Bulk thermodynamics of $SU(N)$ lattice gauge theories at large-N

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Mostly Based on:
BB and Michael Teper, Phys. Lett. B**628**:112, (2005) hep-lat/0506034
But also on:
BB and Michael Teper, PRD (2005) hep-lat/0508021,
BB hep-lat/0511058.

Outline of the talk

I. **Sketch the motivation - the pressure deficit and sQGP** .

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VII. Summary and implications.

I. Motivation:

What replaces Hadronic phase after deconfinement ?

Recent years : evidences that for $T_c < T \lesssim 2T_c$, simple QGP is unsuitable:

- In RHIC : elliptic flow $\longrightarrow$ low viscosity
- Lattice:
  - $10 - 20\%$ deviation from free gas up to $\sim 4T_c$. Boyd et al. '96
  - Small viscosity in pure $SU(3)$ Nakamura et al. '98, '04.
  - Survival of heavy quarkonia states up to $\sim 2T_c$. e.g. the recent Aartz et al. Nov '05, and its references.

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Inspired many theoretical approaches, to mention only a few:

1. Quasi-gluons and quarks with $m_{q,g}(T)$, Peshier et al. '96, Levai and Heinz '97.
2. Perturbation up to $\mathcal{O}(g^6)$ + 3d Euclidean theory Kajantie et al. '02.
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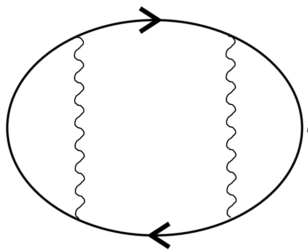
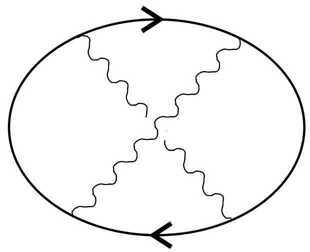
Large N- reminder of basic features:

't Hooft '72 : In $N \to \infty, g^2 N =$fixed limit, use $1/N$ as expansion parameter. Leading diagrams look planar:

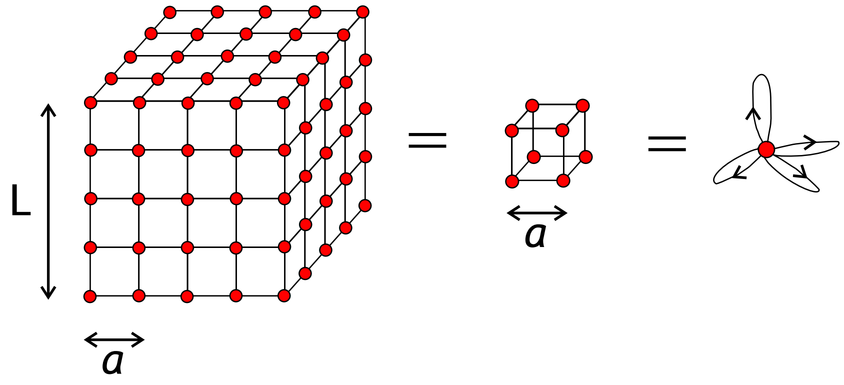
E.g. $\langle J(x)J^\dagger(y)\rangle_c$: $\{$  $\sim g^4 N^3 \sim N \}$

$+\{$  $\sim g^4 N \sim 1/N \}$

Witten '79 : Planarity $\to N = \infty$ is a theory of

- Free mesons (no decays, mixings or scatterings).
- Classical, skryrme-like, baryons.

Lattice and reduction :

- Eguchi-Kawai '82: 

$+\mathcal{O}(1/N^2)$

- But apparently EK reduction works only in bulk phase →Twisted EK, Quenched EK, and

  – Neuberger et al. '02-'05: $[SU(N), V] \equiv [SU(N), \infty] + \mathcal{O}(1/N^2)$ for $V > V_c$.

  – Or: for $V > V_c$, finite V corrections are tiny for larger-N, which coincides with:

- Teper et al. '98-'05 which studied many aspects of $SU(N)$, almost always at smaller lattice volumes.

Maldacena '98 : AdS/CFT - probe $g \gg 1$ limit of large-N (SUS)YM.

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A large–N numerical study can test/constrain these and point to their important ingredients.

II. QCD at nonzero temperature in large-N:

QCD at $T > 0$ simplifies at large-N, again, because :

1. No interactions: Mesons, glueballs, Polyakov-lines are free and stable.
2. No fluctuations: (e.g. in order paramteres) - expected to be $O(1/N)$.

$\rightarrow$ Deconfinement and the deconfined phase simplify, but do not become trivial !

1. No interactions

If hadrons are free-particles in a box then QCD = resosnace gas model with a Hagedorn behavior.

Hagedorn's 'ultimate temperature' '65 - pre-dated QCD

- From spectrum of pp particle multiplicities get that only for hadronic matter is consistent only for $T < T_H \simeq 158$ MeV.
- Cabibo and Parisi '75 - re-interpreted this as a 2nd order phase transition of quark liberation.

A dynamical description for Hagedorn behavior is generic to free-string models :
Banks and Rabinovici '79, noted that in a confining theory the high energy stringy excitations have

$$\left.\begin{array}{l} E(l) = \sigma l \\ \rho(l) = \exp\left(+cl\right) \end{array}\right\} \to Z(T) = \sum_l \rho(l) e^{-E(l)/T} = \sum_l e^{(c-\sigma/T)l}.$$

So $Z(T = \frac{\sigma}{c} \sim \sqrt{\sigma}) = \infty$ and long loops proliferate with $m(T = T_H) = 0$.

But numerical simulations tell us :

For $N = 2$:	2nd order.	McLerran and Svetitsky '81
For $N \geq 3$:	1st order.	Wingate and Ohta '01, Teper et al. '02-05, Kiskis '05
For $N = \infty$:	1st order.	Teper et al. '02-05, Kiskis '05

Is this a contradiction ? Does large-N fail here ? ?
How can we have a 1st order transition with no interactions ?

2. No fluctuations

Imagine that the transition was second order and

$$\mathcal{L}_{LGW}(P;T<T_H) = N^2\left\{\frac{1}{2}|\partial_\mu P|^2 + \frac{1}{2}(T_H-T)|P|^2 + \frac{\lambda}{4!}|P|^4 + \mathcal{O}(P^6)\right\}.$$
$$\xrightarrow{\text{rescaling}} \frac{1}{2}|\partial_\mu P|^2 + \frac{1}{2}(T_H-T)|P|^2 + \mathcal{O}(\lambda/N^2)$$

Which means that the correlation length of $|P|$ is scales like :

$\boldsymbol{\xi \sim (T_H - T)^{-1/2}}$**.**

But it must be in $3DXY$'s universality class !

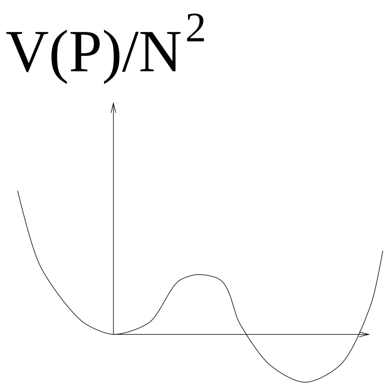
Is this a contradiction ? Does large-N fails here ? ?

How can we have non-MF behavior with no fluctuations ?

Explanation of this apparent contradiction

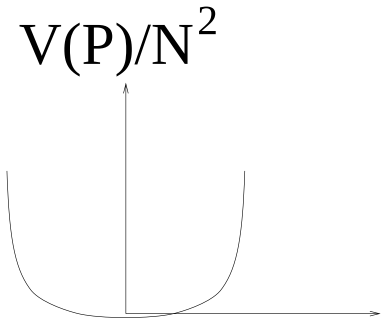
The effective action $\mathcal{L}_{LGW}(P) \sim N_{\mathrm{d.o.f}} \sim N^2$.
$\xrightarrow{\text{rescaling}}$ $\mathcal{L}_{LGW}(P) =$Free theory with $m^2 \sim (T_H - T)$.

If the transition is 1st order :



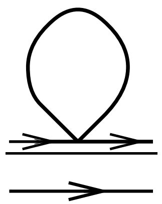
- Fluctuations$\sim O(1/N^2)$ around any stable minimum, but $V(P)$ is still nontrivial.
- Tunnelings are $\sim O(1/N^2)$, but for high enough $T \rightarrow$ tunneling will occur.
- Before they occur, the system 'thinks' its approaching a Hagedorn point.

If transition is 2nd order :



- Fluctuations$\sim O(1/N^2)$ around $P = 0$.
- As $T \to T_c$, IR fluctuations win their suppression $\to$ a nontrivial fixed point. The Ginzburg criterion:

Importance of fluctuations =



$$\sim \frac{\lambda}{N^2} \frac{\frac{1}{m^2} \int \frac{d^3k}{k^2+m^2} \frac{1}{m^2}}{\frac{1}{m^2}} \sim \frac{1}{mN^2} = \frac{1}{(T_H - T)^{1/2} N^2}$$

- So only if $(T_c - T) \sim 1/N^4$ then fluctuations are important.

