbf{k}^2}} \coth \frac{\sqrt{\Delta^2 + c^2 \mathbf{k}^2}}{2x} \qquad x = T/\lambda$$

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Whose solution gives exact phase diagram at $N = \infty$:

- $\Delta = 0$ at $T_c/J = f(\kappa) < \infty$ AA '88, Sarker et al. '89.
- $\frac{\partial}{\partial x}$: What happens at $\Delta \ll 1$? keep $d > 2$

$$\begin{aligned} 0 &= \int \left(\frac{dk}{2\pi}\right)^d \left[\frac{2}{\Delta^2 + c^2\mathbf{k}^2} - \frac{2x}{(\Delta^2 + c^2\mathbf{k}^2)^2} \times \frac{\partial \Delta^2}{\partial x}\right] \\ &= +A - \left(B + C\Delta^{d-4}\right) \times \frac{\partial \Delta^2}{\partial x} \\ &= -A + \frac{\partial \Delta^2}{\partial x} \times \left[\begin{array}{ll} C\Delta^{d-4} & 2 < d < 4 \\ C \log \Delta & d = 4 \\ B & d > 4 \end{array} \right. \end{aligned}$$

Which means that at $N = \infty$:

$$\Delta \sim \left[\begin{array}{cc} (x - x_c)^{1/(d-2)} & 2 < d < 4 \\ (x - x_c)^{1/2} & d > 4 \end{array} \right. \Leftrightarrow \boxed{\nu\text{'s of } CP_N \text{ at } N = \infty}$$

Sarker et al. '89

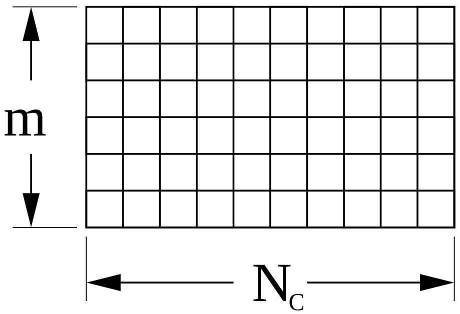
So we solved the system exactly at $N = \infty$, but :

MF only $d > 4$. Critical region finite (depends only on κ)

So where is the critical region suppression ?

IV. Large-N_f, fixed N_c Read & Sachdev '89

Study : $H = \frac{J}{N_c} \sum_{\mathbf{n}\mu} \vec{S}_{\mathbf{n}} \cdot \vec{S}_{\mathbf{n}+\mu}$; $\vec{S}_{\mathbf{n}} = \sum_{a=1}^{N_c} \psi^\dagger_{\mathbf{n}} \vec{M} \psi_{\mathbf{n}} \in$



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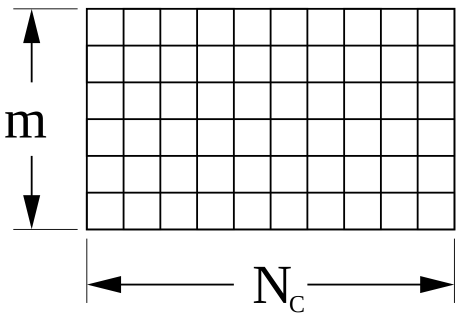
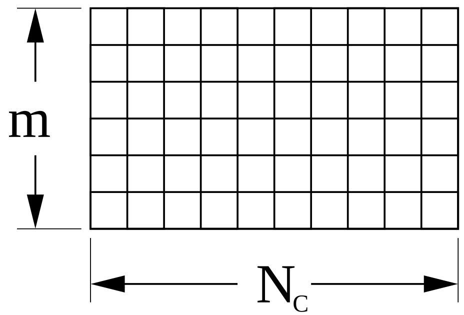


$$\hookrightarrow \frac{J N_f}{N_c} \sim \frac{N_f}{\lambda} \to J$$

IV. Large-N_f, fixed N_c, $\frac{N_f}{\lambda}$ Read & Sachdev '89

Study : $H = \frac{J}{N_f} \sum_{\mathbf{n}\mu} \vec{S}_{\mathbf{n}} \cdot \vec{S}_{\mathbf{n}+\mu}$; $\vec{S}_{\mathbf{n}} = \sum_{a=1}^{N_c} \psi^{\dagger}_{\mathbf{n}} \vec{M} \psi_{\mathbf{n}} \in$



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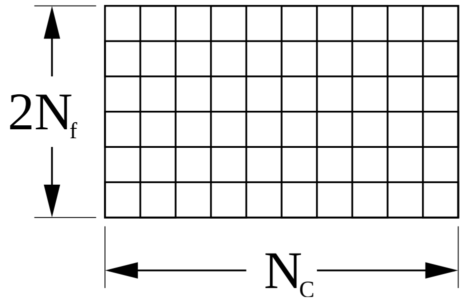

