String tensions of $SU(N)$ gauge theories in $2+1$ dimensions

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Based on:
BB and M. Teper, A. Athenodorou, BB and M. Teper work in progress.

I. Why $2+1$?:

Karabali and Nair '95 : Approach the Hamiltonian of pure gauge (continuum) using a clever transformation of the fields and :

$$\frac{\sqrt{\sigma}}{g^2 N} = \sqrt{\frac{1-1/N^2}{8\pi}}$$

Remarkably close (within a few percent) to lattice data Lucini and Teper '02.

When $N \uparrow$ then discrepancy $\downarrow$ $\Rightarrow$ **Exact at large-N ?**

If correct then Karabali-Nair approach = a significant step forward !

$$(KN - \text{lattice}) \simeq +1\%, \quad 5\sigma$$

Comparison assumes that systematics are small :

- Extracting string tensions

$$m_{string}(L) = \sigma L - \frac{\pi}{6L} \quad - \quad \frac{?}{L^2}$$

Tends to increase σ

Systematics work in the "right direction" $\rightarrow$ towards KN

$\Downarrow$

Need to remove these uncertainties.

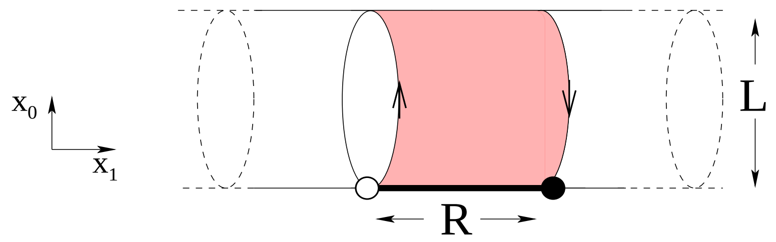
but also

$\frac{?}{L^2}$ **has deeper significance - beyond Luscher term**

$\rightarrow$ **Info on string theory of QCD flux-tube**

II. Background: Strings and QCD

Basic assumption : $q\bar{q}$ Flux-tube $\stackrel{R\to\infty}{\simeq}$ String.



Basic questions :

- open channel - $\boldsymbol{E(R) = ?}$
- closed channel - $\boldsymbol{\tilde{E}(L) = ?}$

Theoretical predictions :

- Nambu-Goto a là Arvis '83, Need D=26.

$$\tilde{E}_n(L) = \sigma L\sqrt{1 + \frac{8\pi}{\sigma L^2}\left(n - \frac{D-2}{24}\right)} = \sigma L + \frac{4\pi}{L}\left(n - \frac{D-2}{24}\right) - \frac{8\pi^2}{\sigma L^3}\left(n - \frac{D-2}{24}\right)^2 + \dots$$

- Luscher and Wiesz '02,'04:
  - phenomenological action, low energy contstants a là χPT.
  - open-closed string duality.

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- Polchinski and Strominger '91, 'phenomenological' (better as $R \to \infty$):
  - Modify NG, quantize at any D.

$$\tilde{E}_n(L) = \sigma L + \frac{4\pi}{L}\left(n - \frac{D-2}{24}\right) + O\left(\frac{1}{L^3}\right) \qquad \forall D$$

  - Recently : Drummond '04, Dass and Matlock '06.

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III. Lattice calculation

Activity in last decade was rich : $3D, 4D$ with $Z_2, U(1), SU(N \leq 6)$. For review Kuti '05, or Caselle and collaborators, Kuti and collaborators, Luscher and Wiesz, Majumdar and collaborators, Teper and collaborators, Meyer.

Unusual to fit $\boxed{\tilde{E}_0(L) = \sigma L - \frac{\pi(D-2)}{6L} - \frac{A}{L^3} + \frac{B}{L^5}}$ to get A, B. (but Teper and Meyer '04)

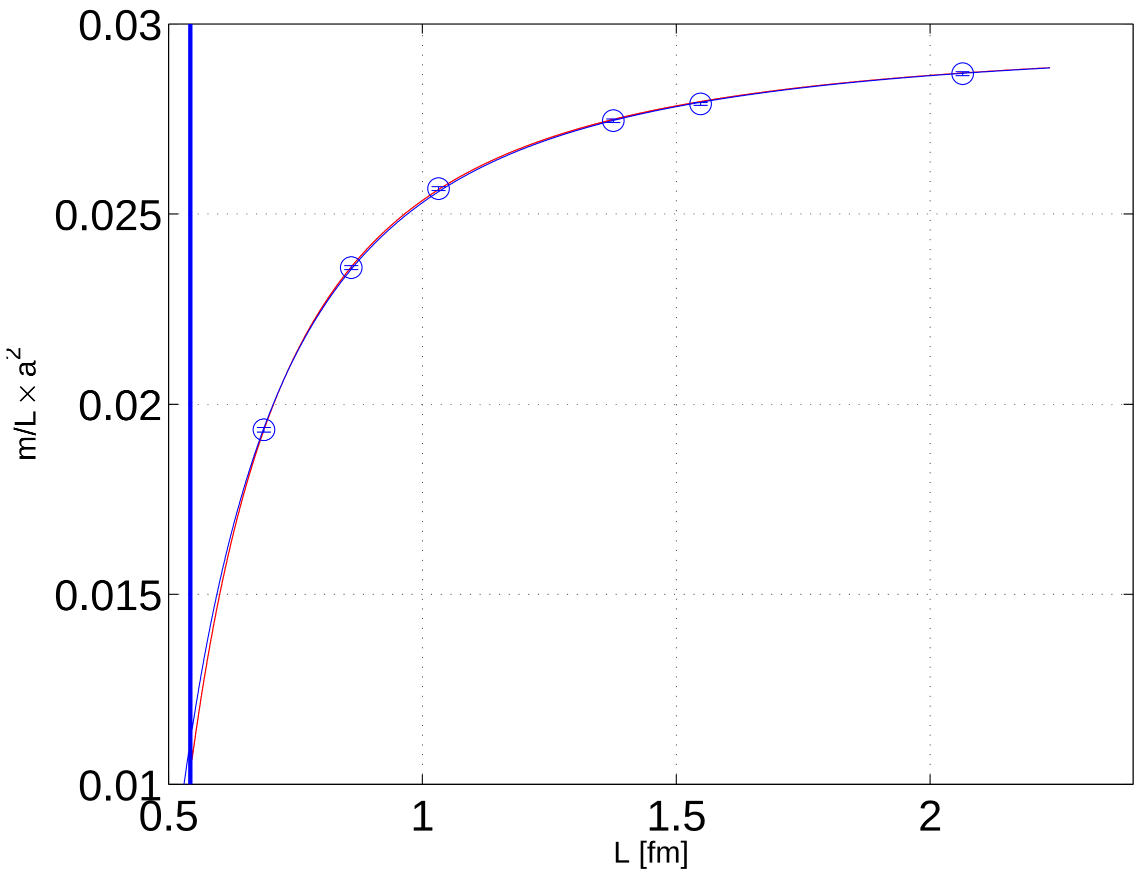
Our approach :