In other words, the critical region shrinks when $N \to \infty$.

There is a crossover in critical exponents with $x \equiv (T_H - T)^{1/2} N^2$ from

$$\text{MF}(x = \infty) \quad \longrightarrow \quad \text{3dXY } (x \sim 1).$$

The suppression of the critical region is found in

- Gross-Neveu, Nambu-Jona-Lasinio, and Yukawa models Rosenstein et al. '94, Kogut et al. '98, Pelissetto et al. '05 .
- cond-mat systems, e.g. classical spin models Pelissetto et al. '05
- Lattice transitions in Hamiltonian strongly-coupled-QCD (scQCD) at large-N_f BB '05 .

But does not happen always, since not always $\mathcal{L}_{LGW} \sim N_{d.o.f.}$, e.g.

- Chiral phase transitions in the 't Hooft limit of euclidean scQCD Chandrasekharan and Strouthos '04 .
- Chiral phase transitions in the 't Hooft and Veneziano limits (N_f/N_c =fixed) of Hamiltonian scQCD BB '05.

Q: What happens for the chiral-phase-transition in real QCD ?

A: Identify N_c-scaling of terms in $\mathcal{L}_{LGW}(\bar{\psi}\psi)$.

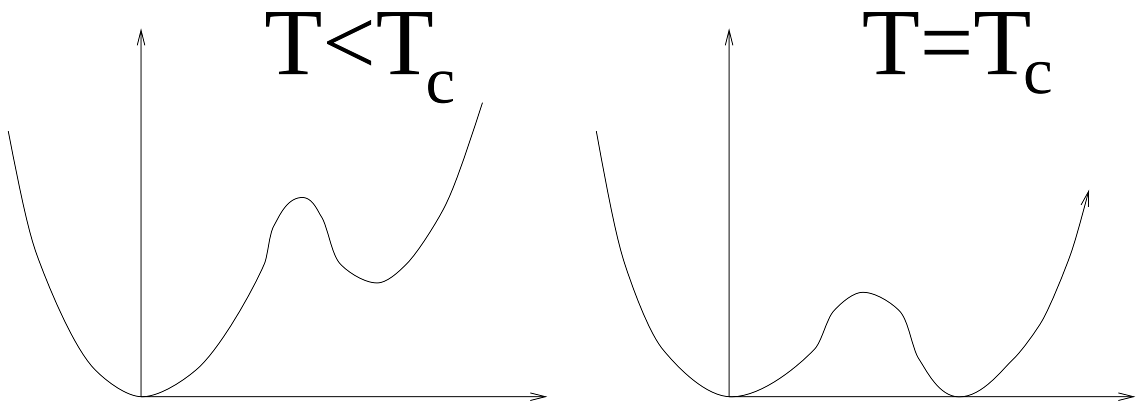
- Identifying the σ resonance with the radial mode of $\mathcal{L}_{LGW}(\bar{\psi}\psi)$.
- Assuming that it has $1/N$ interactions like all other mesons, suggests that indeed $\mathcal{L}_{LGW}(\bar{\psi}\psi) \sim N^2$.

So $SU(\infty)$ is simpler, but nontrivial ! !

In fact one can do *analytical* calculations at $N = \infty$:

Aharony et al. '03-'05

- Find $V(P)$ on $S^{d-1} \times S^1$ as a perturbative series of planar diagrams .
  $\rightarrow$ 1st order at T_c:



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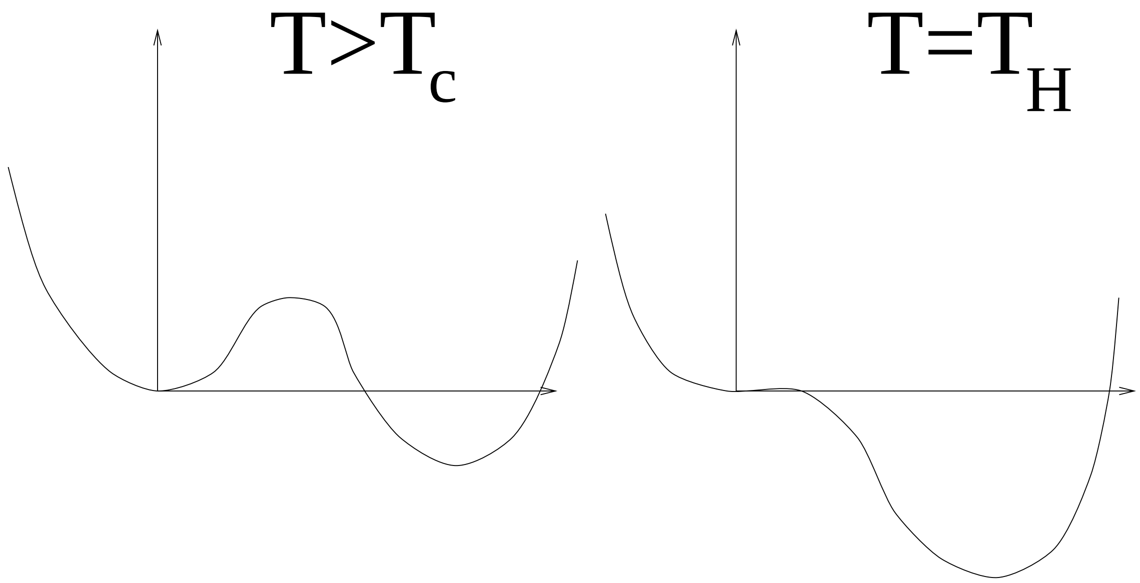

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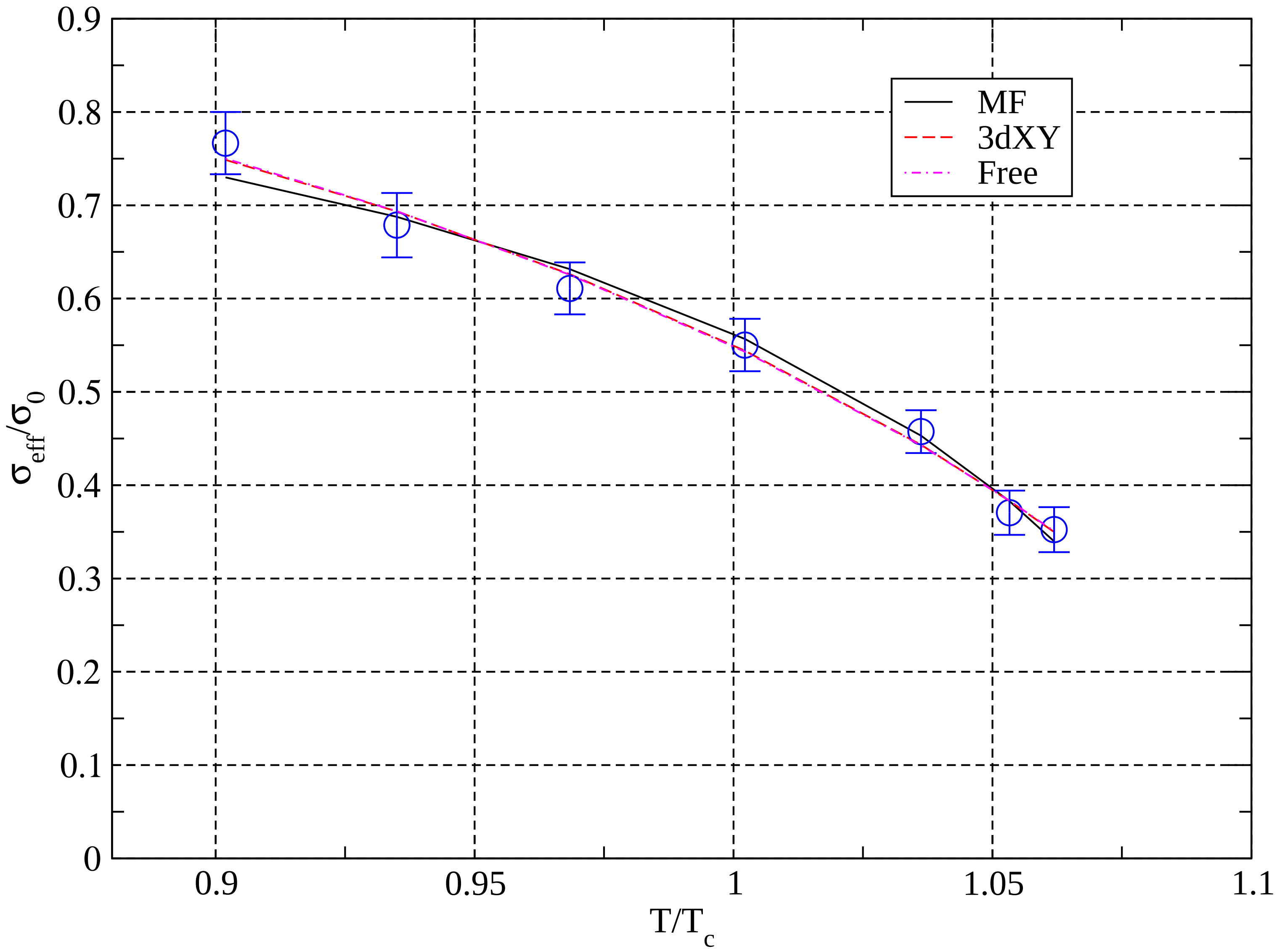
Numerical MC's : Look at Polyakov-loop mass $m_t(T > T_c)$. BB and Teper '05

- Do it in the metastable confined phase - tunnelings are $O(1/N)$.
- $m_t(T)$ decreases with T, extrapolate to $m_t(T^*) = 0$, and identify $T^* = T_H$.