Simplicity : choose $N_c = 1$, $m = \frac{4N_f}{2}$ on all sites, so $\sum_{\alpha=1}^{4N_f} \psi_{\mathbf{n}}^{\alpha\dagger} \psi_{\mathbf{n}}^{\alpha} = 2N_f$

Again : $\delta(\psi^{\dagger}\psi - 2N_f) \to i\lambda(\psi^{\dagger}\psi - 2N_f)$, and $-\frac{J}{N_f}(\psi^{\dagger}\psi)^2 \to \frac{N_f}{J} Q^2 + 2Q\psi^{\dagger}\psi$

$$A = \int dt \left[\sum_{\mathbf{n}\mu} \frac{N_f}{J} |Q_{\mathbf{n}\mu}|^2 + i2N_f \sum_{\mathbf{n}} \lambda_{\mathbf{n}} + \underbrace{\sum_{\mathbf{n}} \bar{\psi}_{\mathbf{n}}^{\dagger} \left[\partial_\tau - i\lambda_{\mathbf{n}}\right] \psi_{\mathbf{n}} + \sum_{\mathbf{n}\mu} \left(\bar{\psi}^{\alpha}{}_{\mathbf{n}} Q_{\mathbf{n}\mu} \psi^{\alpha}_{\mathbf{n}+\mu} + hc \right)} \right]$$

Integration over ψ $\longrightarrow$ $4N_f \mathrm{tr} \log D^{-1}(Q, \lambda)$

$$A_{eff} = 4N_f \left\{ \int dt \left[\frac{1}{4J} \sum_{\mathbf{n},\mu} |Q_{\mathbf{n}\mu}|^2 + \frac{i}{2} \sum_{\mathbf{n}} \lambda_{\mathbf{n}} \right] + \mathrm{tr}\, \log D^{-1}(Q,\lambda) \right\},$$

For any T can take $N_f \gg 1$, fixed J and try $Q_{\mathbf{n}\mu}(t)$, $i\lambda_{\mathbf{n}}(t)$=time-independent,real.

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From now on restrict to $2 + 1$!

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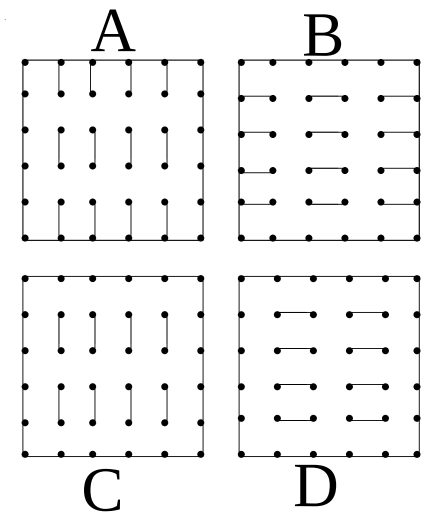
For any T can take $N_f \gg 1$, fixed J and try $Q_{\mathbf{n}\mu}(t)$, $i\lambda_{\mathbf{n}}(t)$=time-independent,real.

From now on restrict to $2+1$!

Ground state is "Spin-Peierls" state :

- $\lambda_{\mathbf{n}} = 0.$
- $Q_{\mathbf{n}\mu} = q \neq 0$ on ladders.



$$\frac{A_{\mathrm{Spin-Peierls}}}{\mathrm{Volume}} = \frac{4N_f}{T} \left[\frac{1}{4J} q^2 \times \frac{1}{4} \times 2 - T \log 2\cosh \frac{q}{2T} \right],$$

MF equation: $$\frac{\delta}{\delta q}\left[\frac{A_{MF}T}{\text{Volume}\cdot 4N_f}\right] = \frac{\delta}{\delta q}\left[\frac{1}{8J}q^2 - T\,\log\,\cosh\frac{q}{2T}\right] = 0$$

$$\boxed{q/2J = \tanh(q/2T)} \quad \Leftrightarrow \quad \text{(like a simple MF Ising !)}$$

$\Rightarrow \quad q(T) \sim (T - \underbrace{J}_{T_c})^{1/2} \Rightarrow$ **MF scaling ! This is exact at $N_f = \infty$.**

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Q: Why ? the universality determined by: dimension, symmetry, degeneracy of g.s.

$\hookrightarrow$ Z_4 model in two dimensions.

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$\hookrightarrow$ Z_4 model in two dimensions.

A: This is the same phenomenon as in Rosenstein et al. '94,Kogut et al. '94-'98,Strouthos et al. '01-'05,Caracciolo '05 :

The suppression of the critical region.

How can you see that ?

Need to calculate the effective action for fluctuations around the "MF" state.