- Measure $\langle P_x P^*_{x+R} \rangle$ of Polyakov loops with $p_\perp = 0$.
- Look at 'large' R mass plateaus.

Perform MC : $L \times L_\perp \times L_t$ with $S = -\beta \sum_p \mathrm{Re}(\mathrm{tr}\, U_\Box)/N$, $\beta = \frac{2N}{ag^2} = \frac{2N^2}{ag^2 N} \sim N^2$.

Do $SU(N)$ with $N = 2, 3, 4, 6, 8$ and ...

III.A. Results - $m(L) = \sigma L - \frac{\pi}{6L} - \frac{A}{L^3} - \frac{B}{L^4}$. $SU(6), a^{-1} = 2.6$ GeV.



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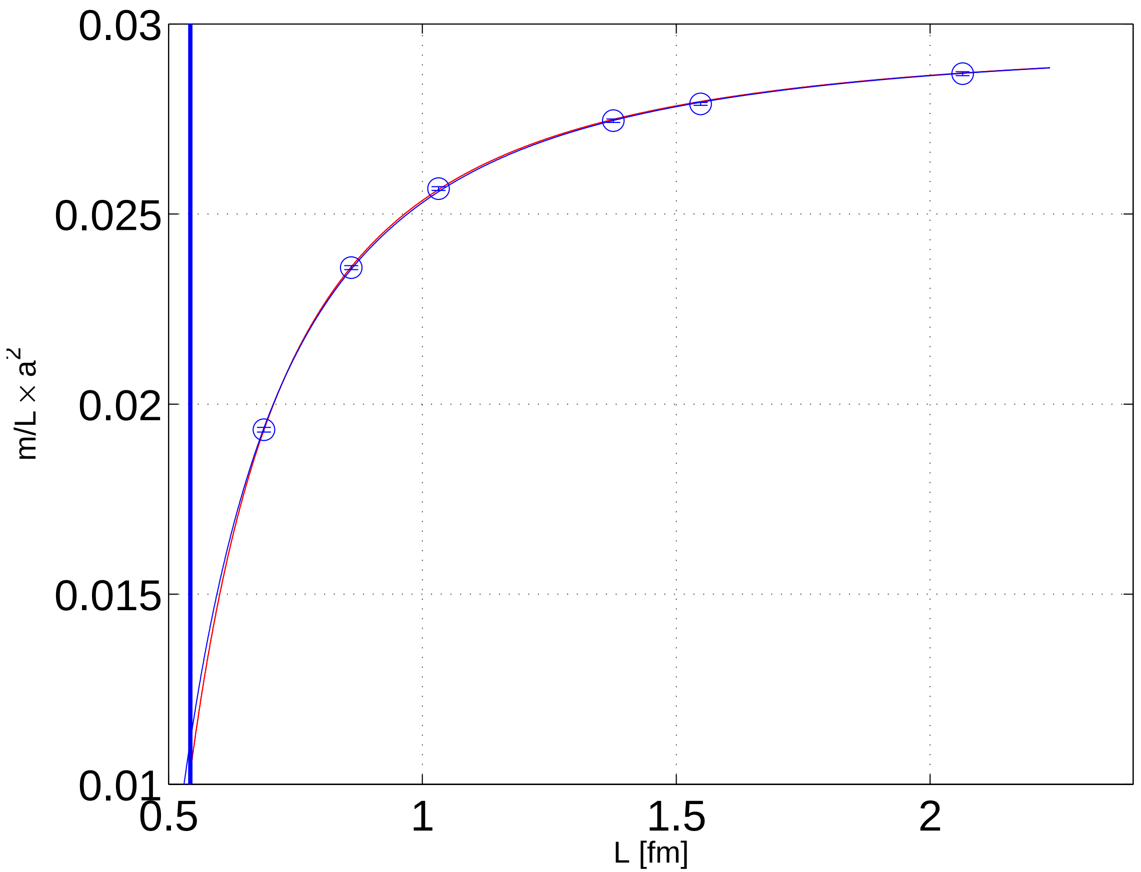