Fit: $m_t(T)/(\sigma/T_c) \equiv \boxed{\sigma_{\text{eff}}/\sigma \times T/T_c}$ with $A(T_H/T_c - T/T_c)^\nu$, $\nu =$MF, 3dXY.

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- Our most statistically reliable result, for $SU(12)$: - loop mass vanishes at

$$T_H/T_c = 1.116(9) \text{ if } \nu = 3dXY, \qquad T_H/T_c = 1.092(6) \text{ if } \nu = MF$$

- The transition is *strongly* first order and $\sigma_{\text{eff}}(T_c) \simeq \frac{1}{2}\sigma$ for $N = 8, 10, 12$.

Finally - more related studies .

1. Loop models Damgaard and Patkos '86,Pisarksi '00, Dumitru et al. '04 ,Eguchi-Kawai inspired models Billo et al. '94, Small volumes Aharony et al. '03,'05.

   → these become soluble at large–N.

2. $\mathcal{N} = 4$ SUSY with large–N, and $g^2 N \gg 1$, where behaviour is reminiscent of QCD's Gusber et al. '98, Policastro et al. '01.

3. Have accurate numerical results of deconfinement in $4d$ pure gauge with $N \leq 8$ by Teper and Lucini '02, Del Debbio et al. '04, Lucini, Teper and Wenger '05, BB and Teper, '05, Bursa and Teper '05 $[T_c, \xi(T_c), L_h, \ldots]$.

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⇓

We wish to numerically calculate bulk thermodynamics of lattice QCD with $N_c > 3$.

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Bad news:

- N_c fermions are very expensive.
- Huge project [no $T = 0$ yet]

Good news:

- Sea quarks are suppressed by $\mathcal{O}(1/N_c)$.
- Deficit is probably in the glue sector.

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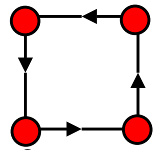
$$\Downarrow$$

We numerically calculate bulk thermodynamics of pure lattice gauge theories for $N > 3$.

III. Bulk thermodynamics on the lattice

Define our pure $SU(N)$ on a 4D Euclidean hypercubic lattice with sides $L_s^3 L_t$.

- $U_\mu(x) \in SU(N)$ live on links, $U_p = U_1 U_2 U_3 U_4 =$ 
- Action $S = \beta \sum_p [1 - \mathrm{Re}(\mathrm{tr}\, U_p)/N]$, $\beta = \frac{2N}{g^2} = \frac{2N^2}{g^2 N} \sim N^2$.
- Evaluate $Z = \int DU \exp(S)$ by MC.
- Lattice spacing $a(\beta)$ in physical units is given by beta function.
- Continuum $a \to 0$ is at $\beta \to \infty$, with fixed $a(\beta) L_t \equiv T^{-1}$, and large enough V.

Want to evaluate the free energy $F(T, V) = -T \log Z$

When $V = \infty$**,** then $F = V f(T)$, and $p(T) = \frac{T}{V} \log Z = \frac{1}{L_s^3 L_t a^4} \log Z$.

Useful to define $\Delta/T^4 = \frac{\partial(p/T^4)}{\partial \log T} \Rightarrow \begin{cases} \epsilon = \Delta + 3p \\ s\,T = \Delta + 4p \end{cases}$

$\Delta = 0$ for free ($P_{\mathrm{Stephan-Boltzmann}} \sim T^4$) or conformal theory (like the $\mathcal{N} = 4$).

On the lattice: express p, Δ in terms of MC averages.

1. "Differential" but larger $\boldsymbol{N} \rightarrow$ smaller $\boldsymbol{L_t}$
   $\rightarrow \boldsymbol{a^{-1} \simeq 1.3}$GeV near $\boldsymbol{T_c} \rightarrow$ can get $p(T_c) < 0$ Svetitsky and Fucito, '83

2. Direct evaluation of density of states $\rightarrow$ modern methods Wang and Landau '01.
   But, preliminary checks did not converge for our largest $N_c = 8$.

3. "Integral method" Boyd et al. '96 (for pure $SU(3)$), Engels et al. '90.

We choose the integral method :

$$
\begin{aligned}
P(T) &= \frac{1}{a^4(\beta) L_t L_s^3} \log Z \overset{[T^{-1}=aL_t]}{=} \frac{L_t^4 T^4}{L_t L_s^3} \log Z, \\
\log Z(\beta) &= \int_{\beta_0}^{\beta} \frac{\partial \log Z}{\partial \beta'} d\beta' + \log Z(\beta_0), \\
\frac{\partial \log Z}{\partial \beta} &= Z^{-1} \frac{\partial Z}{\partial \beta} = \langle \sum_p \mathrm{Re}(\mathrm{tr}\, U_p)/N - 1 \rangle \equiv 6 L_t L_s^3 (u_p - 1).
\end{aligned}
$$

$$
\Rightarrow P(T)/T^4 = 6 L_t^4 \int_{\beta_0}^{\beta} (u_p(\beta') - 1) d\beta' + \left[\left(\frac{L_t}{L_s} \right)^3 \log Z_0 = P(T_0)/T_0^4 \right]
$$

Regularization : Z, and therefore P are UV-divergent, defined up to arbitrary coefficients in DU:

- Define $Z(L_t = L_s = L \gg 1) = 1 \Leftrightarrow P(T=0) = 0$ which means replacing,
- $P(T;\beta) \to P(T;\beta) - P(0;\beta)$ or $u_p \to \delta u_p \equiv u_p(L_s^3 L_t) - u_p(L_s^4)$:

$$P/T^4 = 6L_t^4 \int_{\beta_0}^{\beta} \delta u_p(\beta')\, d\beta' \quad \left[+P/T^4\right]_{T(\beta_0)}$$

$$\Delta/T^4 = \frac{\partial P/T^4}{\partial \log T} = \frac{\partial P/T^4}{\partial \log a^{-1}} = \frac{\partial P/T^4}{\partial \beta} \times \frac{\partial \beta}{\partial \log a^{-1}} = 6\, \delta u_p(\beta) \times \frac{\partial \beta}{\partial \log a^{-1}}.$$

To conclude: Need 2 MC sets on an L_s^4, and an $L_s^3 L_t$ for each $\beta \in [\beta_0, \beta_{\max}]$.

Also choose a low $\beta_0 < \beta_c$ $\to$ $\boldsymbol{\delta u_p(\beta_0) \simeq 0 \to \left(P/T^4\right)_{T_0} \simeq 0}$.

MC's info : Pure gauge with $\boldsymbol{L_t = 5}$: $\boldsymbol{SU(4)}$ on $\boldsymbol{16^3 5}$, $\boldsymbol{SU(8)}$ on $\boldsymbol{8^3 5}$, Compare with $\boldsymbol{SU(3)}$ of Boyd et al. '96, where $\boldsymbol{L_t = 4, 6, 8}$, $\to$ supplement for $\boldsymbol{SU(3)}$ on $\boldsymbol{20^3 5}$.