$\rightarrow$ **Need to calculate the $1/N$ corrections.**

$1/N_f$ corrections

Start from $A_{eff} = 4N_f \left\{ \int dt \left[\frac{1}{4J} \sum_{\mathbf{n},\mu} |Q_{\mathbf{n}\mu}|^2 + \frac{i}{2} \sum_{\mathbf{n}} \lambda_{\mathbf{n}} \right] + \text{tr} \log D^{-1}(Q, \lambda) \right\}$

Write fluctuations $Q_{\mathbf{n}\mu}(t) \to q + \frac{1}{\sqrt{4N_f}} Q_{\mathbf{n}\mu}(t) \quad ; \quad \lambda_{\mathbf{n}\mu}(t) \to \langle \lambda \rangle + \frac{1}{\sqrt{4N_f}} \lambda_{\mathbf{n}}(t)$

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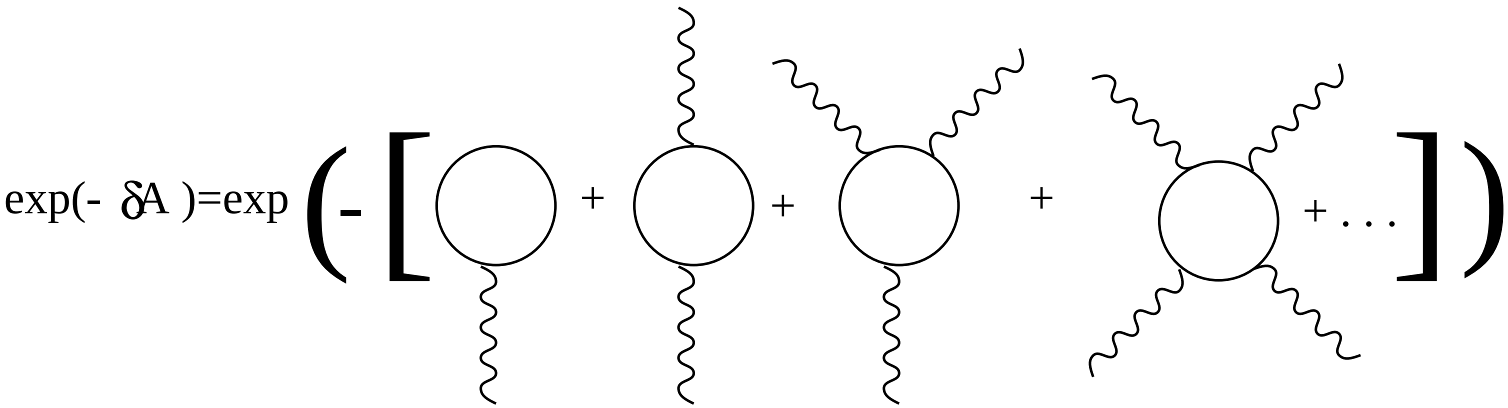
And in Matzubara: $Q_{\mathbf{n}\mu}(\omega) \rightarrow q\delta_{\omega,0} + \frac{1}{\sqrt{4N_f}} Q_{\mathbf{n}\mu}(\omega) \quad ; \quad \lambda_{\mathbf{n}\mu}(\omega) \rightarrow \frac{1}{\sqrt{4N_f}} \lambda_{\mathbf{n}}(\omega)$

$$
\begin{aligned}
A_{\mathrm{eff}} &= A_{MF} + \sqrt{4N_f} \left[\frac{1}{2J\sqrt{T}} \sum_{\mathrm{Bonds}} q\,\mathrm{Re}\,Q_{\mathbf{n}\mu}(\omega=0) + \frac{i}{2T} \sum_{\mathbf{n}} \lambda_{\mathbf{n}} \right] \\
&+ \frac{1}{4J} \sum_{\mathbf{n}\mu} \sum_{\omega} |Q_{\mathbf{n}\mu}(\omega)|^2 + \delta A,
\end{aligned}
$$



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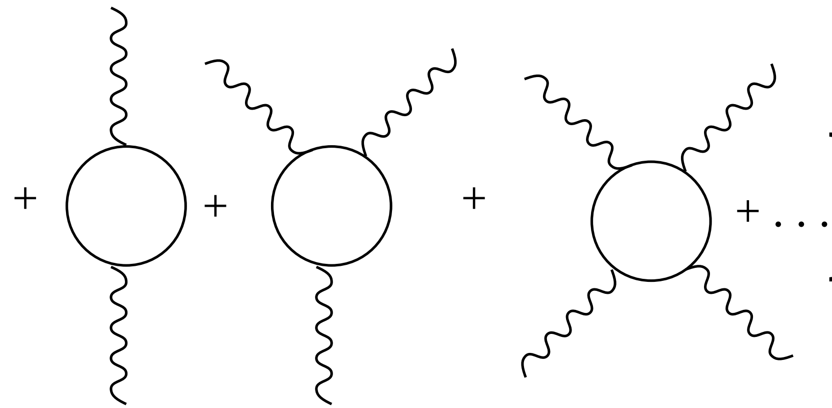
$$\begin{aligned} A_{\mathrm{eff}} &= A_{MF} + \\ &+ \frac{1}{4J} \sum_{\mathbf{n}\mu} \sum_{\omega} |Q_{\mathbf{n}\mu}(\omega)|^2 + \delta A, \end{aligned}$$

exp(- δA)=exp(-[



])