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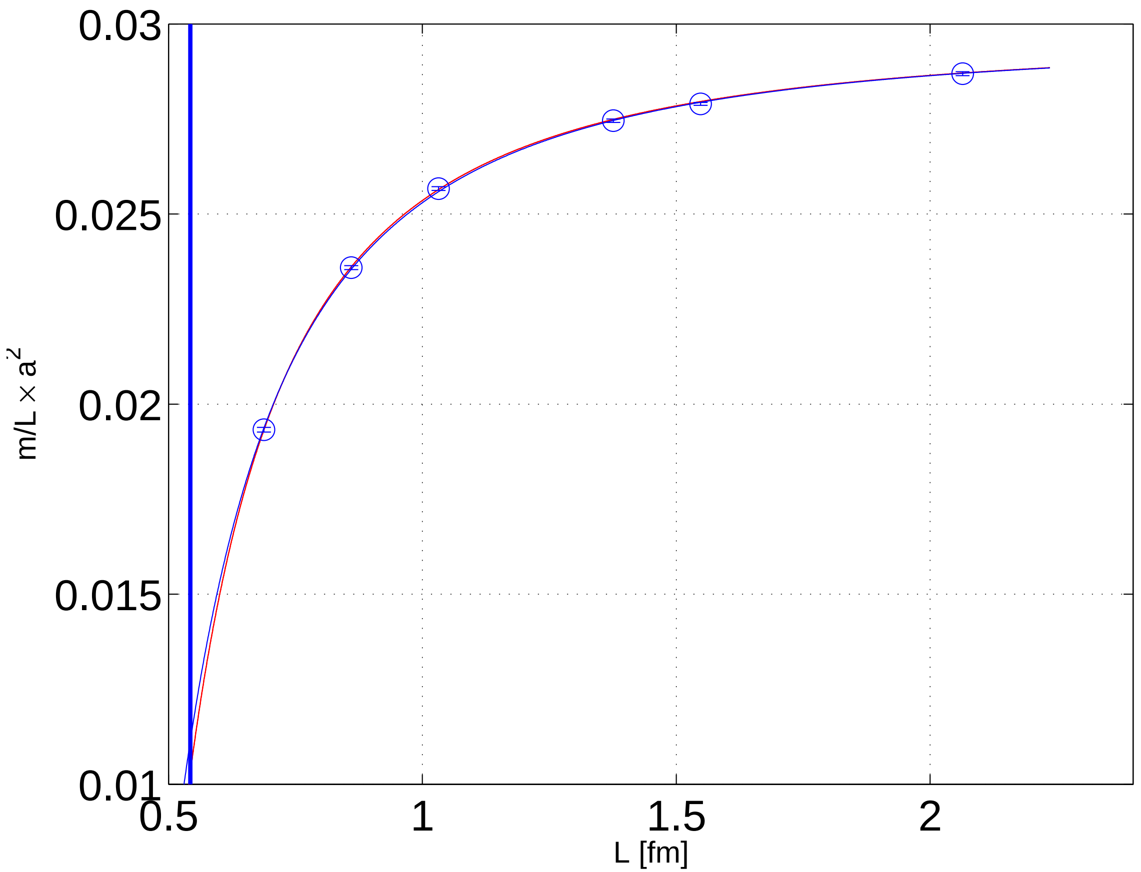

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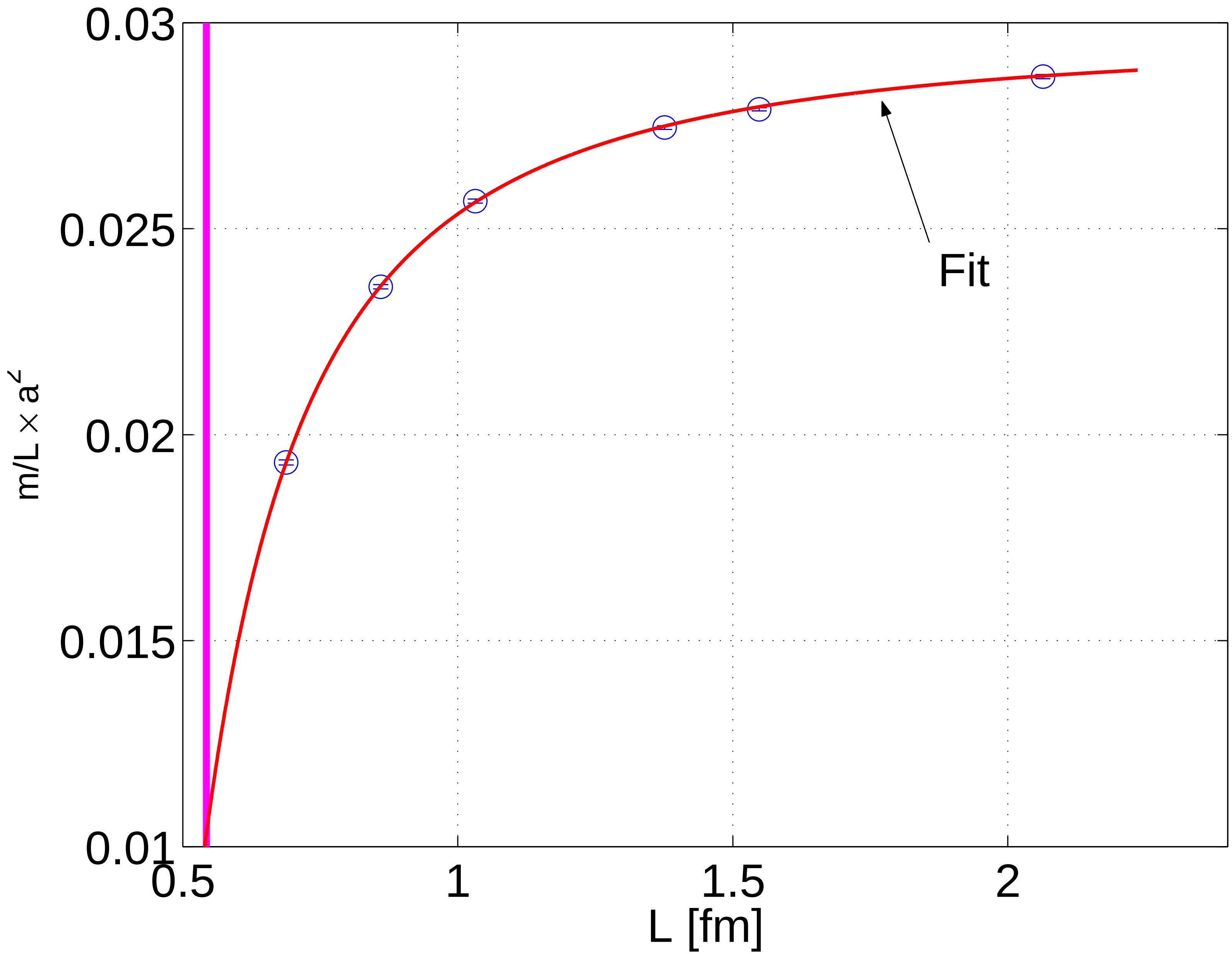
$$CL = 88\% \quad : \quad m(L) = \sigma L - \frac{\pi}{6L} - \frac{\text{N.G.}}{L^3} - \frac{0.168(16)}{\sigma^{3/2} L^4}$$