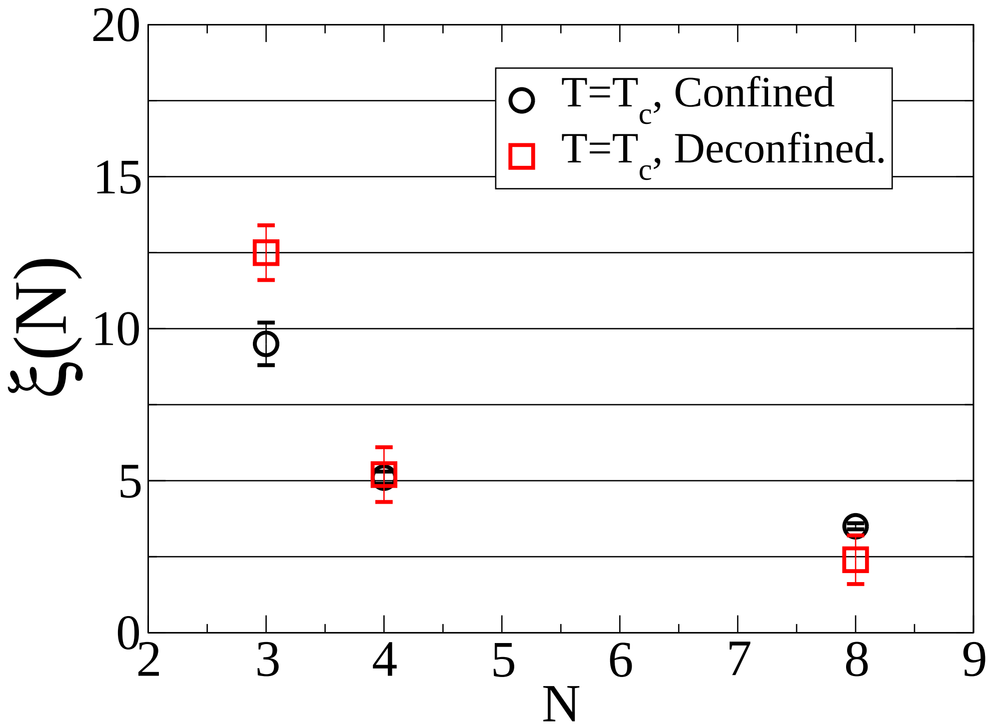
Reduction at large-N or are these volumes really large enough ?

Reduction : suggests fast approach to $V = \infty$ at large-N.

- Lightest Euclidean excitations are Polyakov lines and apart from

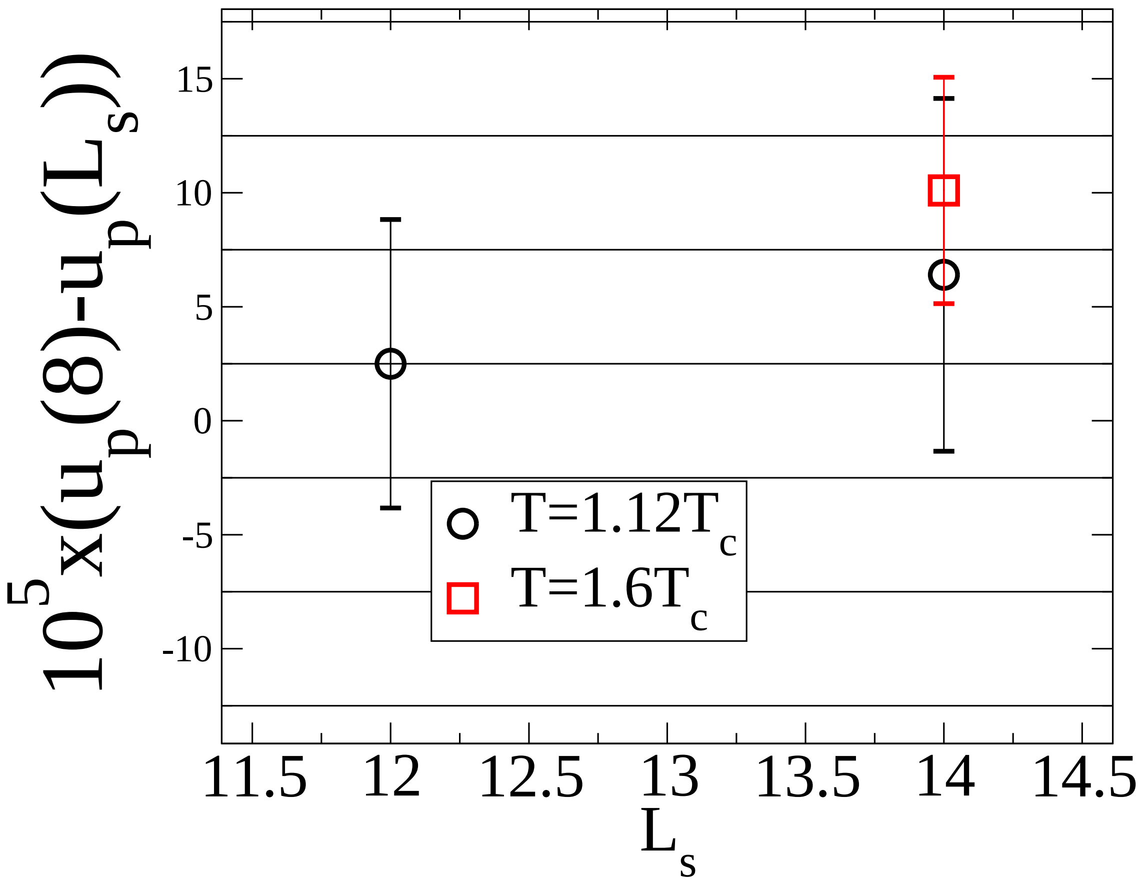
$$\langle P(t)P^{\dagger}(0)\rangle - \langle P(t)\rangle\langle P^{\dagger}(0)\rangle \sim 1/N \times \langle P(t)\rangle\langle P^{\dagger}(0)\rangle$$

- Find: ξ decreases with N (and decreases from T_c) Lucini et al. '05.



Reduction at large-N or are these volumes really large enough ?

Asymmetric lattices, $T \neq 0$: our choices of L_s are **OK** when studying $u_p(L_s)$.
At most 2σ variations (only for one single β, mostly 1σ)



Reduction at large-N or are these volumes really large enough ?

Symmetric lattices, $T = 0$: need $\underline{V^{1/3} \gg 1/T_c}$, since at $N \gg 1$ have phase transitions there e.g. Neuberger and Narayanan '03:

- For $N = 3$: $4 \geq V^{1/3} T_c \geq 2$. $\surd$ (also satisfies empirical bound Boyd et al. '96)
- For $N = 4$: $2 \geq V^{1/3} T_c \geq 1.6$. ?
- For $N = 8$: $1.6 \geq V^{1/3} T_c \geq 1.0$. probably ×

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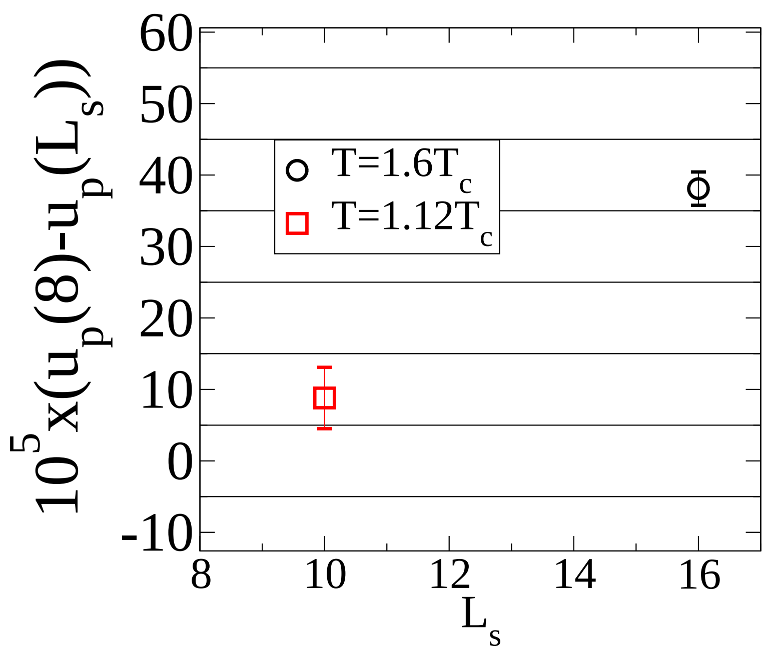
- For $N = 8$: $1.6 \geq V^{1/3}T_c \geq 1.0$. Indeed ✗

Investigating $u_p(L_s)$ we find **a huge 16σ deviation for $SU(8)$**.



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Symmetric lattices, $T = 0$: need $\underline{V^{1/3} \gg 1/T_c}$, since at $N \gg 1$ have phase transitions there e.g. Neuberger and Narayanan '03:

- For $N = 8$: $1.6 \geq V^{1/3} T_c \geq 1.0$. Indeed ✗
- **For $N = 8$, 8^4 is too small** $\rightarrow$ We use (Lucini et al. '05)'s $u_p(L_s)$ from $L_s \leq 16$, to fit $u_p(\beta, L^4) \equiv u_p^0(\beta)$:

$$u_p(\beta) = u_p^{PT}(\beta) + \frac{\pi^2}{12}\frac{G_2}{N} a^4(\beta) + c_4 g^8 + c_5 g^{10}$$

$$\hookrightarrow \mathcal{O}(g^6) \text{ Alles et al. '98}$$

Finally to get $V = \infty$ physics: separate phases with $\beta_c(V = \infty)$ Lucini et al. '05:

- At $V = \infty$ g.s. confining at $T < T_c$, and deconfining at $T > T_c$.
- At $V < \infty$ tunnel with probability $\sim e^{-\delta F} = e^{-V\, L_h\, \delta_T}$, and $\Rightarrow L_h \sim N^2 \rightarrow \delta T \sim 1/N^2$ Lucini et al. '05.