The rest: First lets focus on the action of the Q-fields around $\langle Q \rangle = q > 0$

$$\delta A_{\text{eff}}(Q) = \sum_{\omega \mathbf{n}\mu} \frac{1}{2} m^2 |Q_{\mathbf{n}\mu}(\omega)|^2 + v \left\{ \text{Re}\, Q_{\mathbf{n}\mu}(\omega) Q_{\mathbf{n}\mu}(-\omega) + \sum_{\substack{\omega \\ \mathbf{n}_{1,2}\mu}} 2\text{Re} Q^*_{\mathbf{n}_1\mu}(\omega) Q_{\mathbf{n}_2\mu}(\omega) \right\}$$

$$+ \frac{1}{\sqrt{4N_f}} \sum_{\substack{\text{paths} \\ \omega}} V^{(3)}_{\text{path}}(\omega) \text{Re}\left[Q(\omega_1) Q^*(\omega_2) Q(\omega_2 - \omega_1) \right]$$

$$+ \frac{1}{4N_f} \sum_{\substack{\text{paths} \\ \omega}} V^{(4)}_{\text{path}}(\omega) \text{Re}\left[Q_{\omega_1} Q^*(\omega_2) Q(\omega_3) Q^*(\omega_1 - \omega_2 + \omega_3) \right] + O\left(\frac{1}{N_f^{3/2}}\right)$$

$$\frac{1}{2} m^2 = \frac{1}{4J} - \sum_{\epsilon} \frac{T \cdot \epsilon(\omega + \epsilon)}{(\epsilon^2 + q^2)((\epsilon + \omega)^2 + q^2)} \quad ; \quad v = \sum_{\epsilon} \frac{T \cdot q^2}{(\epsilon^2 + q^2)((\epsilon + \omega)^2 + q^2)}.$$

$$V^{(3)}_{\text{path}}(\omega) = 2T^{3/2} \sum_{\epsilon} \frac{q}{\epsilon^2 + q^2} \frac{\epsilon + \omega_2}{(\epsilon + \omega_2)^2 + q^2} \frac{\epsilon - \omega_1}{(\epsilon - \omega_1)^2 + q^2} \quad ; \quad (\text{path} = \sqcap, -, --)$$

$$V^{(4)}_{\text{path}}(\omega) = T^2 \sum_{\epsilon} \frac{\epsilon}{\epsilon^2 + q^2} \frac{\epsilon + \omega_1}{(\epsilon + \omega_1)^2 + q^2} \frac{\epsilon + \omega_1 - \omega_2}{(\epsilon + \omega_1 - \omega_2)^2 + q^2} \frac{\epsilon + \omega_1 + \omega_3 - \omega_2}{(\epsilon + \omega_1 + \omega_1 - \omega_2)^2 + q^2}.$$

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The rest: First lets focus on the action of the Q-fields around $\langle Q \rangle = q = 0$,

$$\delta A_{\text{eff}}(Q) = \sum_{\omega \mathbf{n}\mu} \frac{1}{2} m^2 |Q_{\mathbf{n}\mu}(\omega)|^2 + \frac{1}{4N_f} \sum_{\substack{\text{paths} \\ \omega}} V^{(4)}_{\text{path}}(\omega) \text{Re}\left[Q_{\omega_1} Q^*_{\omega_2} Q_{\omega_3} Q^*_{\omega_1-\omega_2+\omega_3}\right]$$

$$\frac{1}{2} m^2 = \frac{T - T_c}{4T_c^2} + O(\omega^2)$$

$$V^{(4)}_{\square,-,--}(\omega) = T^2 \sum_{\epsilon} \frac{1}{\epsilon(\epsilon+\omega_1)(\epsilon+\omega_1-\omega_2)(\epsilon+\omega_1+\omega_3-\omega_2)}$$

The order parameter : $\boldsymbol{\phi \equiv Q(\omega = 0)}$ has a mass $\boldsymbol{m \sim (T - T_c)^{1/2}}$.