III.A. Results - still $m(L)$. Comparing fit with NG and with Luscher term



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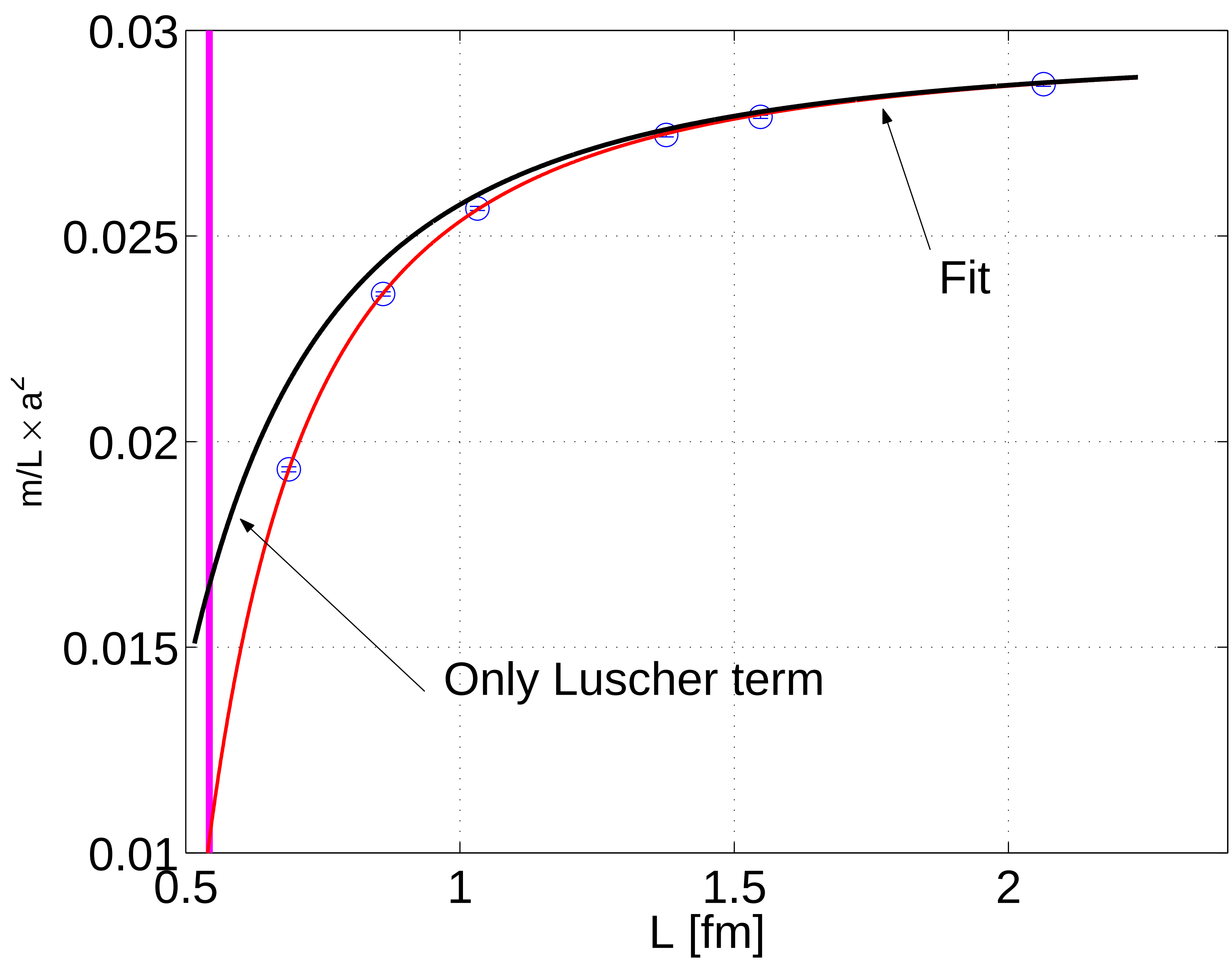