  $\rightarrow$ Only when $T \simeq T_c$ for $SU(4)$, and $SU(8)$.

  $\rightarrow$ For $N = 3$ the volume is too small to separate (very weakly 1st order)

Another aspect of reduction at large-N

Reduction a lá Eguchi-Kawai: at $N = \infty$

- Wilson loops do not depend on V if $V > V_c$ at $T = 0$ Recent Neuberger et al. '02-'05.
- Wilson loops do not depend on T if $T < T_c$ Gocksch and Neri '84.

$\longrightarrow$ Lead them to conclude that $SU(\infty)$ is 1st order.

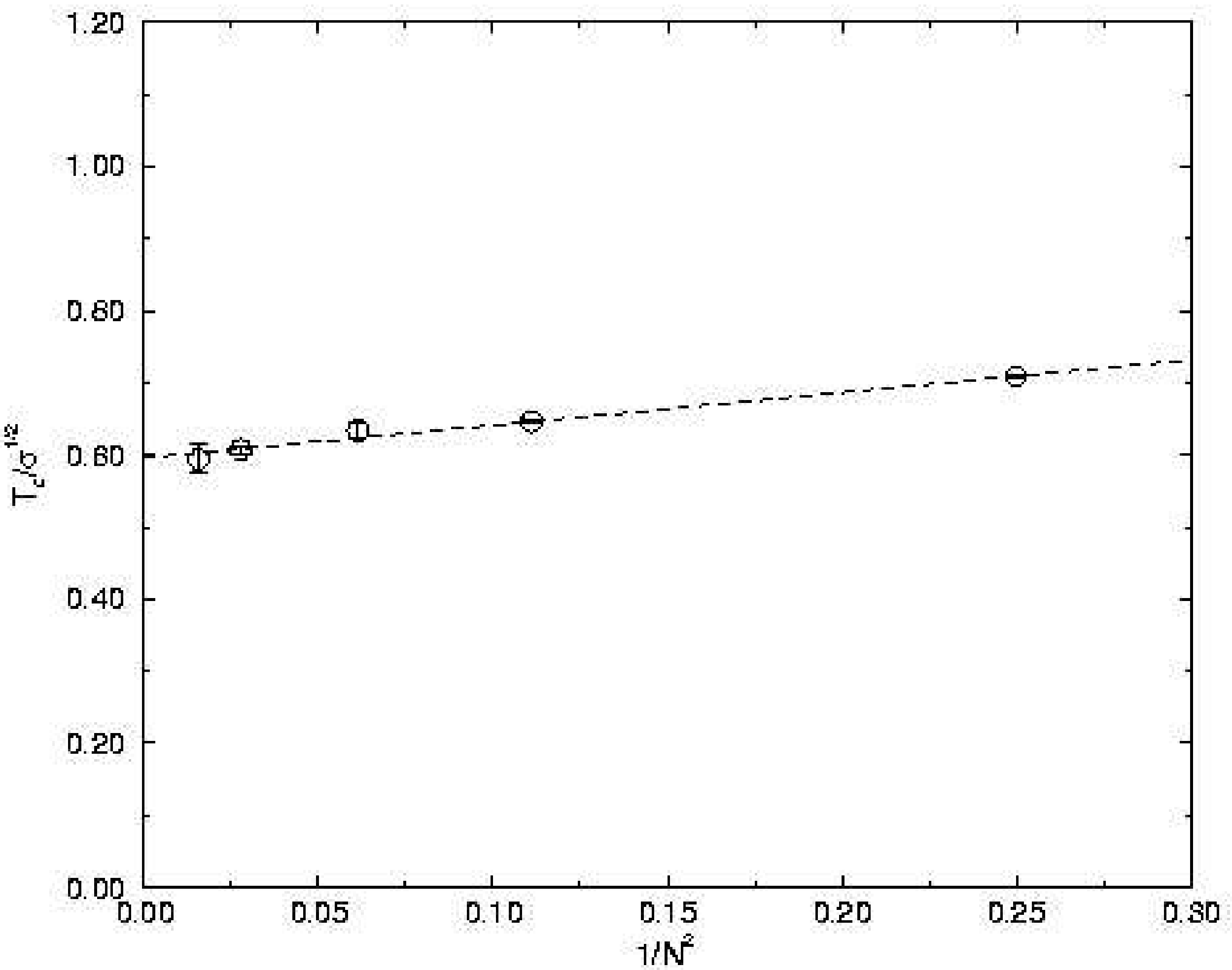
Is $u_p(T_c^-) = u_p(0)$? :

- $SU(3)$: $\mathbf{15\sigma}$ ✕ data from Boyd et al. '96
- $SU(4)$: $\mathbf{1.7\sigma}$ much better.
- $SU(8)$: $\mathbf{2.1\sigma}$ much better.

Which means that $[p/T^4]_{T<T_c} \simeq 0$ for larger $N \to$ very small systematic error from integration for larger N.

V. The physical scale, what is $T/T_c(\beta) = ?$

The value of $T_c/\sqrt{\sigma}$ Lucini et al. '03,'05 $(a \to 0, V \to \infty)$



The most natural scale here is T_c.

→ this requires $\beta_c(L_t, L_s)$ → a very large-scale project (need many L_t, L_s).

One can also fix the scale with $\sqrt{\sigma}$ using the $(a\sqrt{\sigma})_\beta$ interpolation of Lucini et al. '05

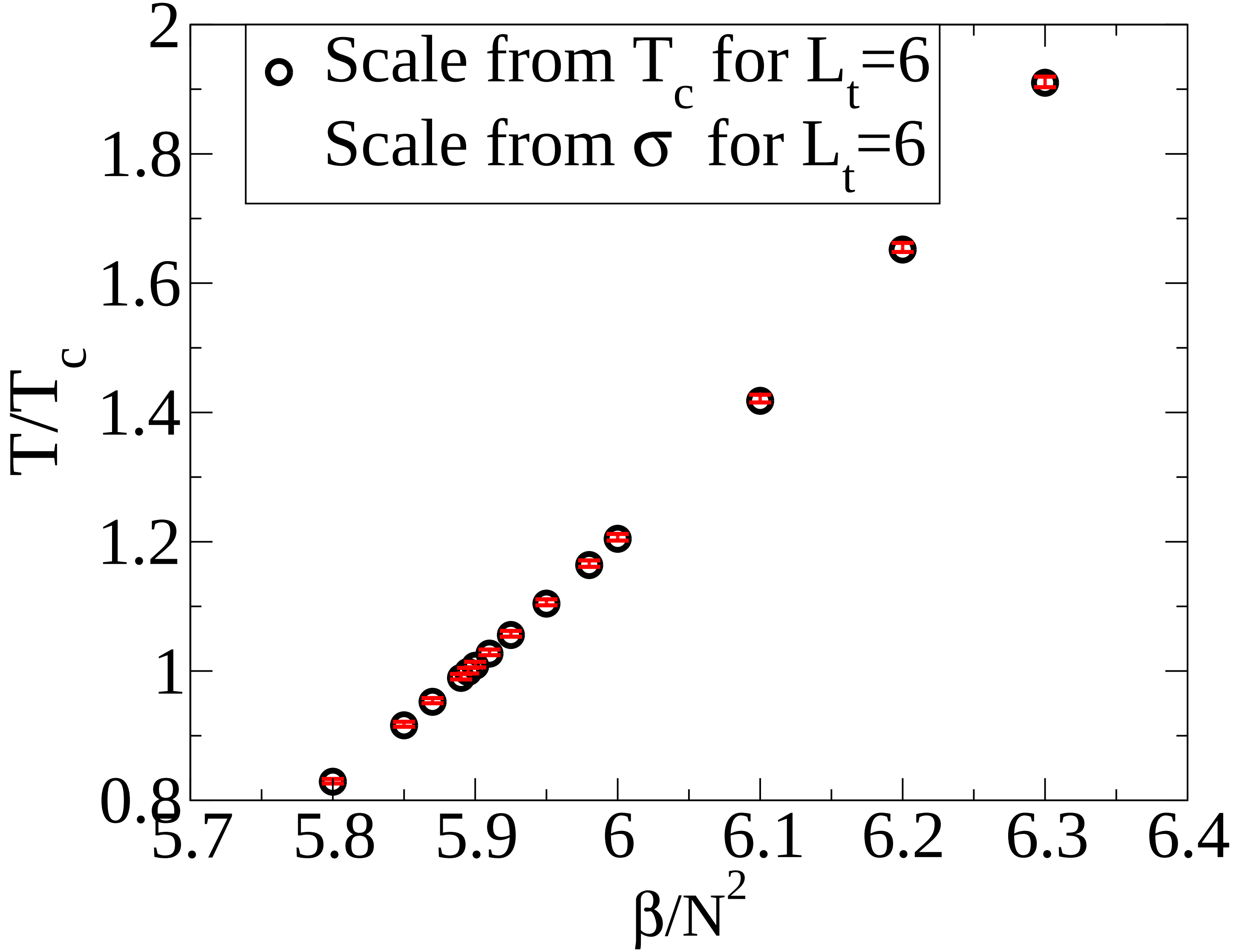
$$^*\beta = \frac{2N}{g^2} = \frac{N^2}{g^2 N} \sim N^2 \qquad \text{in 't Hooft limit } (N \to \infty, g^2 N = \text{fixed}).$$