The rest are massive (including λ's) so integrate them out :

$$A_0 = \sum_{\mathbf{n}\mu} \frac{1}{2} m^2 |\phi_{\mathbf{n}\mu}|^2 + \frac{(48T_c^2)^{-1}}{4N_f} \left\{ \sum_{\mathbf{n}\mu} \frac{1}{2} |\phi_{\mathbf{n}\mu}|^4 + \sum_{\substack{\mathbf{n} \\ \mu \neq \nu}} |\phi_{\mathbf{n}\mu}|^2 |\phi_{\mathbf{n}+\mu,\nu}|^2 + 2\text{Re} \sum_{\square} \phi_{\square} \right\}$$

Now the 'puzzle' becomes clearer !

Effective action of the order-parameter is the Landua-Ginzburg-Wilson action :

$$\mathcal{L}_{LGW} = \sum_{\mathbf{n}\mu} \frac{1}{2} m^2 |\phi_{\mathbf{n}\mu}|^2 + \frac{A}{4N_f} \left\{ \sum_{\mathbf{n}\mu} \frac{1}{2} |\phi_{\mathbf{n}\mu}|^4 + \sum_{\substack{\mathbf{n} \\ \mu \neq \nu}} |\phi_{\mathbf{n}\mu}|^2 |\phi_{\mathbf{n}+\mu,\nu}|^2 + 2\mathrm{Re} \sum_{\Box} \phi_{\Box} \right\}$$

where close to T_c you have $\frac{1}{2} m^2 = \frac{T - T_c}{4T_c^2}$, $A = (48T_c^2)^{-1}$

Can approach the transition in several ways :

- first $\boldsymbol{N_f \rightarrow \infty, T =}$**fixed**, $\Rightarrow$ $\boldsymbol{m \sim (T - T_c)^{1/2}}$ **- the 'MF' puzzle.**

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Can approach the transition in several ways :

- first $N_f \to \infty, T =$ fixed, $\Rightarrow m \sim (T - T_c)^{1/2}$ - the 'MF' puzzle.
- first $T \to T_c$ while N_f is fixed $\Rightarrow$ flow to nearest stable fixed point.
- Take $\boldsymbol{N_f \to \infty}$ **with** $\boldsymbol{T - T_c \sim \frac{1}{N_f^p}}$ **with some** $\boldsymbol{p > 0}$ - **crossover** behavior.

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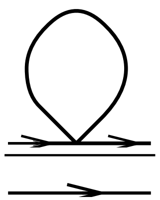
Can approach the transition in several ways :

- first $N_f \to \infty$, and then $T \to T_c \quad \Rightarrow m \sim (T - T_c)^{1/2}$ - the 'MF' puzzle.
- first $T \to T_c$ while N_f is fixed $\quad \Rightarrow$ flow to nearest stable fixed point.
- If you take $N_f \to \infty$ with $T - T_c \sim \dfrac{1}{N_f^p}$ with some $p > 0$ - crossover behavior.

What is p ? Evaluate importance of fluctuations $\quad \longrightarrow \boxed{p = 1/2}$



$$\sim \frac{A}{4N_f} \times \frac{\frac{1}{\frac{1}{2}m^2} \times \frac{1}{\frac{1}{2}m^2} \times \frac{1}{\frac{1}{2}m^2}}{\frac{1}{\frac{1}{2}m^2}} \sim \frac{T_c^2}{N_f (T - T_c)^2} \sim O(1) \text{ when } \frac{T - T_c}{T_c} \sim \frac{1}{N_f^{1/2}}$$

V. Partial summary - strong-coupling LQCD in different large-N limits

In $1/\lambda$ expansion of LQCD you get an effective H for meson dynamics.

In large-N_c, fixed N_f, λ - a classical magnet.

- g.s. breaks $U(4N_f) \rightarrow \chi$ symmetry breaking.
- $T_c \sim O(N_c)$.
- Scaling is not MF and not trivially calculable at $N_c = \infty$.
- No suppression of the critical region.

Believe this is what Chandrasekharan and Strouthos '04 saw numerically.

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In large-N_c, large-N_f, and fixed $\frac{N_c}{N_f}, \lambda$ - a CP_N model.

- g.s. breaks $U(4N_f) \to \chi$ symmetry breaking.
- $T_c \sim f\left(\frac{N_c}{N_f}\right) < \infty$.
- Scaling is easily calculated at $N = \infty$, and is not MF.
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In fixed N_c, large-N_f, and fixed λ/N_f a spin-Peierls state.

- g.s. breaks lattice symmetries, but not $U(4N_f) \rightarrow$ no χ symmetry breaking.
- The critical region is suppressed $\Rightarrow$ if $\dfrac{|T - T_c|}{T_c} \gtrsim \dfrac{1}{\sqrt{N_F}}$ get MF.