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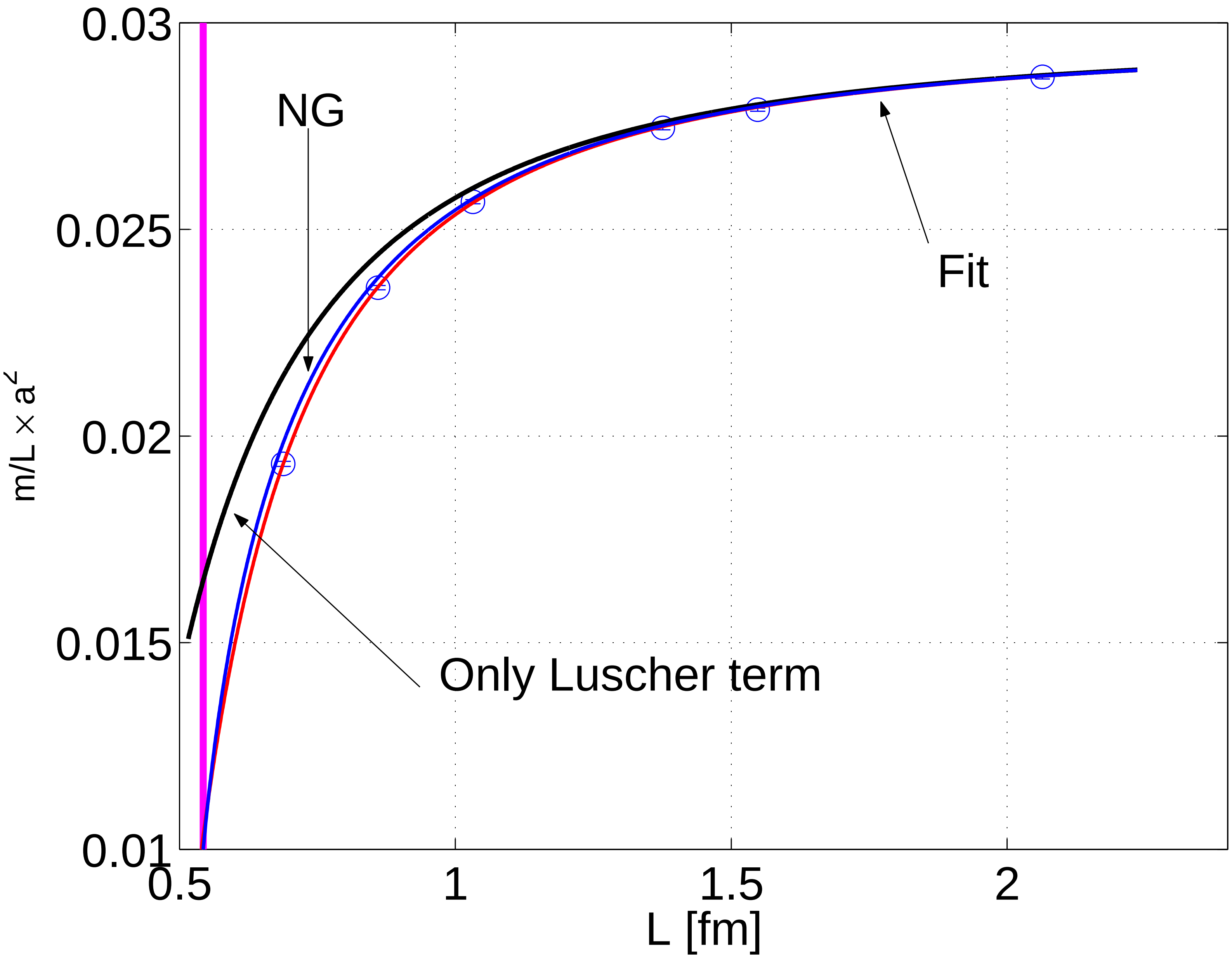

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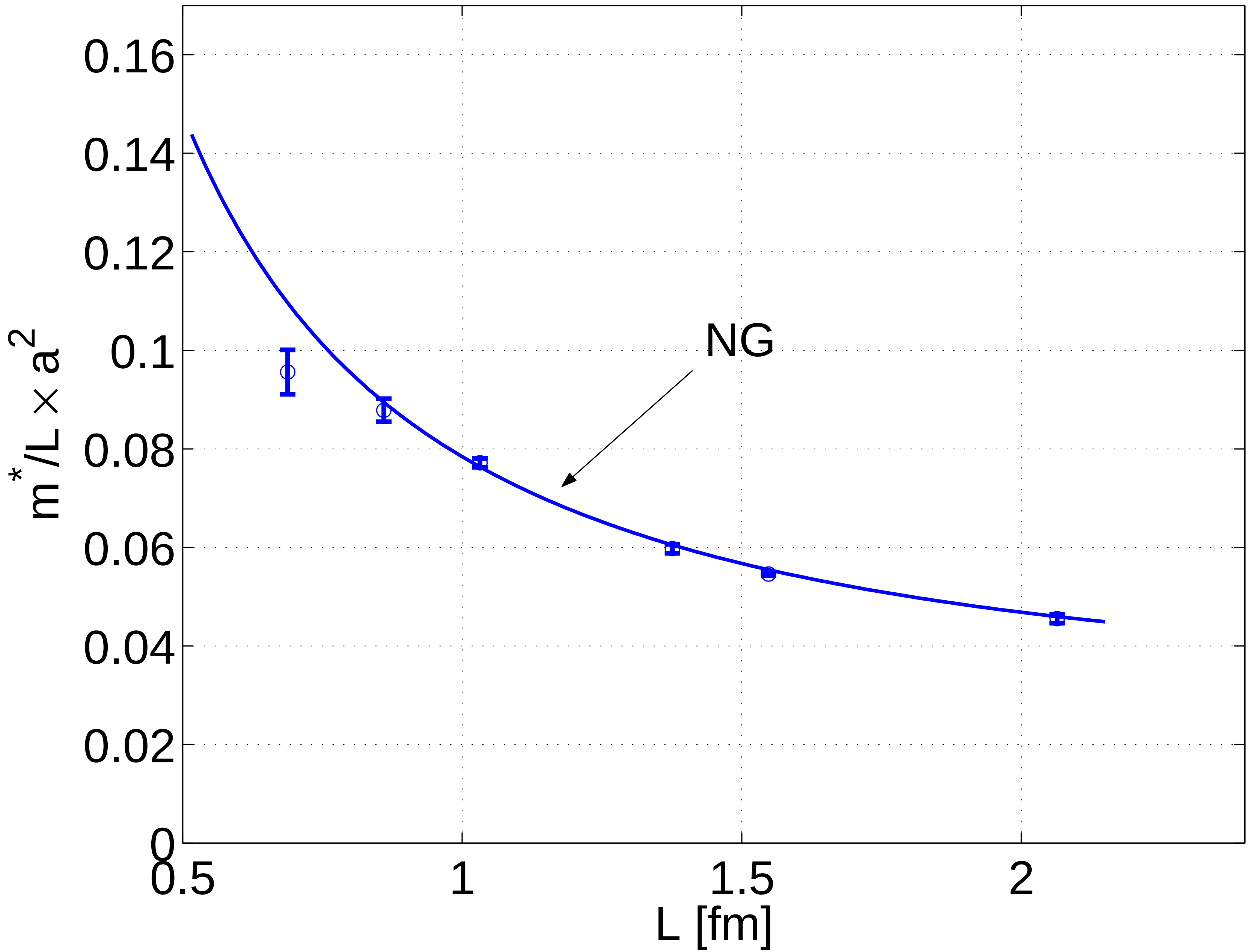