Possible differences are due to $\mathcal{O}(a^2)$ corrections to $\frac{T_c}{\sqrt{\sigma}}$, and

- For $SU(3), SU(8)$: $\frac{T_c}{\sqrt{\sigma}}$ at $a = 1/(5T_c), 1/(8T_c)$ are the same within errors.
- For $SU(4)$: there is a 5σ difference between $a = 1/(5T_c), 1/(8T_c)$, which is however only $\sim 2\%$ → a small overestimate of T/T_c when $T/T_c \simeq 8/5$.

→ We fix the scale with $\sqrt{\sigma}$

As a consistency check : compare $T/T_c(\beta)$ from Boyd et al. '96 made with T_c for $SU(3)$:



VI. Results

We work at fix lattice spacing $a = 1/5T$:

- Have $\mathcal{O}(a^2) = \mathcal{O}((aT)^2) = \mathcal{O}(1/L_t^2)$ to the free-gas Engels et al. '99. If $L_s \to \infty$, then

$$P_{\text{free}}/T^4 \quad = \quad (N^2-1)\frac{\pi^2}{45}\left[1 + \underbrace{\frac{8}{21}\left(\frac{\pi}{L_t}\right)^2 + \frac{5}{21}\left(\frac{\pi}{L_t}\right)^4}_{\sim 19\% \text{ for } L_t = 5.} + \mathcal{O}(L_t^{-6})\right].$$

We normalize to the lattice free gas, to get $p/T^4 \to 1$, $\epsilon/T^4 \to 3$, $s/T^3 \to 4$, and therefore we normalize Δ/T^4 accordingly.

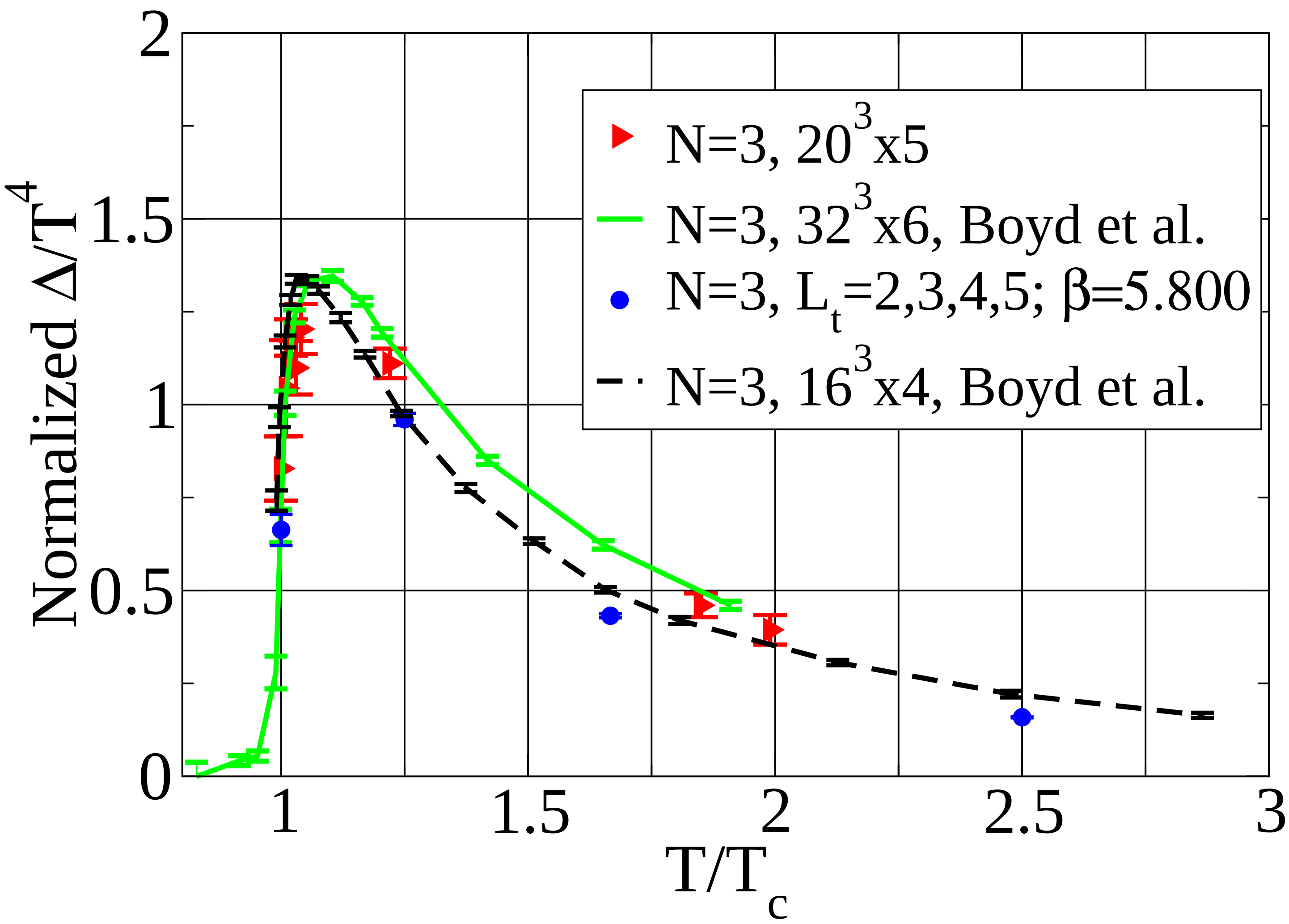
To see that finite a corrections are systematic compare the $\boldsymbol{\Delta}$, for $\boldsymbol{SU(3)}$, we:

- Compare (Boyd et al.)'s $L_t = 4, 6$, and ours $L_t = 5$.
- Add data from $L_t = 2, 3, 4$ that go up to $2.5T_c$.

And get :

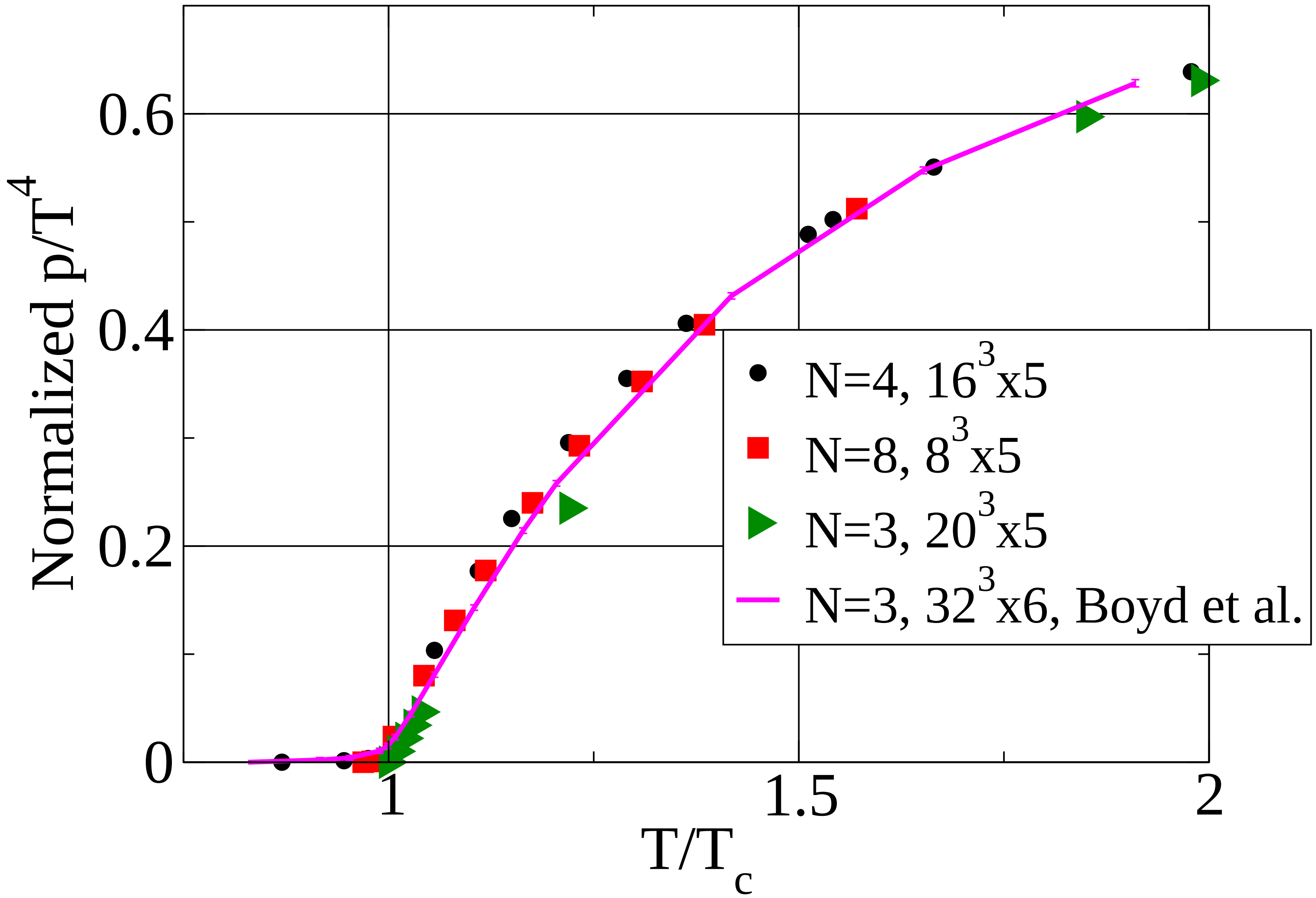
Results-Normalized Δ for $SU(3)$, $L_t = 2, 3, 4, 5, 6$ dep.