VI. Conclusions and gauge realizations in the continuum

The exact solution was possible because when :

- Their action looked like $A(Q,\lambda) = N\left\{Q^2 + \lambda + \mathrm{tr}\,\log G^{-1}(Q,\lambda)\right\}$
- Rescaling : $A(Q,\lambda) = A_{MF} + \left\{m_1^2 Q^2 + m_2^2 \lambda^2 + \frac{1}{\sqrt{N}}(Q^3,\lambda^3) + \frac{1}{N}(Q^4,\lambda^4)\right\}$

But the results were completely different !

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In the combined large-(N_f, N_c) limit : order parameter is the boson $\langle b \rangle$
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Planar QCD : Order $= \langle \bar{\psi} \psi \rangle \rightarrow$ fluctuation is the σ. Its interactions are $O(1/N_c)$?

- Mesons interact with $O(1/N_c)$.
- The $\mathcal{L}_{LGW}$ - analyzed by many, starting from e.g. Wilzcek and Pisarski '84
  $\rightarrow$ only discuss the the scaling of the anomaly term, which is $\sim 1/N_c$.

VII. Realizations in the continuum - the Hagedorn temperature

Other realizations in the continuum include NJL,GN,Yukawa Kogut et al. '98, Strouthos et al. '01-'05

All these involve fermions :

But what about 2nd order phase deconfinement ? 'cheap' numerically.
Similar to the QCD discussion above:

- A discrete symmetry is broken - Z_N.
- Order-parameter is Polyakov loop, whose mass $\to 0$ in 2nd order.
- Interactions of the Polyakov loops are $1/N_c^2$ suppressed.

'Problem' : Have 1st order for $N \geq 3$.

'No Problem' : Approach hidden Hagedorn point at $T_H > T_c$ which may be 2nd order.

Hagedorn's 'ultimate temperature' **'65** - pre-dated QCD

- Spectrum of pp collisions $\to$ hadrons are consistent only for $\boldsymbol{T < T_H \simeq 158}$ MeV.
- Cabibo and Parisi '75 - re-interprate as 2nd order PT of quark liberation.

2nd order because there is a condensation of fluxed with effectively zero mass .

But in $SU(N_c \gg 1)$ deconfinement is 1st order: protects theory from Hagedorn.

Seek T_H in the meta-stable confined phase - tunnelings are $O(1/N_c)$.

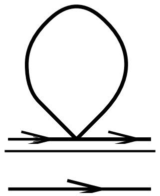
Imagine that the transition was second order and

$$\mathcal{L}_{LGW}(P; T < T_H) = \frac{1}{2}|\partial_\mu P|^2 + \frac{1}{2}(T_H - T)|P|^2 + (\lambda/N^2)|P|^4 + \dots$$

AT $N = \infty$ correlation length of $|P|$ is scales like : $\xi^{-1} \sim (T_H - T)^{1/2}$.

As $T \to T_c$, IR fluctuations win their suppression $\to$ a nontrivial fixed point.

Importance of fluctuations =



$$\sim \frac{\lambda}{N^2} \frac{\frac{1}{m^2} \int \frac{d^3k}{k^2+m^2} \frac{1}{m^2}}{\frac{1}{m^2}} \sim \frac{1}{mN^2} = \frac{1}{(T_H - T)^{1/2} N^2}$$

If $(T_c - T) \sim 1/N^4$ then fluctuations are important.