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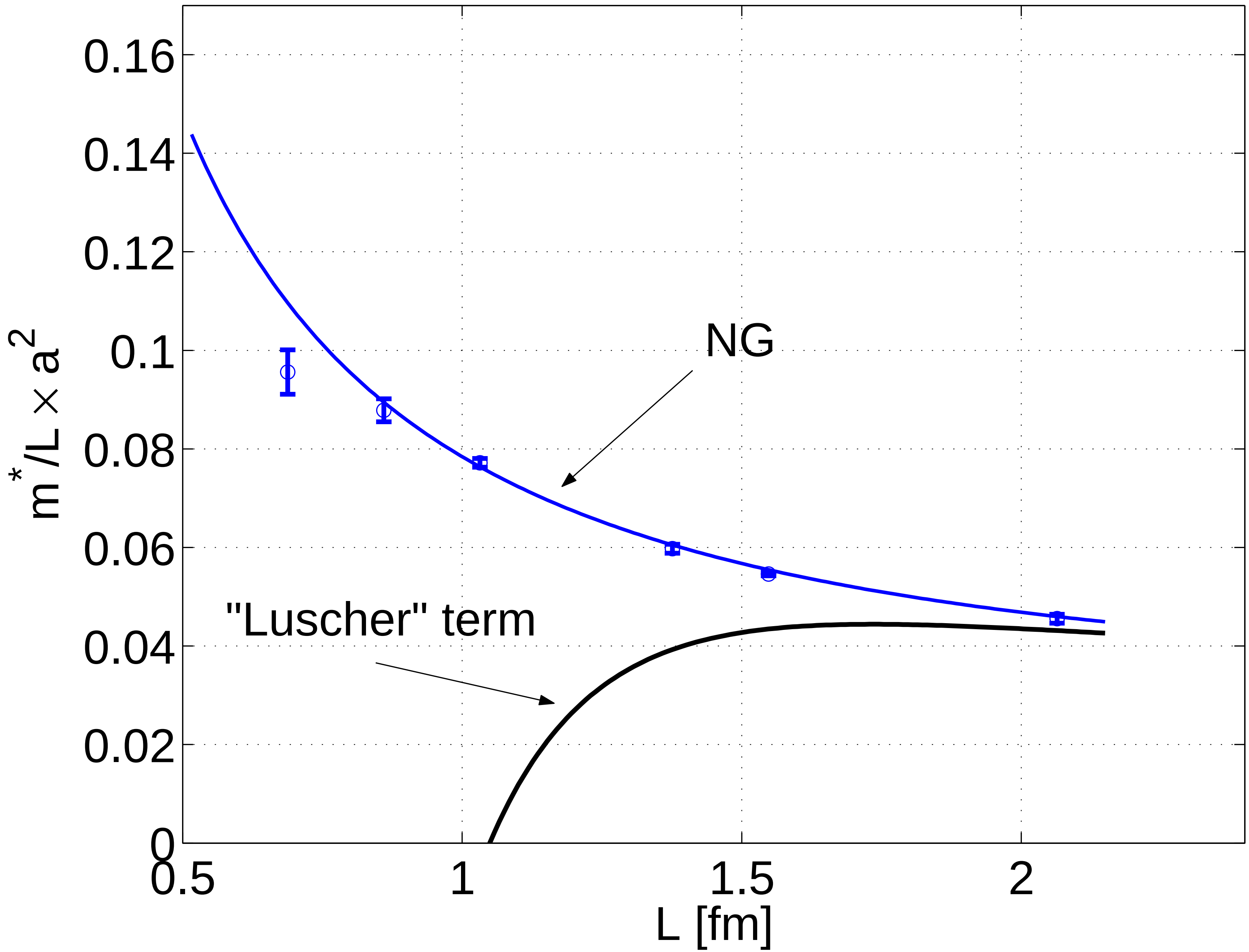