See systematic L_t dep.



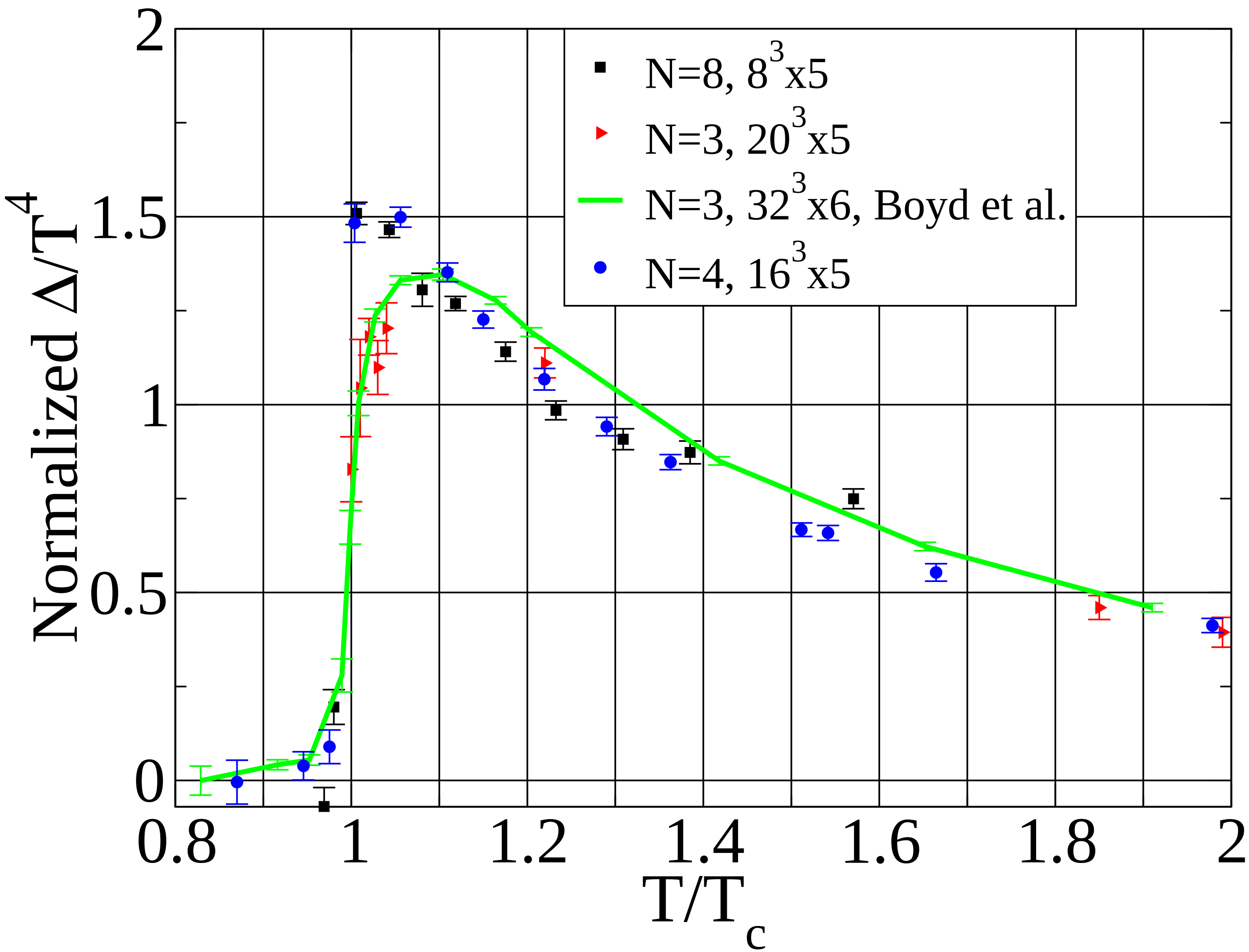
VII.a. Results-Normalized pressure (symbols' size=errors)

Pressure plots lie almost on top of each other.

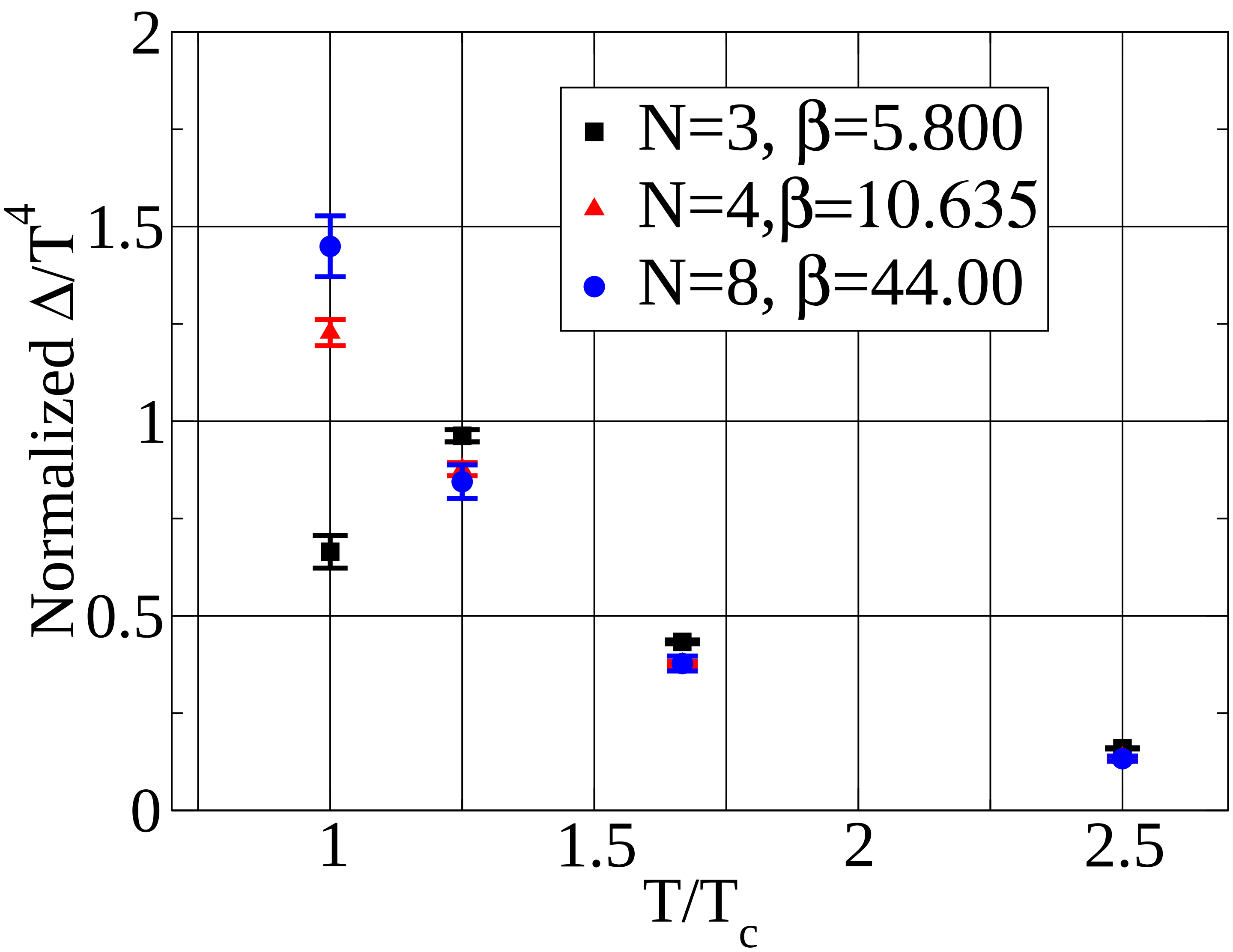


VII.b. Results-Normalized Δ

(1) Δ has modest $O(1/N)$ corrections.
(2) At $T = T_c$ Δ is different, possibly due a weak 1st order in $SU(3)$.