Check with MC's : Look at Polyakov-loop mass $m_t(T > T_c)$. BB and Teper '05

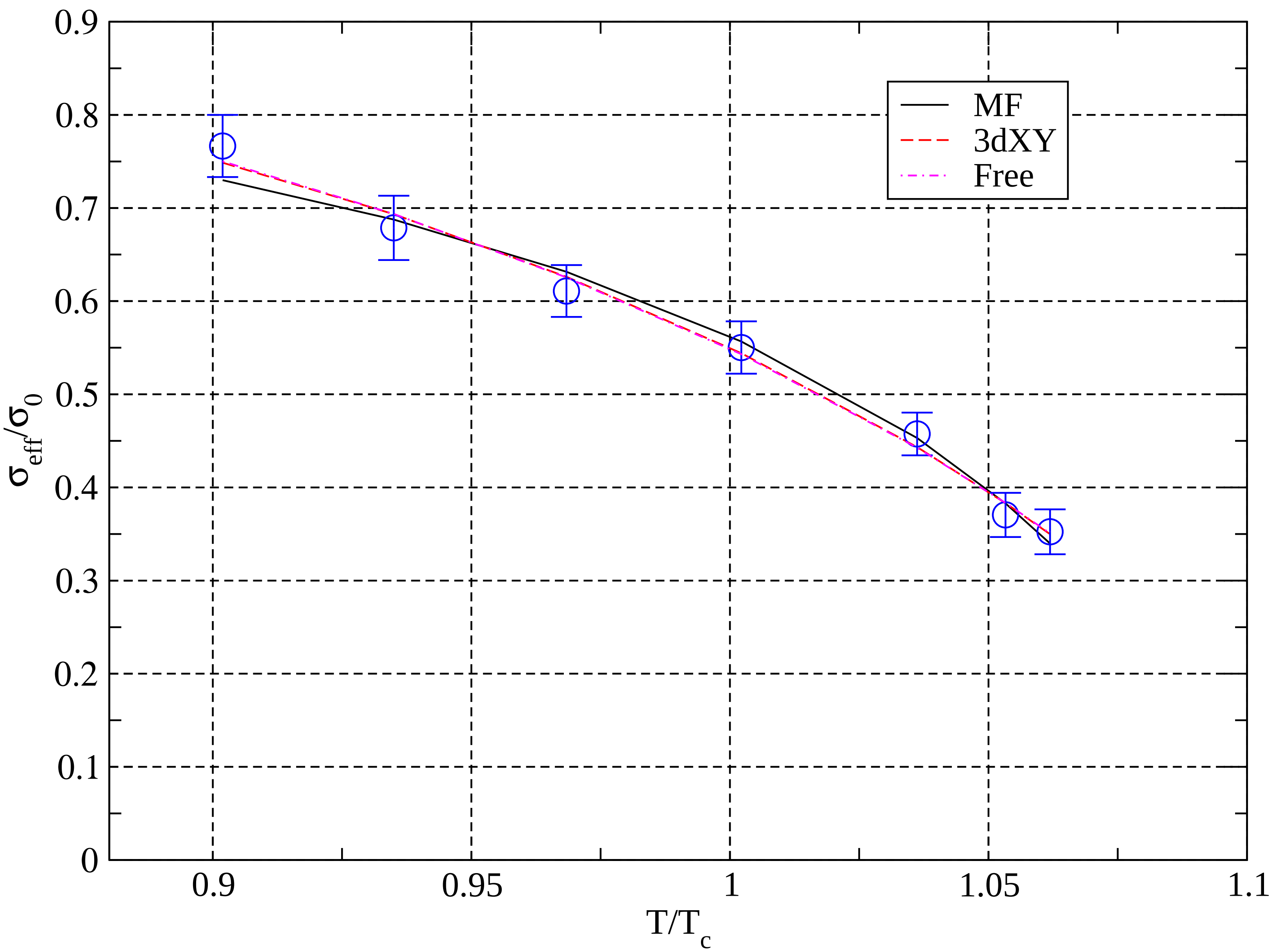
- $m_t(T)$ decreases with T, extrapolate to $m_t(T^*) = 0$, and identify $T^* = T_H$.

$\underline{SU(12), 12^3 \times 5}$

- $\frac{\sigma_{\text{eff}}(T_c)}{\sigma} \simeq \mathbf{0.5}$

- **MF:**
  $\frac{\chi^2}{dof} \simeq \mathbf{0.5}$
  $\frac{T_H}{T_c} = \mathbf{1.092(6)}$

- **3dXY:**
  $\frac{\chi^2}{dof} \simeq \mathbf{0.3}$
  $\frac{T_H}{T_c} = \mathbf{1.116(9)}$



Fit: $m_t(T)/(\sigma/T_c) \equiv$ $\boxed{\sigma_{\text{eff}}/\sigma \times T/T_c}$ with $A(T_H/T_c - T/T_c)^{\nu}$, $\nu =$MF, 3dXY.

II. Large N lattice strong-coupling expansions - what exactly do I mean ?

Standard strong-coupling expansions for $SU(N_c)$ were performed like for $SU(3)$:

- Fix N_c and perform an expansion around $\lambda = \infty$ in $1/\lambda \equiv 1/(g^2 N_c)$.
- If the observable $\hat{C} \sim N_c^0$ then $\hat{C} \sim \sum_n c_n(N_c)\, \lambda^{-n}$. (up to possible $\log$'s)
- Fix λ, take $N_c \to \infty$ and find that $\forall n$, $c_n(\infty)$ are finite.
- Both in Hamiltonian Kogut et al. '80, Smit '80, Grignani '03 and Euclidean de Wit & 't Hooft '79, Munster '81, Smit et al. '81, Ichinose '85, Aoki '86, Green and Samuel '81.