Finally for $SU(3)$ Have more accurate data so perform a **3** parameter fit of

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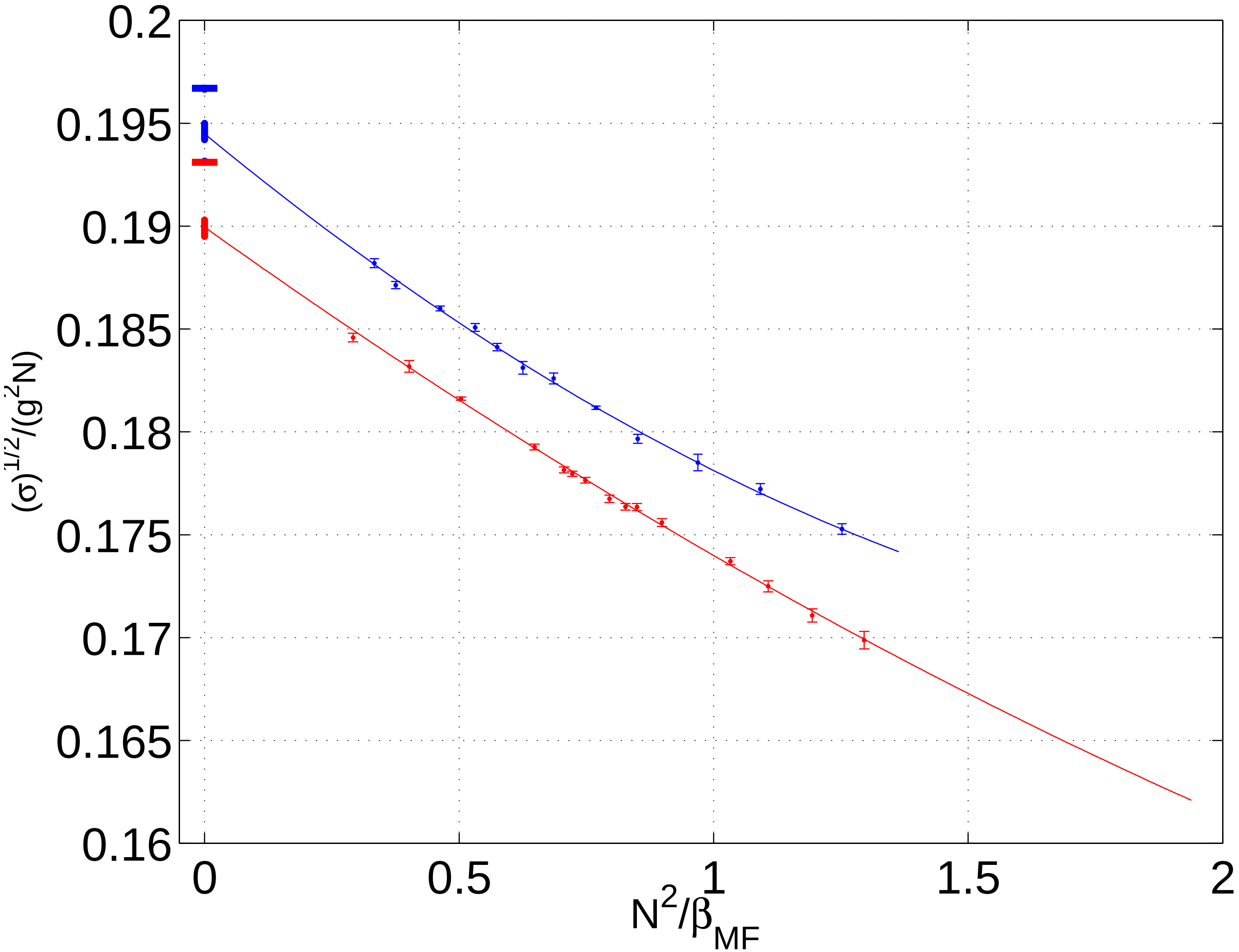
Now can extract σ given $E(L)$ and find

$$\frac{O(1/L^3)}{\sigma L - (\text{Luscher term})} \simeq 0.32\% \quad \text{at usual} \qquad L\sqrt{\sigma} \simeq 3 \quad (1.5 \text{ fm})$$

Use that for the continuum, and e.g. $SU(4,6)$ with $L\sqrt{\sigma} \simeq 3 - 3.5$ we find ...

III.B. Results - continuum string tensions



Doing that for $SU(3,4,6,8)$ we are ready for the $N \to \infty$ extrapolation, and . . .

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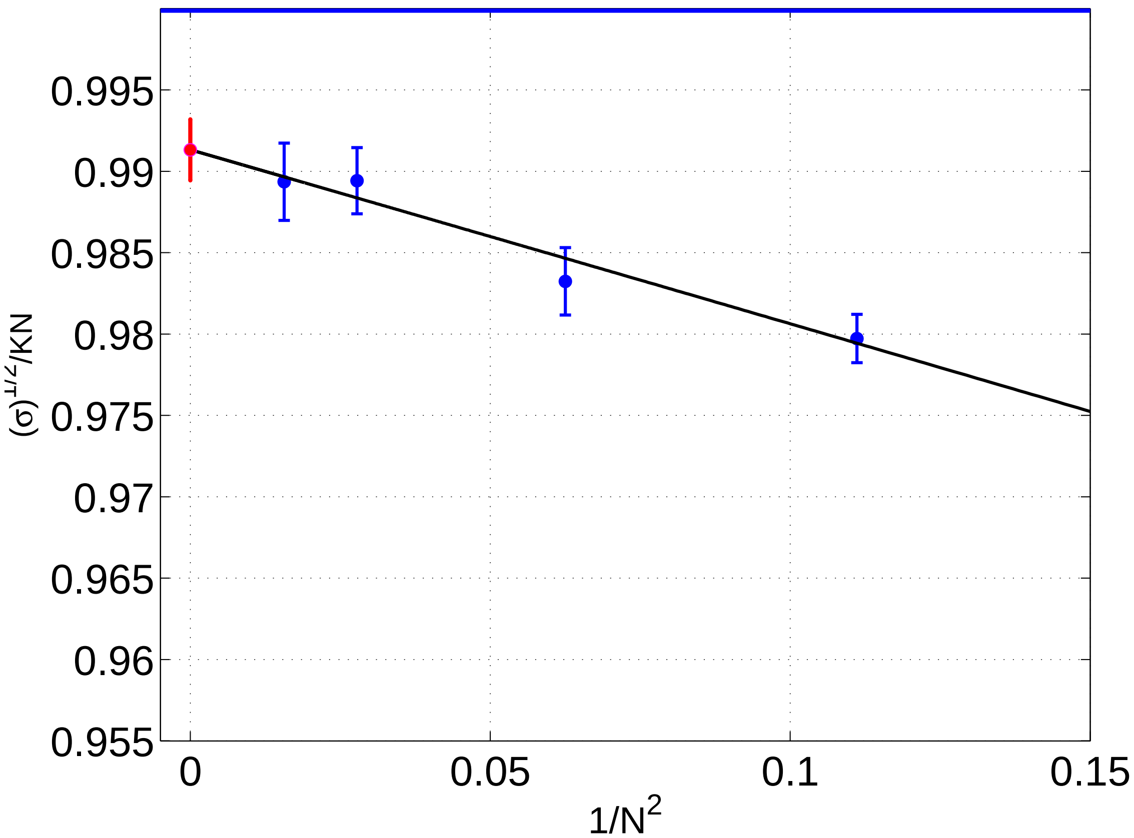