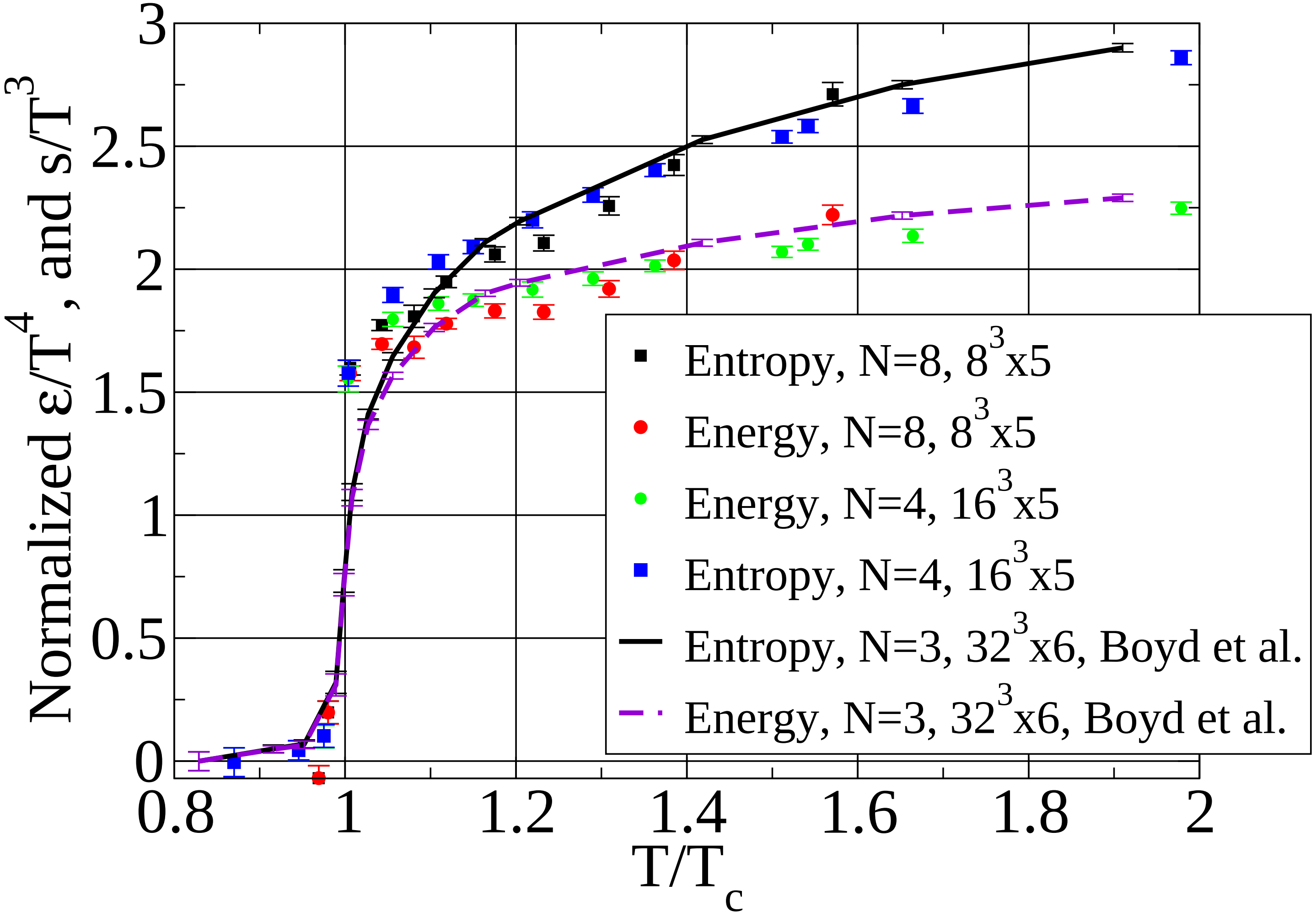
VII.d. Results-Normalized Δ, $L_t = 2, 3, 4, 5$ (higher temperatures.)



Here we see Δ up to $T = 2.5T_c$, again modest $O(1/N)$ corrections.

VII.e. Results-Normalized $\epsilon, s,\ L_t = 5$

$s/T^3, \epsilon/T^4$ have modest $O(1/N)$ corrections.



IX. Summary and Implications

We calculated $\boldsymbol{p, \Delta, \epsilon, s}$ at lattice spacing $a^{-1} = 5T$ with the integral method:

- $\boldsymbol{T \leq 2T_c}$ **for** $\boldsymbol{SU(4)}$.
- $\boldsymbol{T \leq 1.6T_c}$ **for** $\boldsymbol{SU(8)}$.

We calculated $\boldsymbol{\Delta}$ **for** $\boldsymbol{SU(4)}$ **and** $\boldsymbol{SU(8)}$ $\boldsymbol{T_c \leq T \leq 2.5T_c}$ at $a^{-1} = (2-5)T_c$, .

We find that for all quantities the $\boldsymbol{\mathcal{O}(1/N)}$ **corrections are modest/small.**

An exception is with Δ (and therefore also ϵ, s): where the **discontinuity at** $\boldsymbol{T_c}$ **is sharper for** $\boldsymbol{SU(4,8)}$ **than for** $\boldsymbol{SU(3)}$.

Any calculation of $\boldsymbol{P, \Delta}$ **deficit must also survive the large-**$\boldsymbol{N}$ **limit which is easier to approach analytically**.

Which means ...

Diagrammatic methods - **Planar diagrams are most important.**

→ Up to $\mathcal{O}(g^6)$ a lá Kajantie et al. '02 everything is planar ! Non-planar diag. maybe at $\mathcal{O}(g^7)$ or $\mathcal{O}(g^8)$ Schroder priv. comm.

Models (loops, quasi-particles, bound states)- **Models' parameters must have weak N-dependence.**

Singlet excitations have no role, as there are $\mathcal{O}(1)$ of them.

No role for topology: as at large-N and $T > T_c$ have no instantons Lucini et al. '04, Del Debbio et al. '04

QCD vs. **large-N SUSY models.**

→ the difference is not due to $\mathcal{O}(1/N)$ corrections.

→ Understanding what SUSY dynamics imply on nonSUSY gauge theory is sufficient in the large-N limit.

IX. Future studies on thermodynamics

Interesting analytical work on $SU(N)$ deconfinement with large-$\boldsymbol{N}$ is Aharony et al. '03-'05.
Using

- Absence of zero modes on spherical spatial volumes.
- Asymptotic freedom make perturbation theory reliable at tiny volumes.
- Large-$\boldsymbol{N}$ techniques (otherwise no transition at small volume).

$\rightarrow$ calculate on $S^1(\mathrm{time}) \times S^d(\mathrm{space})$:

(1) Order of transition. (2) $P(T)/T^d$ in the range $T \simeq T_c$.

They conjecture : $V \simeq 0$ transition continuously connected to $V = \infty$ one.

Is it ? Perform MC's on spherical lattices and check behaviour of different observables as a function of V. BB and J. Wheater, work in progress.

Suppression of critical region in scalar-QCD with bi-fundamentals at deconfinement and/or chiral restoration.

X. And on a different note: quanrkonia at large-N and phenomenology

$SU(\infty)$ **is attractive since** :

- Meson sector has no decays, no mixings, no scatterings.
- Sea quarks are $\mathcal{O}(1/N_c) \rightarrow$ quenched QCD $=$ full QCD (apart from $m_q = 0$).
- Baryon spectrum is 'simpler' - has an additional spin-flavor symmetry.
- and more: χPT is tree level, weak matrix elements, etc.

Can use these features to shed light on QCD spectra issues ?

Do quenched QCD at light masses, and extrapolate to $N \rightarrow \infty$, and:

- m of lightest $q\bar{q}$ scalar, which mixes with glueballs at $SU(3)$.
- Excited states.
- No decays $\rightarrow$ spectral function $= \sum_i \delta(E - m_i)$.

Project with M. Teper, F. Bursa, U. Wenger and ???