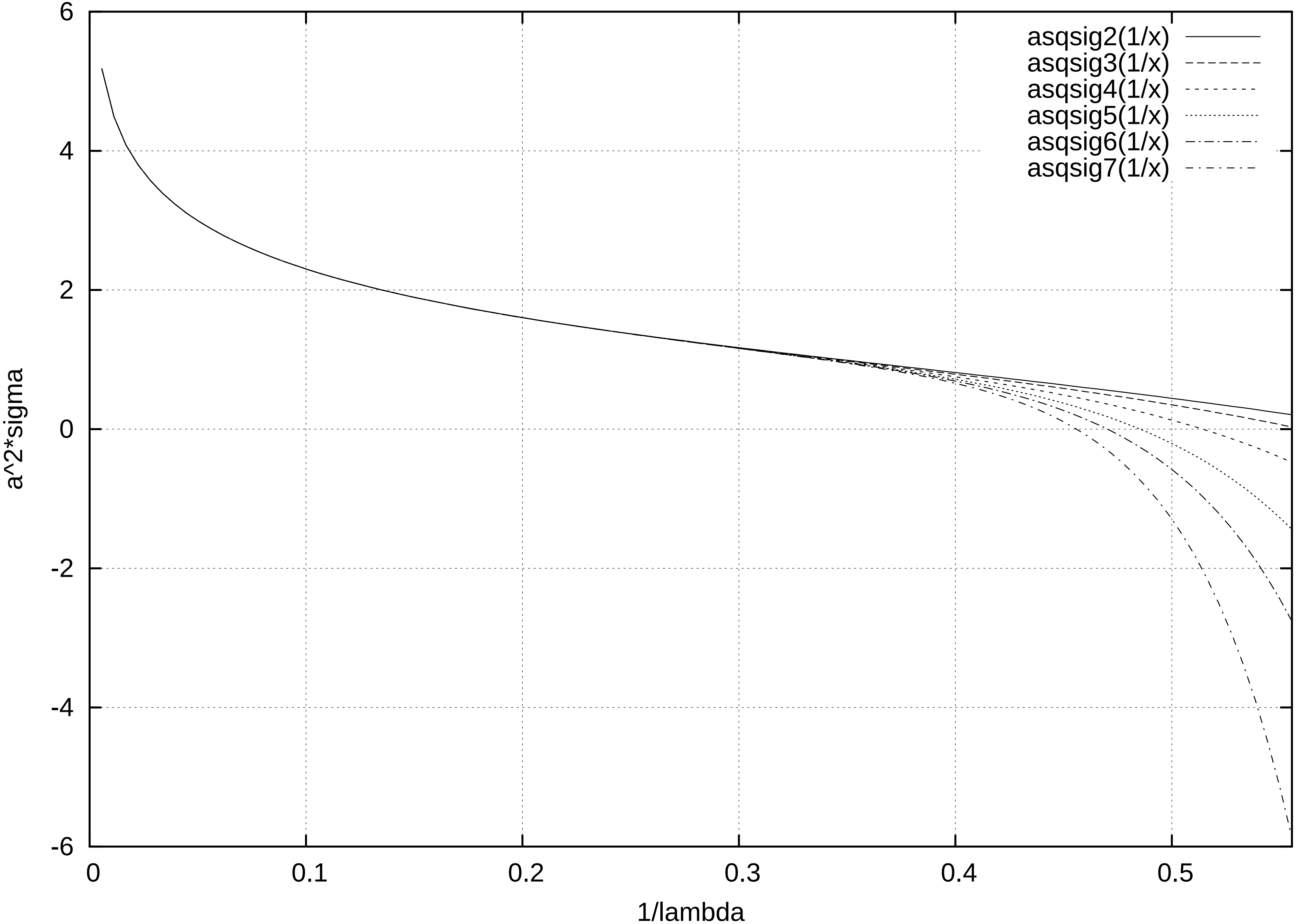
$$(a\sqrt{\sigma})_{N_c=\infty} = \log(\lambda) - 4/\lambda^4 - 6/\lambda^6 - 56/\lambda^8 - 344\lambda^{10} - 4588/(3\lambda^{12}) - 11688/\lambda^{14} + \ldots$$

So in strong-coupling expansions in lattice: take $\lambda \to \infty$ *before* $N_c \to \infty$.

In standard numerical MC's : Same order of limits e.g. Teper's and Neuberger and Narayanan's Lattice '05 contributions.

Gross and Witten '80 : large-λ can (but need not) commute with large-N_c, and

- In $1+1$ they commute for $\lambda > 2$. Gross & Witten '80
- In $3+1$ have a strong-coupling phase for $\lambda > 2.7809(15)$, Okawa '82,Campostrini '98 .


asqsig2(1/x)
asqsig3(1/x)
asqsig4(1/x)
asqsig5(1/x)
asqsig6(1/x)
asqsig7(1/x)
6
4
2
0
-2
-4
-6
a^2*sigma
0
0.1
0.2
0.3
0.4
0.5
1/lambda