Karabali-Nair are 0.87% and 4.6σ away from lattice.

$$\left(\frac{\sqrt{\sigma}}{g^2 N}\right)_{N=\infty} = 0.1977(4) \quad \text{vs.} \quad \frac{1}{\sqrt{8\pi}} = 0.1995, \qquad \frac{\text{Lattice}}{\text{Karabali}-\text{Nair}} = 0.9913(19)$$

IV. Summary

We calculated $E(L)$ in $SU(2-8)$ in $2+1$ for 0.55 fm $\leq L \leq 2$ fm.

- $C_{\mathrm{eff}}(L) \to 1$ at about $L = 1.7 - 2$fm.
- $O(1/L^3)$: Lattice mildly confirmed theory in $SU(4,6,8)$, stronger in $SU(3)$.
- 'Nambu-Goto' is a **very** good.

$a \longrightarrow 0$: removing systematics $\longrightarrow$ **(Karabali-Nair) – Lattice $= 5\sigma$ at $N = \infty$**

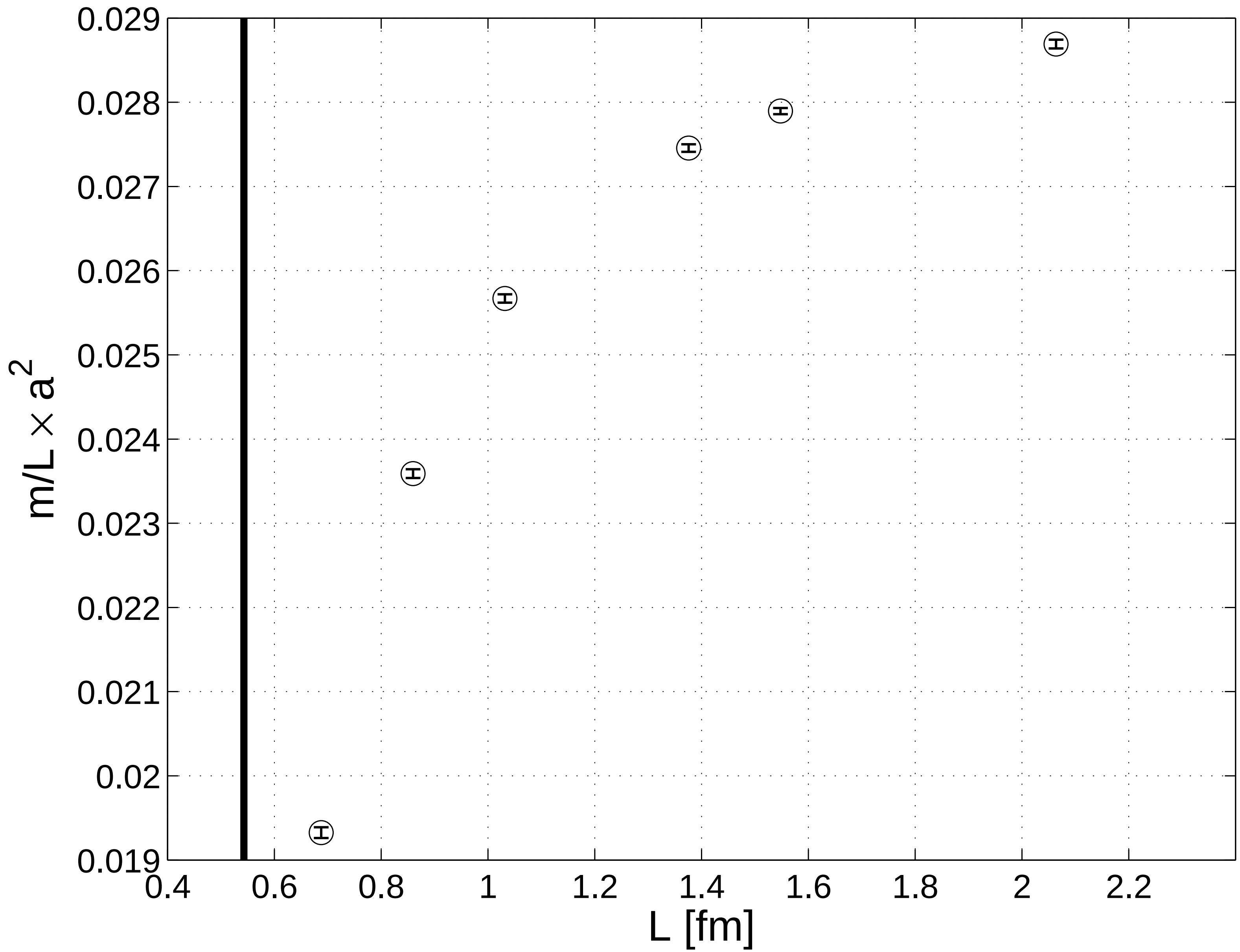
V. What next

Remove excited states systematics :

- E.g., k-strings in $SU(8)$: $\sigma_2/\sigma_1 = 1.739(12) \to 1.706(14)$

Glueballs : aim for accurate spectra to test predictions a lá Karabali-Nair Minic et al.

Appendix - $\frac{m(L)}{L} = \sigma - \frac{\pi}{6L^2} \times C + \ldots$. $SU(6), a^{-1} = 2.6$ GeV.



III.A. Results - for $C_{\rm eff}$. Fitting $m(L) = \sigma L - \frac{\pi}{6L} \times C_{\rm eff}(L)$